

The Line



Lesson 11

Lesson Eleven Concepts

Overall Expectations

- Apply data-management techniques to investigate relationships between two variables;
- Determine the characteristics of linear relations;
- Demonstrate an understanding of the constant rate of change and its connection to linear relations;
- Connect various representations of a linear relation, and solve problems using the representations.

Specific Expectations

- Construct tables of values and graphs, using a variety of tools;
- Construct tables of values, scatter plots, and lines or curves of best fit as appropriate using a variety of tools;
- Identify through investigation, some properties of linear relations;
- Determine values of a linear relation by using a table of values, by using the equation of the relation, and by interpolating or extrapolating from the graph of the relation;
- Express a linear relation as an equation in two variables, using the rate of change and the initial value.

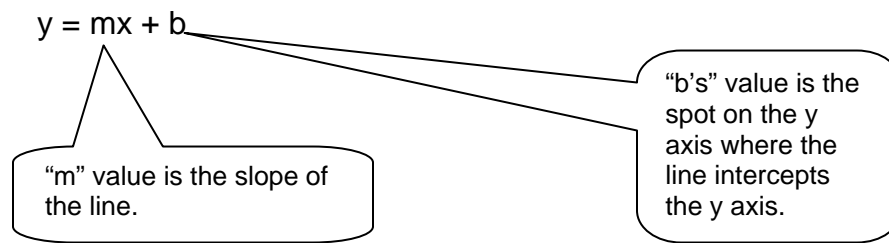
Equation of a Line

The equation of a line has 2 basic forms. One being the y-intercept form and the other is called standard form.

y - intercept form takes the form $y = mx + b$

Y-intercept Form

It is called y-intercept form because we can use the y-intercept in the equation to help us understand and graph the equation.



Example

State the slope and y-intercept for the following equations.

a) $y = 3x + 6$

b) $y = -\frac{1}{2}x - 1$

Solution

a) $y = 3x + 6$

$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{3}{1} = 3 \quad y\text{-intercept} = b = +6$$

b) $y = -\frac{1}{2}x - 1$

$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{-1}{2} \quad y\text{-intercept} = b = -1$$

Example

Graph the following equations using the slope and y-intercept.

a) $y = -2x + 3$

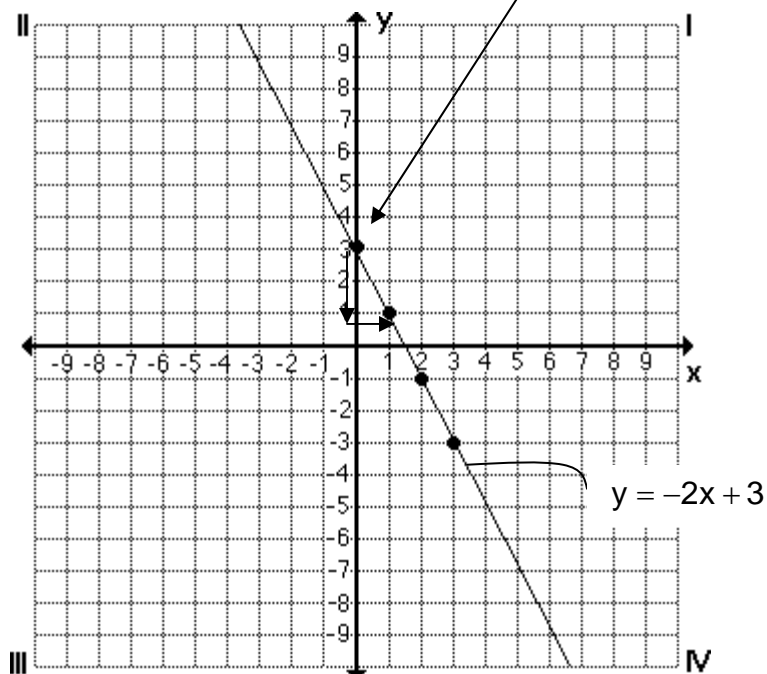
b) $y = \frac{2}{3}x - 1$

Solution

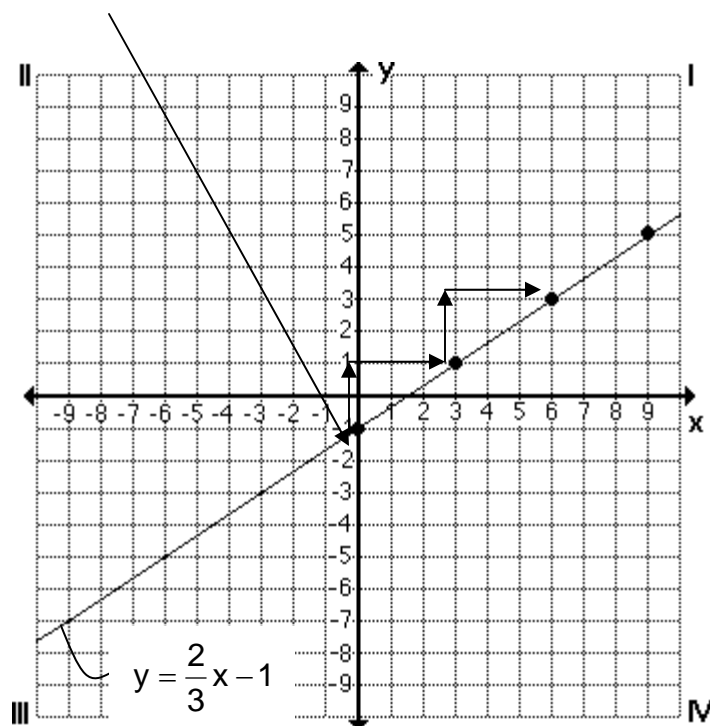
a) $y = -2x + 3$ $m = -2 = \frac{\text{rise}}{\text{run}} = \frac{-2}{1}$ and $b = +3$

Starting at the y-intercept go down 2 and right 1 then down 2 again and right one again and so on... Then draw a line through the points plotted to create your line.

First plot the y-intercept.



b) $y = \frac{2}{3}x - 1$



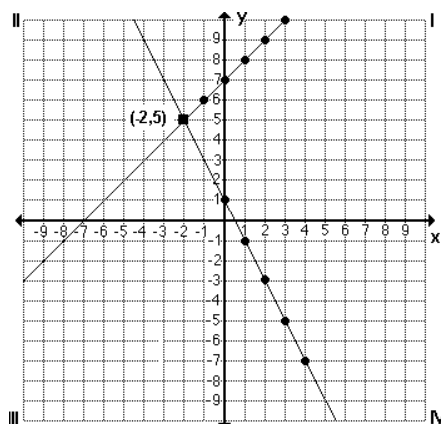
Example

Find the point of intersection of these two equations. (Solve the system of equations graphically)

$$y = -2x + 1$$

$$y = x + 7$$

Solution



The point of intersection is $(-2, 5)$ which solves the system of equations.
 $(-2, 5)$ is the only two ordered pairs that will satisfy both equations.

$$y = -2x + 1$$

$$5 = -2(-2) + 1$$

$$5 = 5$$



$$y = x + 7$$

$$5 = -2 + 7$$

$$5 = 5$$



Support Questions

1. State the slope and y-intercept of the equations below, then properly graph and label the equations using the slope and y - intercept.

a) $y = -4x + 3$

b) $y = x - 4$

c) $y = -\frac{2}{3}x + 7$

d) $y = \frac{3}{5}x - 1$

2. Show by substitution which of the order pairs satisfies that equation.

a) $y = -2x + 4$

$(-3, -5), (4, -4), (-1, 7), (3, -10)$

b) $y = -\frac{3}{4}x + 1$

$(9, 5), (8, -5), (3, -7), (12, -8)$

c) $y = -x - 2$

$(3, -2), (-4, 6), (4, -6), (-1, -1)$

3. Solve for the system of equations by graphing. (Find the point of intersection)

a) $y = -3x + 4$
 $y = -2x + 5$

b) $y = \frac{2}{3}x + 6$
 $y = x + 4$

c) $y = x - 1$
 $y = 4x + 2$

Parallel and Perpendicular Lines

Lines that are **parallel** to each other have the same slope.

Lines that are **perpendicular** to each other will have inverted slopes with opposite polarity.

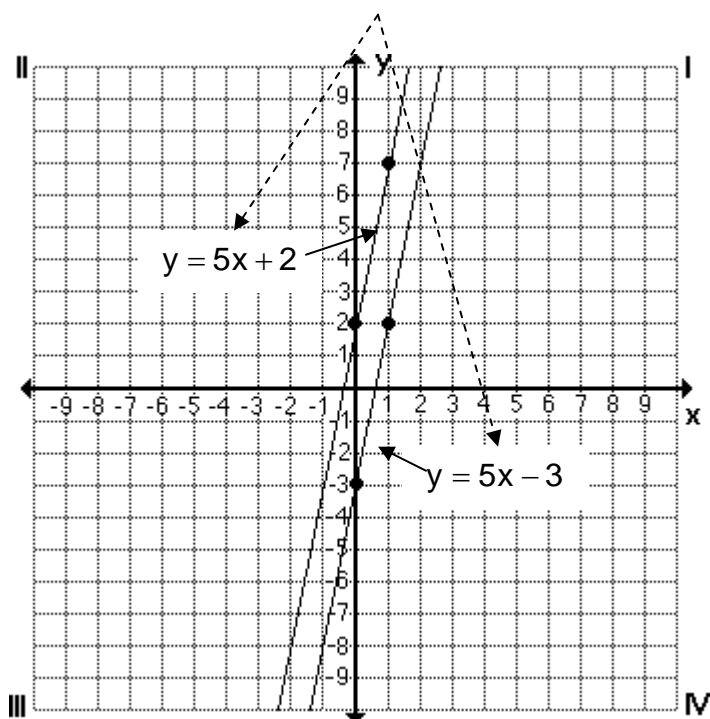
Example

Which of the following equations are parallel to each other?

$$y = 5x - 3 \quad y = -5x + 2 \quad y = 3x - 3 \quad y = 5x + 2$$

Solution

$y = 5x - 3$ and $y = 5x + 2$ are parallel to each other because they have the same slope of $5x$.



Example

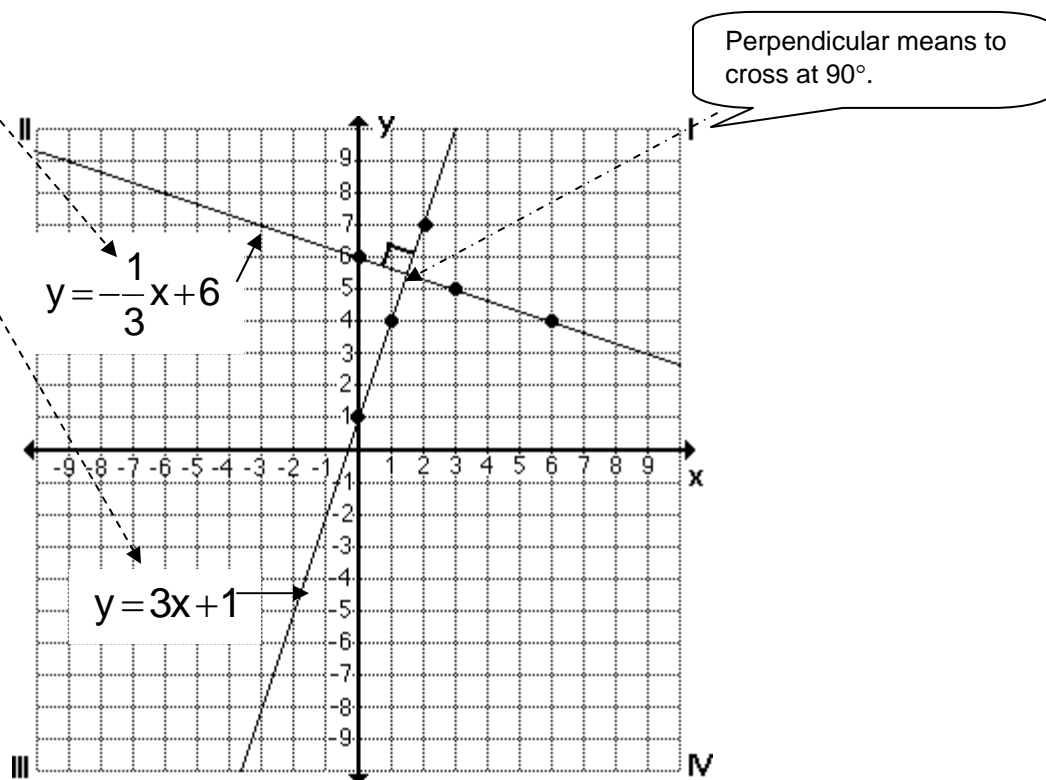
Which of the following equations are perpendicular to each other.

$$y = 3x + 1 \quad y = -2x + 2 \quad y = -\frac{1}{3}x + 6 \quad y = -\frac{1}{2}x - 1$$

Solution

$y = 3x + 1$ and $y = -\frac{1}{3}x + 6$ are perpendicular to each other because they have slope inverted of each other and opposite polarity.

3 and $-\frac{1}{3}$

**Support Questions**

4. Circle then graph the pair of equations that are parallel.

$$y = -x + 2$$

$$y = \frac{2}{3}x + 2$$

$$y = \frac{2}{3}x - 5$$

$$y = x - 6$$

$$y = -\frac{2}{3}x - 7$$

5. Circle then graph the pair of equations that are perpendicular.

$$y = 3x + 2$$

$$y = 2x + 7$$

$$y = \frac{1}{2}x - 5$$

$$y = -\frac{1}{2}x - 6$$

$$y = \frac{1}{3}x + 2$$



Key Question #11

1. State the slope and y-intercept of the equations below, then properly graph and label the equations using the slope and y - intercept. (8 marks)

a) $y = 3x - 2$ b) $y = -x + 4$ c) $y = -\frac{1}{2}x + 1$ d) $y = \frac{2}{3}x - 3$

2. Show by substitution which of the order pairs satisfies that equation. (3 marks)

a) $y = -5x - 2$

$(-3, -5), (4, -4), (-1, 3), (3, -10)$

b) $y = \frac{1}{3}x + 1$

$(9, 5), (8, -5), (3, -7), (12, 5)$

c) $y = x + 7$

$(3, -2), (-4, 3), (3, -6), (-1, -1)$

3. Solve for the system of equations by graphing. (Find the point of intersection) (3 marks)

a) $y = x + 4$
 $y = -x + 5$

b) $y = \frac{1}{2}x - 1$
 $y = x + 4$

c) $y = x + 1$
 $y = 2x - 5$

4. Circle then graph the two pairs of equations that are parallel. (4 marks)

$y = -x + 2$

$y = \frac{2}{3}x + 2$

$y = -\frac{2}{3}x - 5$

$y = -x - 6$

$y = -\frac{2}{3}x - 7$

5. Circle then graph the pairs of equations that are perpendicular. (4 marks)

$y = -3x + 2$

$y = 2x + 7$

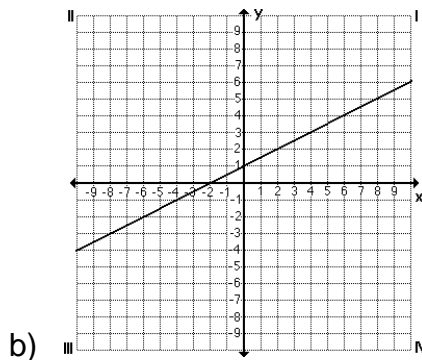
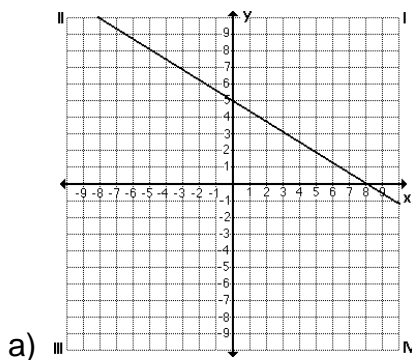
$y = \frac{1}{2}x - 5$

$y = \frac{1}{2}x - 6$

$y = \frac{1}{3}x + 2$

**Key Question #11**

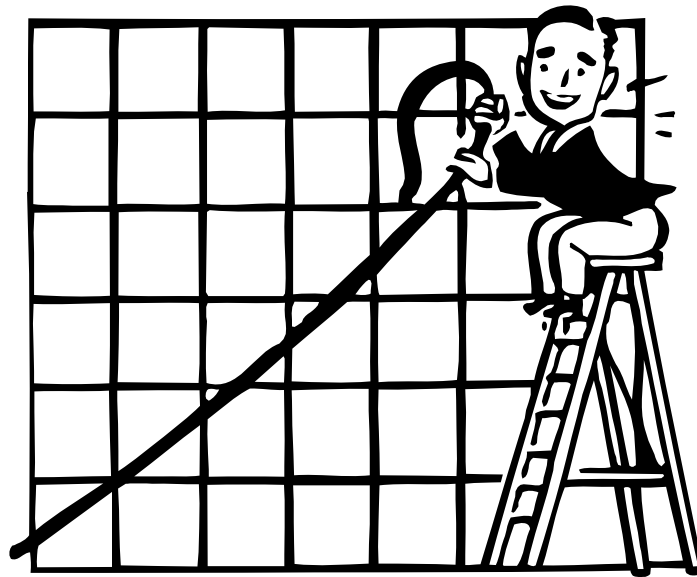
6. State the x - intercept, y - intercept and slope for each line. (4 marks)



7. Explain the steps needed to graph a line using an equation in y - intercept form and without using a table of values. (4 marks)



Direct and Partial Variation



Lesson 12

Lesson Twelve Concepts

Overall Expectations

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Specific Expectations

- Construct tables of values and graphs, using a variety of tools;
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- Determine other representations of a linear relation arising from a realistic situation, given one representation;
- Determine values of a linear relation by using a table of values, by using the equation of the relation, and by interpolating or extrapolating from the graph of the relation;
- Express a linear relation as an equation in two variables, using the rate of change and the initial value;
- Describe the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied;
- Determine graphically the point of intersection of two linear relations, and interpret the intersection point in the context of an application.

Direct and Partial Variation

Direct Variation

Direct Variation is a relation that is of the form $y = mx$.

The graph of $y = mx$ is a straight line with the slope of m .

The line $y = mx$ always passes through the order pair $(0, 0)$. $(0, 0)$ is called the origin.

The relation $y = mx$ represents direct variation because there is a direct relationship.

Example

Graph each equation which models direct variation.

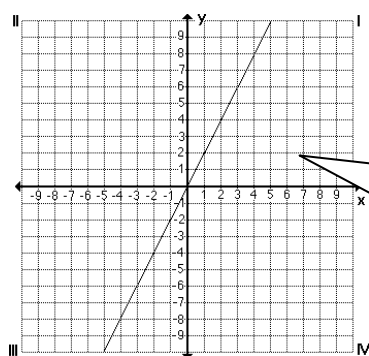
a) $y = 2x$

b) $y = -\frac{1}{2}x$

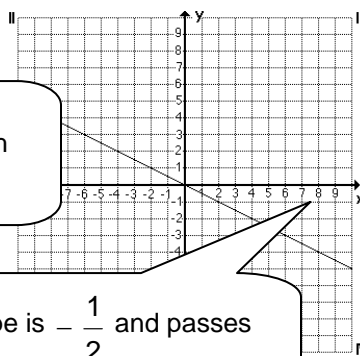
Solution

a) $y = 2x$

b) $y = -\frac{1}{2}x$



Slope is 2 and passes through the origin.



Slope is $-\frac{1}{2}$ and passes through the origin.

**Support Questions**

1. State the value of m in each equation of the form $y = mx$.

a) $y = -4x$

b) $y = x$

c) $y = -\frac{1}{3}x$

d) $y = \frac{2}{5}x$

2. Write an equation of the line through the origin with each slope.

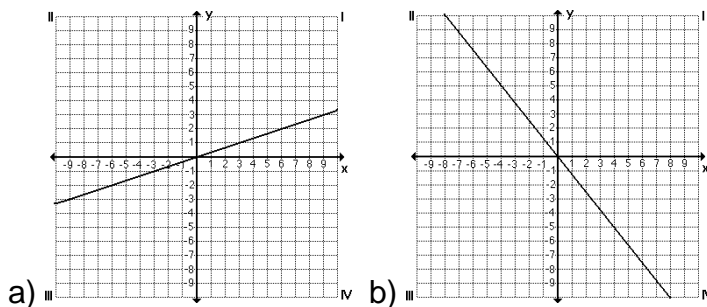
a) $m = -5$

b) $m = -\frac{3}{7}$

c) $m = \frac{1}{4}$

d) $m = 1$

3. Find the slope of each line, and then write its equation.

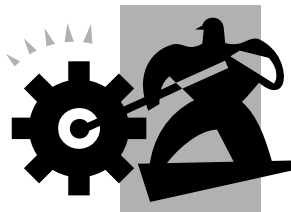


4. Graph each line.

a) $y = -\frac{1}{5}x$

b) $y = \frac{2}{3}x$

5. Johnny earns \$6 for every hour worked. Write an equation for this statement, then create a table of values and graph.



Partial Variation

Partial Variation is a relation that is of the form $y = mx + b$.

The graph of $y = mx + b$ is a straight line with the slope of m and a y -intercept of b .

The line $y = mx + b$ does not pass through the origin.

The relation $y = mx + b$ represents partial variation because the value of y varies partially with the value of x .

Example

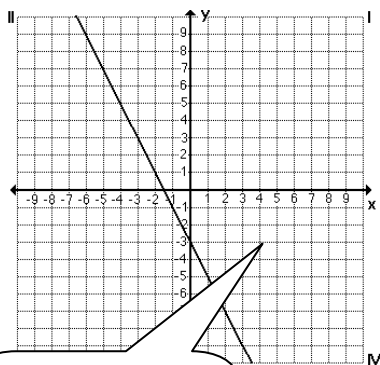
Graph each equation which models partial variation.

a) $y = -2x - 3$

b) $y = \frac{4}{5}x + 1$

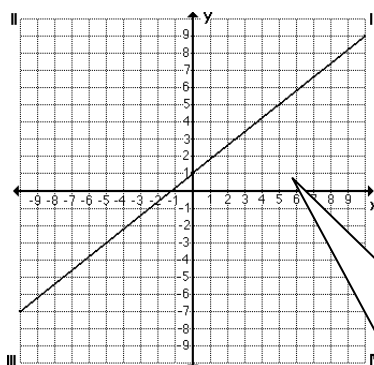
Solution

a) $y = -2x - 3$



Slope is -2 and intersects the y-axis at -3.

b) $y = \frac{4}{5}x + 1$



Slope is $\frac{4}{5}$ and intersects the y-axis at +1.

**Support Questions**

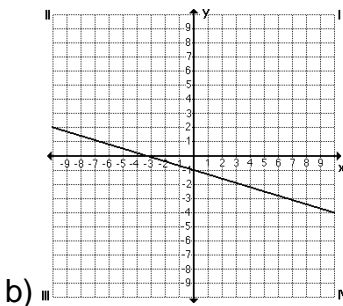
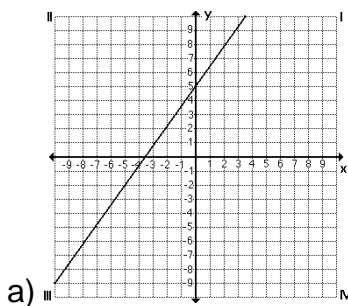
6. Write an equation of the line with each slope and y-intercept.

a) $m = -5$, $b = 1$

b) $m = -\frac{3}{7}$, $b = -4$

c) $m = \frac{1}{4}$, $b = +6$

7. Find the slope and y-intercept of each line, and then write its equation.



8. Graph each line.

a) $y = -2x + 1$ b) $y = \frac{2}{3}x - 6$ c) $y = x - 4$

9. Noah earns a \$100 a week and \$2 for every hour worked. Write an equation for this statement, then create a table of values and graph.



Key Question #12

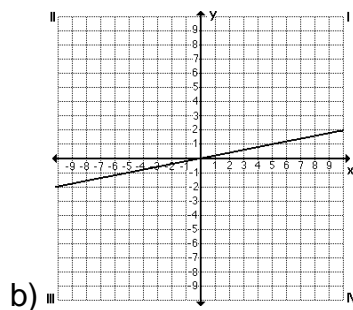
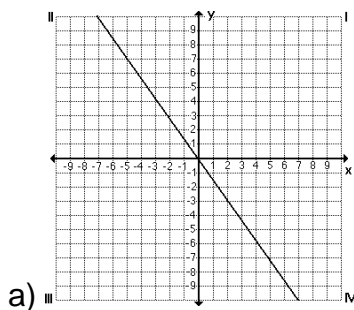
1. State the value of m in each equation of the form $y = mx$. (2 marks)

a) $y = 7x$ b) $y = -x$ c) $y = \frac{2}{7}x$ d) $y = -\frac{3}{8}x$

2. Write an equation of the line through the origin with each slope. (2 marks)

a) $m = 7$ b) $m = \frac{1}{2}$ c) $m = -3$ d) $m = -\frac{3}{8}$

3. Find the slope of each line, and then write its equation. (2 marks)



4. Graph each line. (3 marks)

a) $y = -3x$ b) $y = \frac{3}{5}x$ c) $y = -x$



5. Brook spends \$4.00 each day for lunch on work days. Write an equation for this statement, then create a table of values and graph. (3 marks)

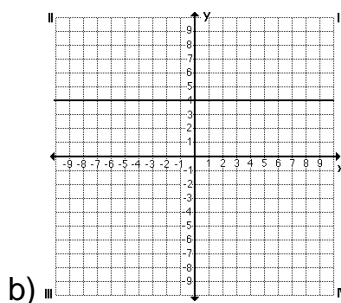
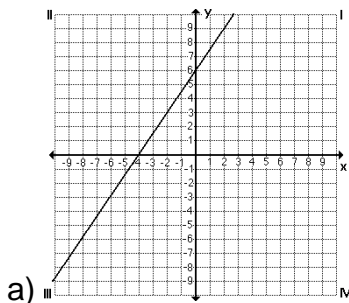
6. Write an equation of the line with each slope and y -intercept. (3 marks)

a) $m = 3$, $b = 0$ b) $m = -2$, $b = -3$ c) $m = \frac{3}{4}$, $b = +7$



Key Question #12

7. Find the slope and y-intercept of each line, and then write its equation. (4 marks)



8. Graph each line. (3 marks)

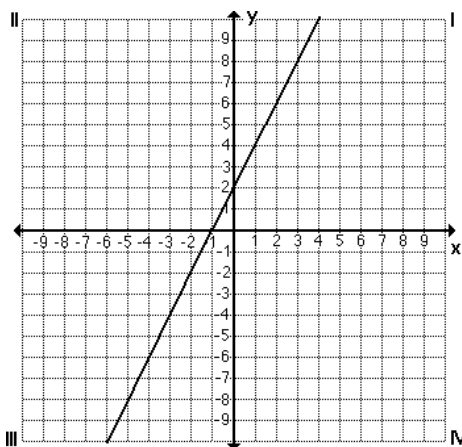
a) $y = 4x - 2$

b) $y = -\frac{1}{4}x + 1$

c) $y = -x + 3$

9. The length of time to set up is 200 min. The time to make paint each sign is 25 min. Write an equation for this statement, then create a table of values and graph. (3 marks)

10. Stephen graphed the equation $y = 2x - 1$ on a Cartesian plane. When he checked with a classmate, he realized the graph was different. Is Stephen's graph correct? If so, explain how you know. If not, explain what Stephen did wrong. (4 marks)



Scatter Plots



Lesson 13

Lesson Thirteen Concepts

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Specific Expectations

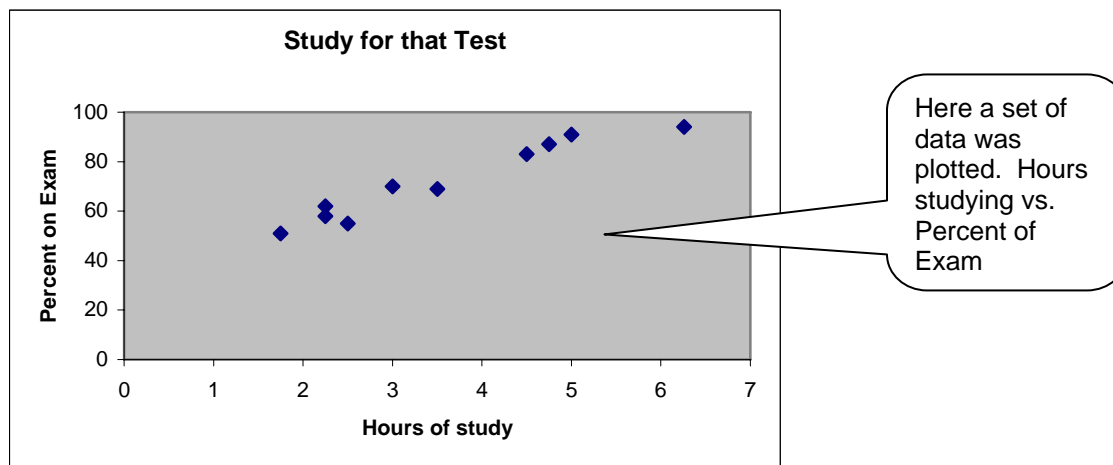
- Construct tables of values and graphs, using a variety of tools;
- Construct tables of values, scatter plots, and lines or curves of best fit as appropriate using a variety of tools;
- Identify through investigation, some properties of linear relations;
- Determine other representations of a linear relation arising from a realistic situation, given one representation;
- Determine values of a linear relation by using a table of values, by using the equation of the relation, and by interpolating or extrapolating from the graph of the relation;

Scatter Plots and Line of Best Fit

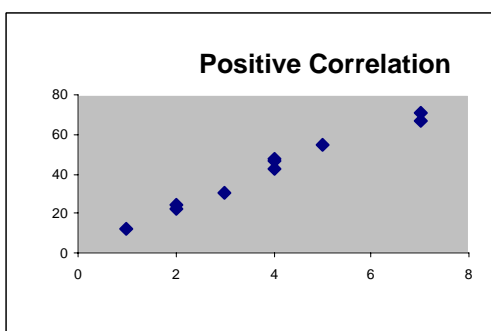
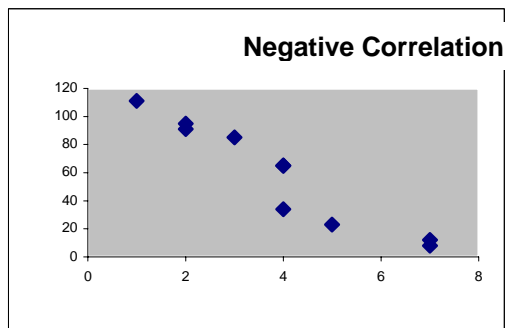
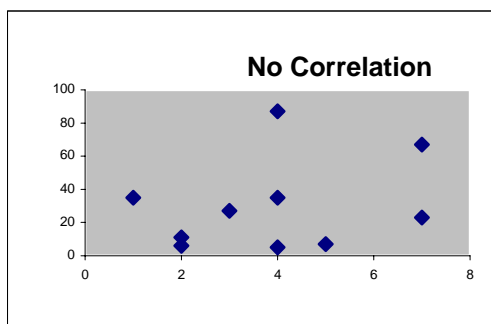
Scatter plots

A **scatter plot** is a graph of data that is a series of points.

Example



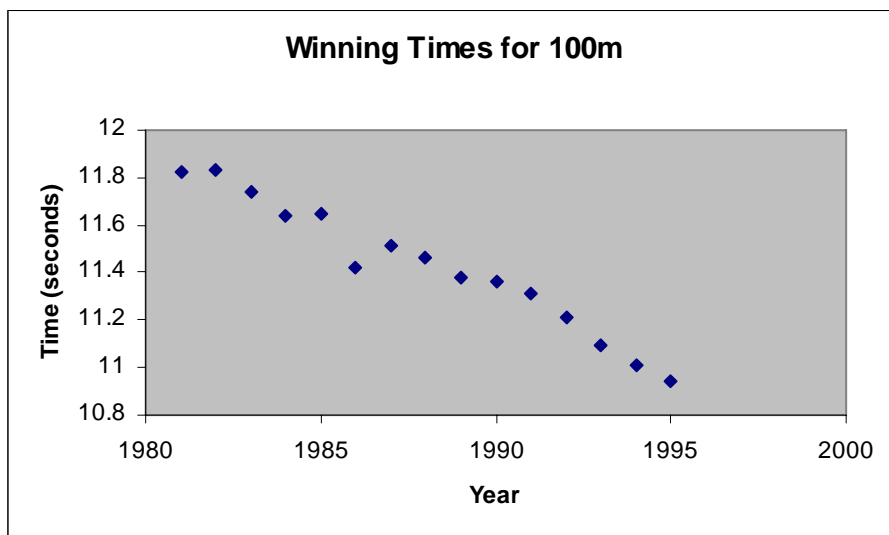
Data in a scatter plot can have a positive/negative or no correlation.



Example

Make a scatter plot for the men's times. Plot the year on the x-axes and the times on the y-axes.

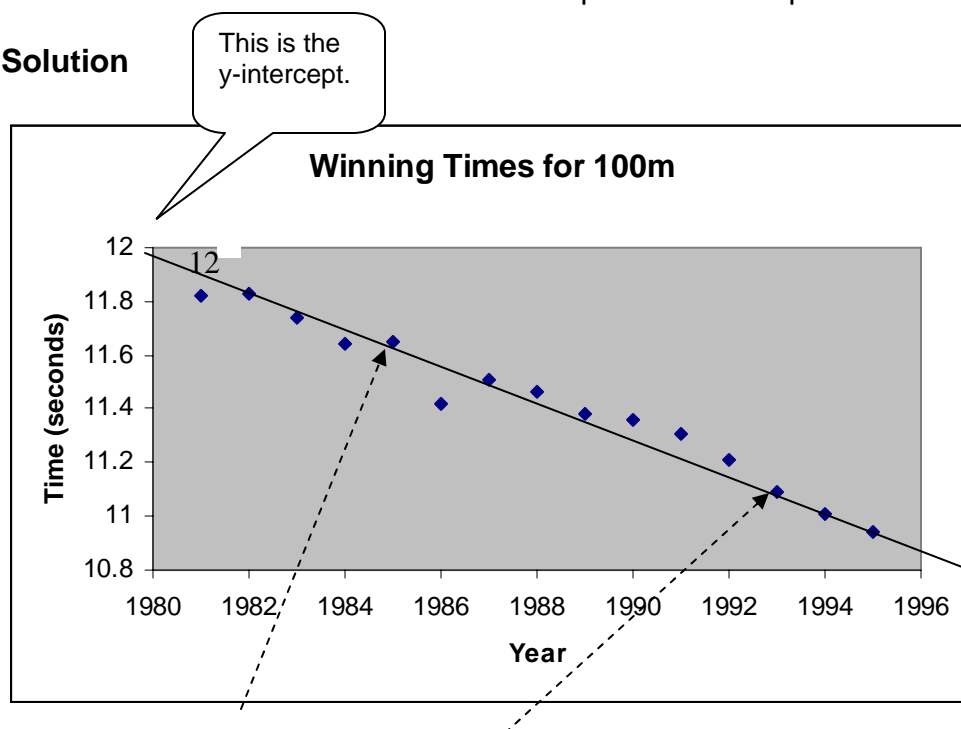
Year	Time	Year	Time	Year	Time
1981	11.82	1986	11.42	1991	11.31
1982	11.83	1987	11.51	1992	11.21
1983	11.74	1988	11.46	1993	11.09
1984	11.64	1989	11.38	1994	11.01
1985	11.65	1990	11.36	1995	10.94

Solution**Line of Best Fit**

Line of best fit is a line that passes as close as possible to a set of plotted points.

Example

Find the line of best fit for the data in the previous example.

Solution

First pick two points that represent the general center of the points plotted.

Then draw the line of best fit through those points generally dividing the plotted points evenly on both sides.

Example

What is the approximate equation for the line of best fit in the previous example?

Solution

Choose the coordinates of the two previously chosen points used to draw the line of best fit.

(1985, 11.7) and (1989, 11.4)

$$\underline{\text{slope}} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11.4 - 11.7}{1989 - 1985}$$

$$m = \frac{-0.3}{4} = -0.075$$

$$\underline{y - \text{intercept}} = b = +12$$

Therefore the equation of the line of best fit is $y = -0.075x + 12$.

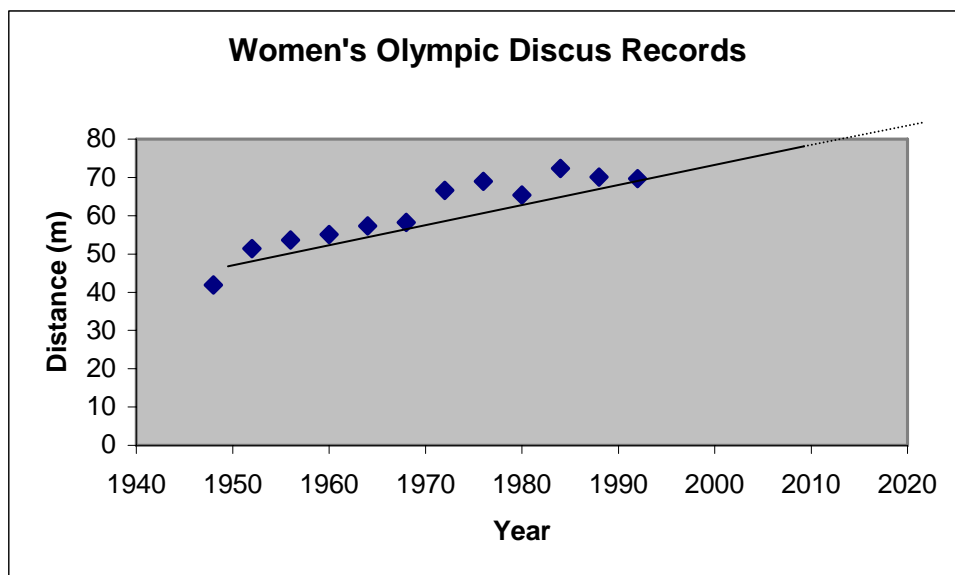
Extrapolation and Interpolation

Extrapolation is extending a graph to estimate the values that are beyond the table of values.

Interpolation is using a graph to estimate the value that are not in the table of values but are within the range of the lowest and largest values presented in the table of values.

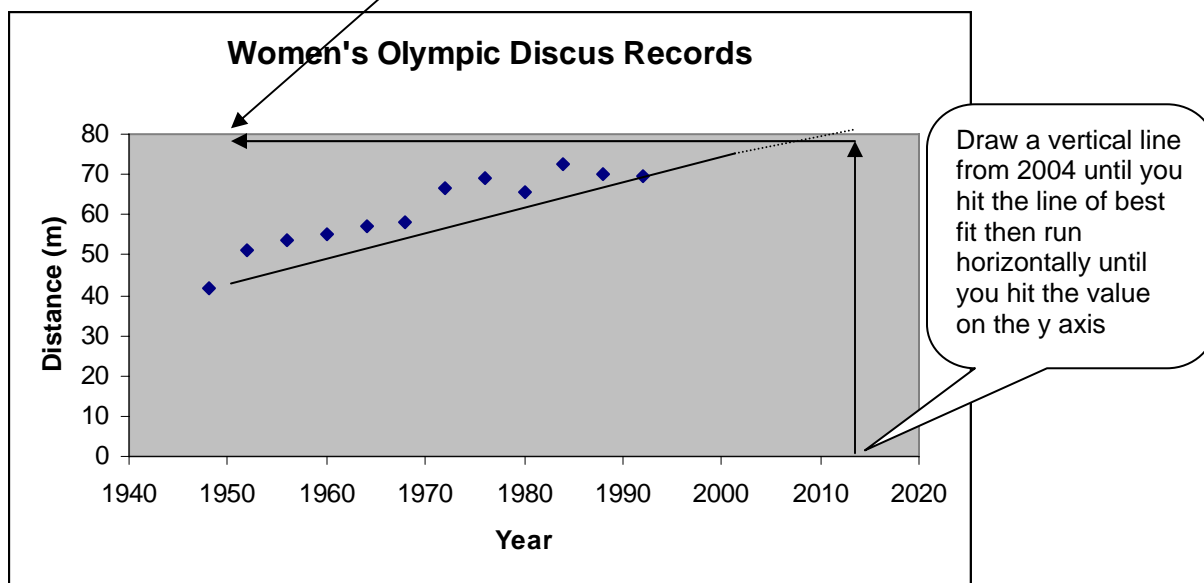
Example

- a) Using extrapolation what would be the approximate distance for the discus will be thrown during the 2004 Summer Olympics?
- b) Using Interpolation how far was the discus thrown in 1952?

**Solution**

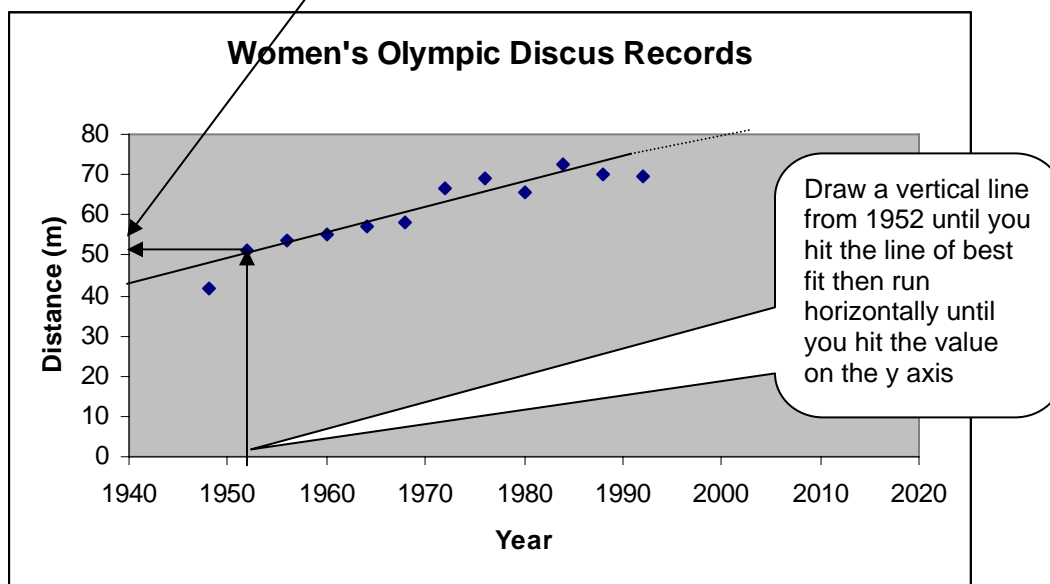
- a) Using extrapolation, what would be the approximate distance for the discus will be thrown during the 2004 Summer Olympics?

approximately 78 metres



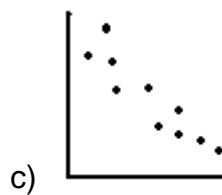
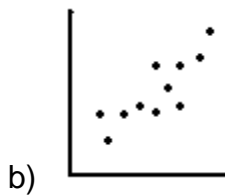
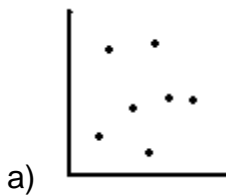
b) Using Interpolation, how far was the discus thrown in 1952?

Approximately 51 metres



Support Questions

1. State whether the scatter plot has a positive, negative or no correlation.



2. The table following shows the winning times for the 800-m race at the Olympic Summer Games.

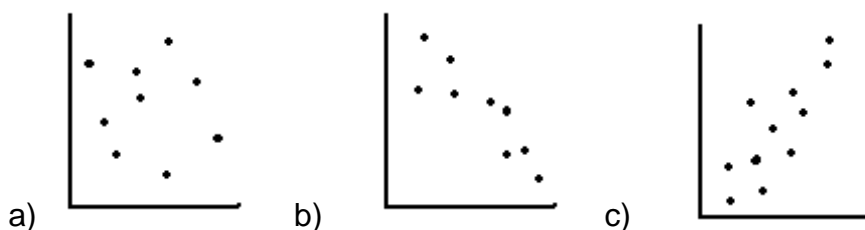
Year	Men's Time
1960	106.3
1964	105.1
1968	104.3
1972	105.9
1976	103.5
1980	105.4
1984	103
1988	103.45

- Construct a scatter plot and line of best fit for the data.
- What is the equation of the line of best fit?
- What is the approximate winning time in the year 2020?
- What approximate year does the time drop below 104 seconds?



Key Question #13

- State whether the scatter plot has a positive, negative or no correlation. (3 marks)



- The table below shows the winning heights in a men's high jump competition. (8 marks)

Year	Winning Country	Jump in Height (m)
1912	Canada	1.93
1932	United States	1.96
1952	England	1.99
1972	United States	2.16
1982	United States	2.23
2002	Australia	2.41

- Create a properly labelled Scatter Plot using the data given in the table.
- Draw a line of best fit.
- What is the approximate slope of the line of best fit?
- Using extrapolation, what would the approximate winning height be in 2022?



Key Questions #13

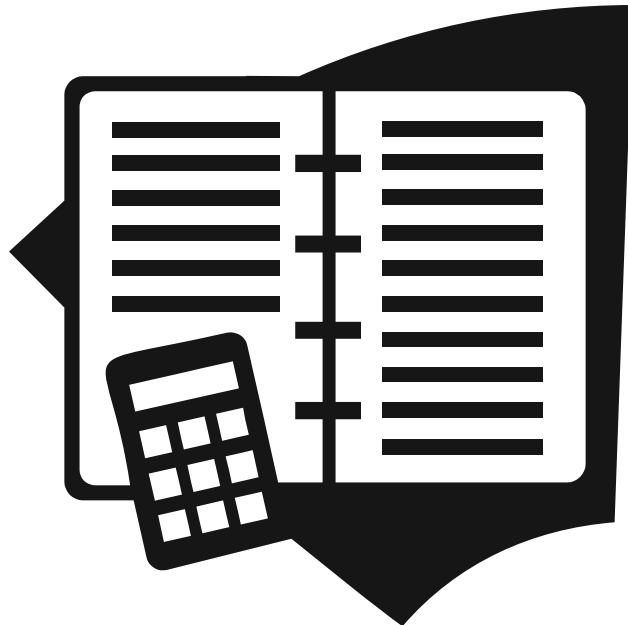
3. The table below shows the percentage of high school girls who smoked more than one cigarette during the previous year. (8 marks)

Year	1981	1983	1985	1987	1989	1991	1993	1995	1997
Percent	34.3	29.5	25.9	25.1	24.4	22	21.3	21.6	20.4

- Graph the data in a scatter plot.
- Determine the equation of the line of best fit.
- Use your equation to predict the percent of high school girls who smoke more than one cigarette in 2000.
- Approximately, what percent of girls smoked more than one cigarette in 1982 and 1994?



Averages



Lesson 14

Lesson Fourteen Concepts

Overall Expectations

- Apply data-management techniques to investigate relationships between two variables;

Specific Expectations

- Pose problems, identify variables, and formulate hypotheses associated with relationships between two variables;
- Describe trends and relationships observed in data, make inference from data, compare the inferences with hypotheses about the data, and explain any differences between the inferences and the hypotheses.

Measures of Central Tendency

There are three different types of averages: **Mean**, **Median** and **Mode**.

Mean Average

Mean is a single number that is used to represent a set of numbers.

To find the mean average, all the numbers in the data set are added together and the sum is divided by the number of entries in the data set.

Example

Find the mean of the following sets of numbers.

a) {1, 3, 4, 7, 7, 8, 9, 11}

b) {4, 7, 3, 5, 1, 8, 3, 9, 4, 7, 5, 3}

Solution

a) {1, 3, 4, 7, 7, 8, 9, 11}

$$\text{mean} = \frac{1 + 3 + 4 + 7 + 7 + 8 + 9 + 11}{8}$$

$$\text{mean} = \frac{50}{8}$$

$$\text{mean} = 6.25$$

Divide by 8 because there are 8 numbers in the set.

50 is the sum of all the values in the numerator.

b) {4, 7, 3, 5, 1, 8, 3, 9, 4, 7, 5, 3}

$$\text{mean} = \frac{4 + 7 + 3 + 5 + 1 + 8 + 3 + 9 + 4 + 7 + 5 + 3 + 1}{13}$$

$$\text{mean} = \frac{60}{13}$$

$$\text{mean} \approx 4.62$$



Support Questions

- Calculate the mean average for each set of numbers.
 - {3, 6, 2, 7, 3, 5, 8}
 - {23, 28, 32, 15, 28, 32, 35, 29, 12}
 - {75, 43, 57, 69, 84, 88, 94, 97, 51, 43, 45, 62, 61, 57}
- The salaries of the 2003-2004 Toronto Maple Leafs is given below. What is the mean average for a player's salary?

\$2,750,000	\$950,000	\$522,500
\$2,200,000	\$925,000	\$500,000
\$2,000,000	\$900,000	\$450,000
\$2,000,000	\$850,000	\$400,000
\$2,000,000	\$700,000	
\$1,600,000	\$650,000	
\$1,500,000	\$585,640	
\$1,400,000	\$575,000	



Median Average

Median average is the middle number of a set of numbers arranged in numerical order.

If there are an even number of values in the set of data then there will be two middle numbers and the mean of those two numbers is the median average.

Example

Find the median of the following sets of numbers.

a) {7, 9, 12, 1, 7, 3, 11, 4}

b) {4, 7, 3, 5, 8, 3, 9, 4, 7, 5, 1}

Solution

a) {7, 9, 12, 1, 7, 3, 11, 4}

Arrange in numerical order. {1, 3, 4, 7, 7, 9, 11, 12}



There are an equal number of values on both sides of the median. In this example there are 4 on both sides.

The middle value of this set of data is 7, therefore the median average is 7.

Median = 7

b) {4, 7, 3, 5, 8, 3, 9, 4, 7, 5, 1}

Arrange in numerical order. {1, 3, 3, 4, 4, 5, 5, 7, 7, 8, 9}

$$\text{mean} = \frac{4 + 5}{2}$$

$$\text{mean} = \frac{9}{2}$$

$$\text{mean} = 4.5$$

This time there are two middle numbers so the median is the mean of 4 and 5.

Therefore the median of the set of data is 4.5.

Median average = 4.5



Support Questions

3. Calculate the median average for each set of numbers.
- {3, 6, 2, 7, 3, 5, 8}
 - {23, 28, 32, 15, 28, 32, 35, 29, 12}
 - {75, 43, 57, 69, 84, 88, 94, 97, 51, 43, 45, 62, 61, 57}
4. The salaries of the 2003-2004 Toronto Maple Leafs is given below. What is the median average for a player's salary?

\$4,750,000	\$950,000	\$522,500
\$4,200,000	\$925,000	\$500,000
\$3,000,000	\$900,000	\$450,000
\$2,500,000	\$850,000	\$400,000
\$2,000,000	\$700,000	
\$1,600,000	\$850,000	
\$1,500,000	\$785,640	
\$1,400,000	\$475,000	



Mode Average

Mode average is the most common value in a set of numbers.

Example

Find the mode of the following sets of numbers.

a) {7, 9, 12, 1, 7, 8, 3, 11, 4}

b) {4, 7, 3, 5, 1, 8, 3, 8, 4, 7, 5, 3, 8, 8, 9}

Solution

a) {7, 9, 12, 1, 7, 8, 3, 11, 4}

Arrange in numerical order because it is easier to see the repeated numbers.
{1, 3, 4, 7, 7, 8, 9, 11, 12}

7 is the most common number in the set so therefore, the mode average = 7.

b) {4, 7, 3, 5, 1, 8, 3, 8, 4, 7, 5, 3, 8, 8, 9}

Arrange in numerical order because it is easier to see the repeated numbers.
{1, 3, 3, 3, 4, 4, 5, 5, 7, 7, 8, 8, 8, 8, 9}

8 is the most common number in the set so therefore, the mode average = 8.



Support Questions

5. Calculate the mode average for each set of numbers.
- {3, 6, 2, 7, 3, 5, 8}
 - {23, 28, 32, 15, 28, 32, 35, 29, 12, 28}
 - {75, 43, 57, 69, 84, 88, 94, 97, 51, 43, 45, 62, 61, 57, 82, 43, 51}
6. The salaries of the 2003-2004 Toronto Maple Leafs is given below. What is the mode average for a player's salary?

\$4,750,000	\$950,000	\$522,500
\$4,200,000	\$925,000	\$525,000
\$3,000,000	\$900,000	\$400,000
\$2,500,000	\$850,000	\$400,000
\$2,000,000	\$850,000	
\$1,600,000	\$850,000	
\$1,500,000	\$785,640	
\$1,400,000	\$475,000	

7. Find the mean, median and mode for the given set of numbers.
- {63, 74, 77, 68, 71, 74, 70, 65}
8. Determine each measure of central tendency (mean, median and mode) based on the table given below.

Annual Salary (\$)	Frequency
35 600	3
42 750	5
51 000	6
99 000	1
150 000	1



Key Question #14

1. Find the mean, median and mode for each set of numbers. (3 marks)

- a) {34, 47, 12, 36, 26, 34, 28, 26, 48}
- b) {21, 23, 26, 34, 21, 29, 36, 45, 32, 26, 28, 26}
- c) {8, 6, 2, 4, 7, 6, 2, 5, 3, 7, 9, 7}

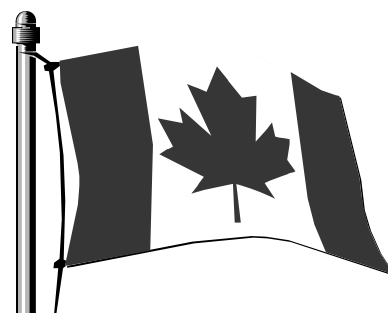
2. Two friends went golfing. Their score card is given below. (6 marks)

Hole	1	2	3	4	5	6	7	8	9	Total
Steve	6	5	4	4	3	4	6	8	3	43
Mary	6	6	3	3	4	5	6	8	4	45

- a) Calculate the mean score for each player.
- b) Calculate the mean score for the two players together.
- c) Calculate the median score for each player.
- d) Calculate the median score for the two players together.
- e) Determine the mode for each player.
- f) Determine the mode for the two players together.

3. The average price of gas in cents per litre is given for each province. (2 marks)

Province	Cost of Gas per Litre (cents)
<i>British Columbia</i>	75.2
<i>Alberta</i>	68.5
<i>Saskatchewan</i>	69.3
<i>Manitoba</i>	71.3
<i>Ontario</i>	75.3
<i>Quebec</i>	74.5
<i>New Brunswick</i>	74.6
<i>Nova Scotia</i>	76.3
<i>Prince Edward Is.</i>	75.4
<i>Newfoundland</i>	77.9



- a) What is the mean price of gas for all the Canadian Provinces?
- b) What is the median price of gas for all the Canadian Provinces?



Key Question #14

4. Is each statement always true, sometimes true or sometimes false? Explain and provide an example to illustrate your understanding. (8 marks)
- If a list of numbers has a mode, it is one of the numbers in the list?
 - The median of a list of whole numbers is a whole number.
 - The mean of a list of numbers is one of the numbers in the list?
 - The mean, median, and mode of a list of numbers are not equal.
5. The April batting statistics of the 2004 Toronto Blue Jays is given below. (6 marks)

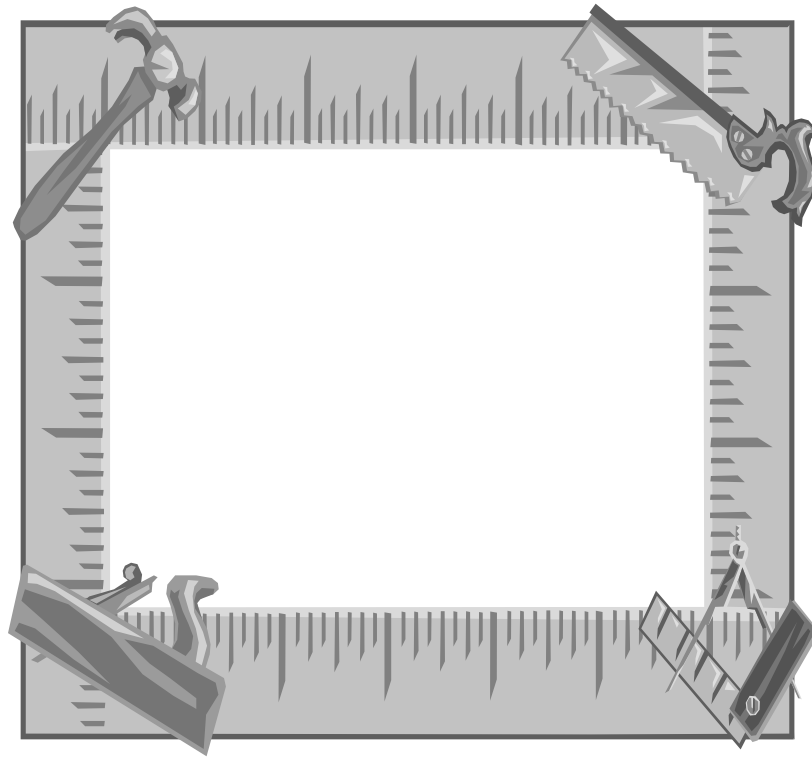


Player	TEAM	POS	G	AB	R	H	2B	3B	HR	RBI	TB	BB	SO	SB	CS	OBP	SLG	AVG
1. V Wells	TOR	OF	22	95	15	20	7	0	1	6	30	7	15	1	0	.265	.316	.211
2. J Phelps	TOR	DH	22	85	11	22	3	1	1	9	30	6	24	0	0	.337	.353	.259
3. C Delgado	TOR	1B	22	83	13	22	7	0	4	13	41	12	16	0	0	.367	.494	.265
4. E Hinske	TOR	3B	21	77	12	18	3	0	2	12	27	12	15	0	1	.330	.351	.234
5. F Catalanotto	TOR	OF	21	75	5	21	6	1	0	9	29	6	14	0	0	.333	.387	.280
6. R Johnson	TOR	OF	22	73	8	18	1	1	2	10	27	6	13	0	1	.337	.370	.247
7. O Hudson	TOR	2B	21	72	11	18	6	1	1	6	29	8	15	1	1	.341	.403	.250
8. K Cash	TOR	C	19	64	7	17	6	0	2	13	29	4	19	0	0	.324	.453	.266
9. C Woodward	TOR	SS	18	58	7	18	8	2	0	6	30	6	10	0	1	.369	.517	.310
10. H Clark	TOR	OF	8	27	4	9	3	0	0	3	12	1	4	0	0	.357	.444	.333
11. C Gomez	TOR	SS	13	23	6	8	1	0	1	6	12	5	6	0	0	.464	.522	.348
12. G Myers	TOR	C	8	18	0	4	2	0	0	1	6	2	4	0	0	.300	.333	.222
13. D Berg	TOR	OF	8	15	0	1	0	0	0	1	1	0	3	0	0	.067	.067	.067

- What is the team's mean batting average (AVG)?
 - What is the team's mode for doubles(2B)?
 - What is the team's mean at bats (AB)?
 - What is the team's median for strikeouts (SO)?
 - What is the team's mean slugging percents (SLG)?
 - What is the team's mean, median and mode for home runs (HR)?
6. Determine each measure of central tendency based on the table given below. (6 marks)

Annual Salary (\$)	Frequency
35 600	2
42 750	4
51 000	7
99 000	2
150 000	3

Perimeter



Lesson 15

Lesson Fifteen Concepts

Overall Expectations

- Determine, through investigation, the optimal values of various measurements of rectangles;
- Solve problems involving the measurements of two-dimensional shapes and volumes of three-dimensional figures.

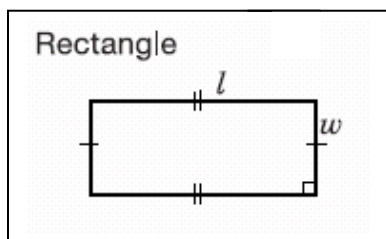
Specific Expectations

- Determine the minimum perimeter of a rectangle with a given area by constructing a variety of rectangles, using a variety of tools;
- Solve problems that require maximizing the area of a rectangle for a fixed perimeter or minimizing the perimeter of a rectangle for a fixed area

Perimeter

Perimeter is the distance around an object. If that object is a circle then the perimeter is called **circumference**.

Formulas to be used to calculate perimeter.



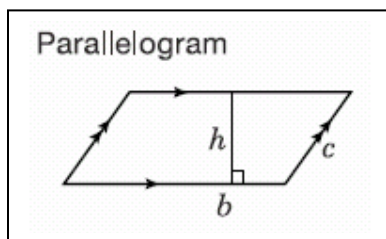
$$P = 2l + 2w$$

or

$$P = 2(l + w)$$

or

$$P = l + w + l + w$$



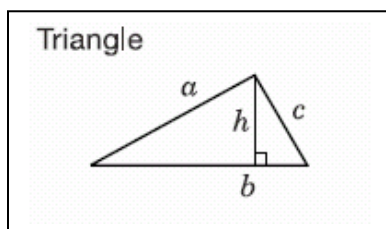
$$P = 2b + 2c$$

or

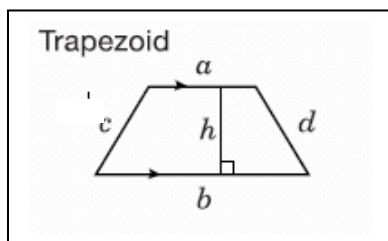
$$P = 2(b + c)$$

or

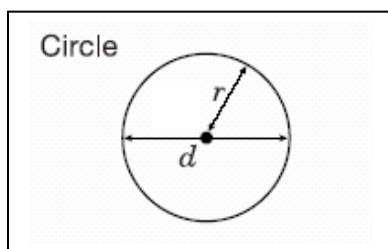
$$P = b + c + b + c$$



$$P = a + b + c$$



$$P = a + b + c + d$$



$$P = d\pi$$

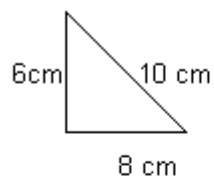
$$\pi \approx 3.14$$

$d = 2r$
 d: diameter
 r: radius
 or $P = 2\pi r$

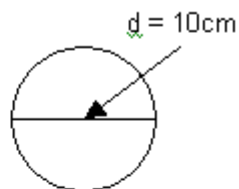
Example

Find the perimeter of each of the following shapes.

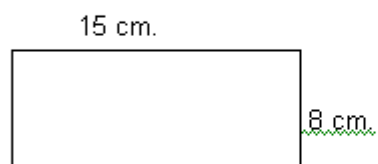
a)



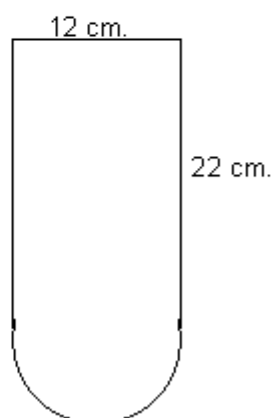
b)



c)

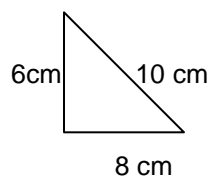


d)



Solution

a)

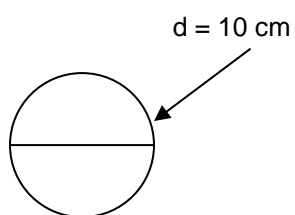


$$P = a + b + c$$

$$P = 6 + 8 + 10$$

$$P = 24 \text{ cm}$$

b)

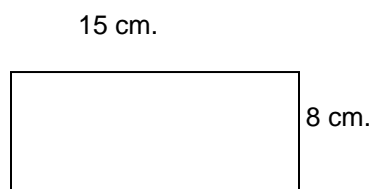


$$P = \pi d$$

$$P = (3.14)(10)$$

$$P = 31.4 \text{ cm}$$

c)



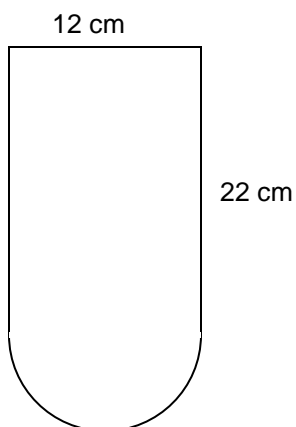
$$P = 2(l + w)$$

$$P = 2(8 + 15)$$

$$P = 2(23)$$

$$P = 46 \text{ cm}$$

d)



Only include 3 sides instead of 4 sides of the rectangle since the 4th side is $\frac{1}{2}$ of the circle

Circumference of circle but only $\frac{1}{2}$ of it since the perimeter does not include the inner portion

$$P = l + w + l + \frac{\pi d}{2}$$

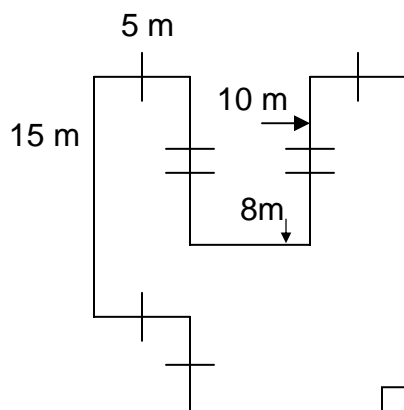
$$P = 22 + 12 + 22 + \frac{(3.14)(12)}{2}$$

$$P = 56 + 18.84$$

$$P = 74.84 \text{ cm}$$

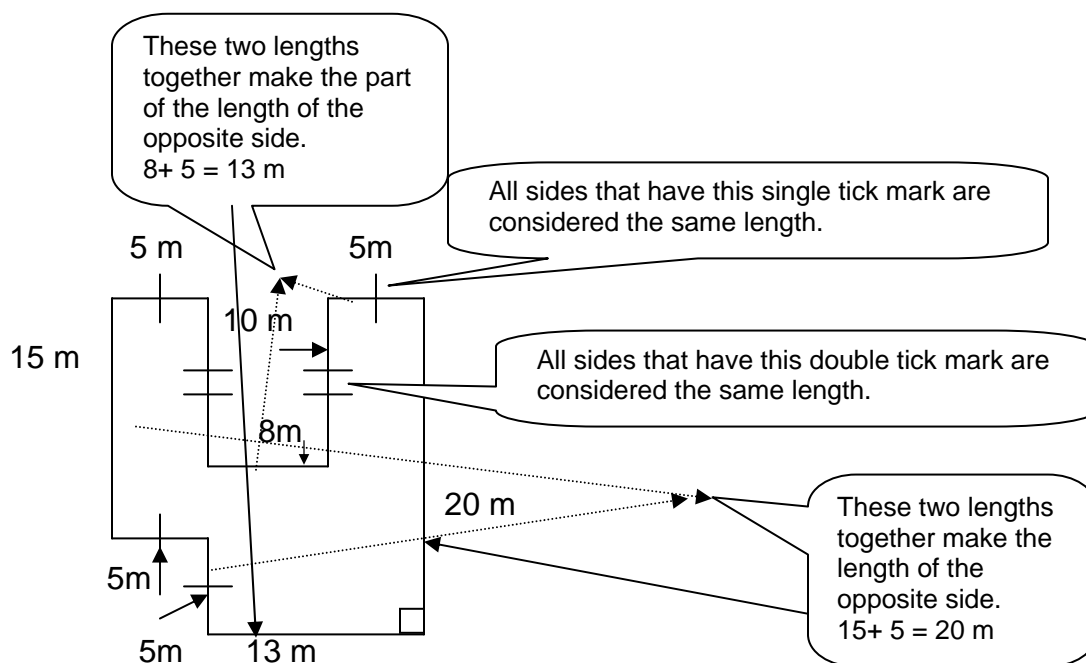
Example

Find the perimeter.



Solution

It is important to recognize the lengths of all side of the object.



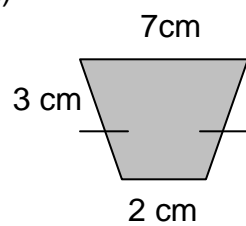
So the Perimeter is $P = 5 + 10 + 8 + 10 + 5 + 20 + 13 + 5 + 5 + 15$
 $= 96 \text{ m}$

or $P = 4(5) + 2(10) + 20 + 15 + 13 + 8$
 $= 96 \text{ m}$

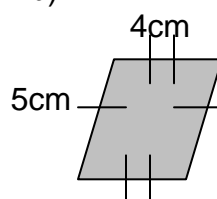
**Support Questions**

1. Calculate the perimeter for each of the following objects.

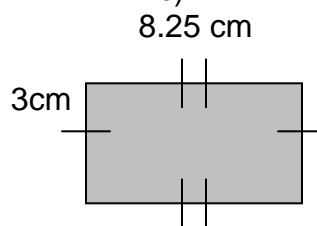
a)



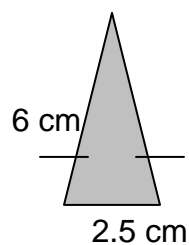
b)



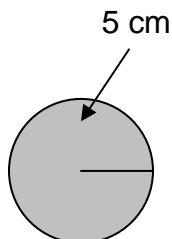
c)



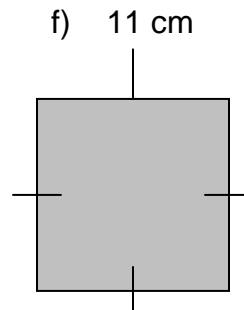
d)



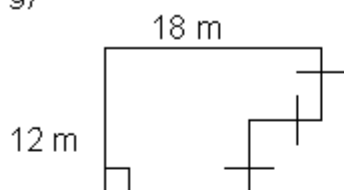
e)



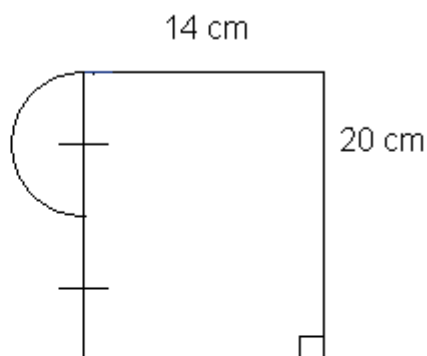
f)



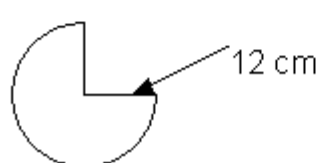
g)



h)



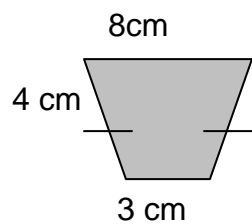
i)



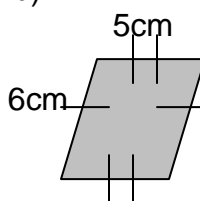
**Key Question #15**

1. Calculate the perimeter for each of the following objects. (9 marks)

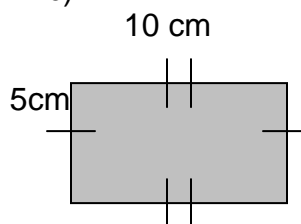
a)



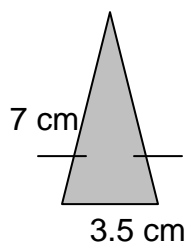
b)



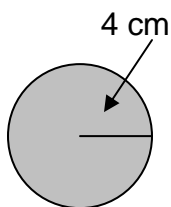
c)



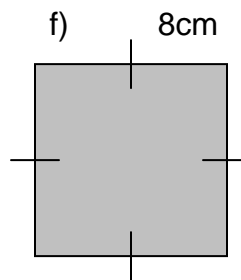
d)



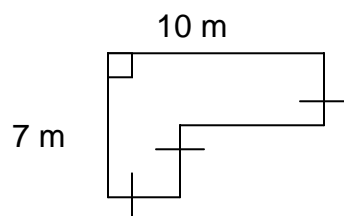
e)



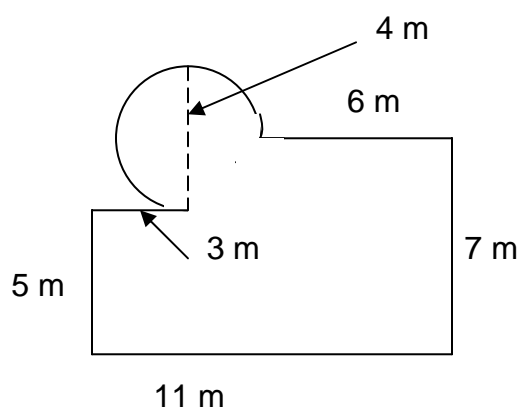
f)



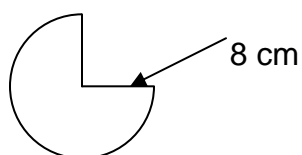
g)



h)



i)



**Key Question #15**

2. Explain how you could find the circumference of a circle given its area. (3 marks)
3. A rectangular yard has an area of 24m^2 . What are four possible perimeter's for this yard? (4 marks)
4. When a circle's radius doubles so does its circumference. True or False? Prove your answer with an example. (4 marks)
5. What is the total amount of edging that would be required to cover all edges of a cube measuring 10 cm on all edges? (4 marks)

Support Question Answers

Lesson 11

1. a. slope = -4 , y-int = 3 b. slope = 1 , y-int = -4 c. slope = $-\frac{2}{3}$, y-int = 7
 d. slope = $\frac{3}{5}$, y-int = -1

2. a. $(4, -4)$

$$\begin{aligned} y &= -2x + 4 \\ (-4) &= -2(4) + 4 \\ -4 &= -8 + 4 \\ -4 &= -4 \end{aligned}$$

- b. $(12, -8)$

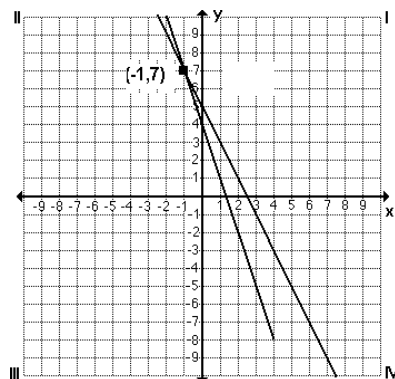
$$\begin{aligned} y &= -\frac{3}{4}x + 1 \\ (-8) &= -\frac{3}{4}(12) + 1 \\ -8 &= -9 + 1 \\ -8 &= -8 \end{aligned}$$

- c. $(-1, -1)$

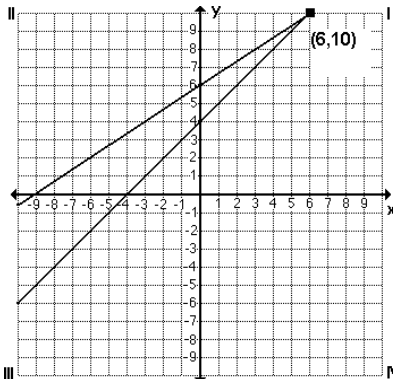
$$\begin{aligned} y &= -x - 2 \\ -1 &= -(-1) - 2 \\ -1 &= 1 - 2 \\ -1 &= -1 \end{aligned}$$

Note: $(4, -6)$ also works

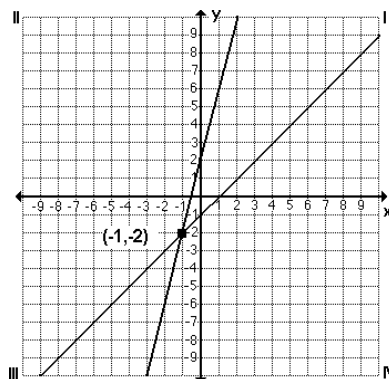
3. a.



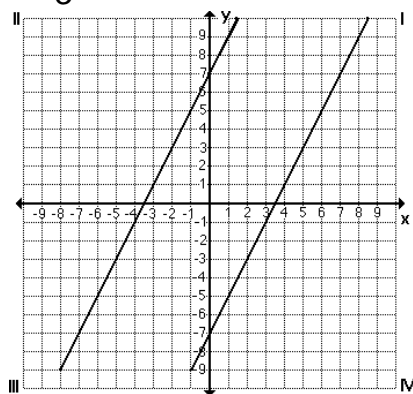
- b.



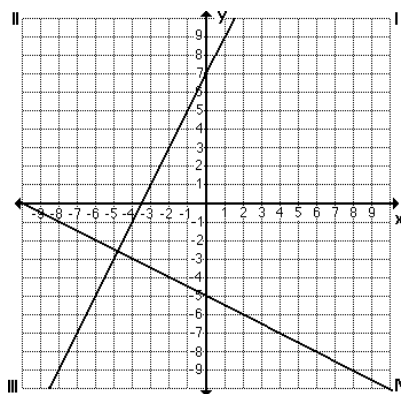
- c.



4. $y = \frac{2}{3}x + 2$
 $y = \frac{2}{3}x - 5$



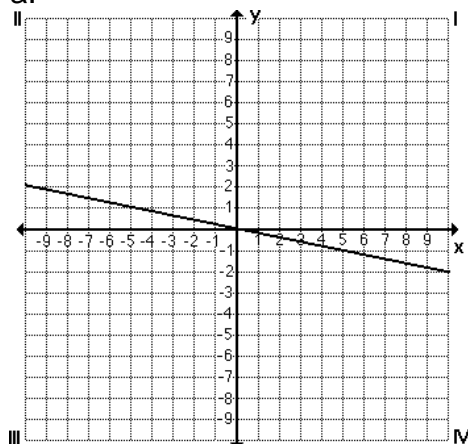
5. $y = 2x + 7$
 $y = -\frac{1}{2}x - 6$



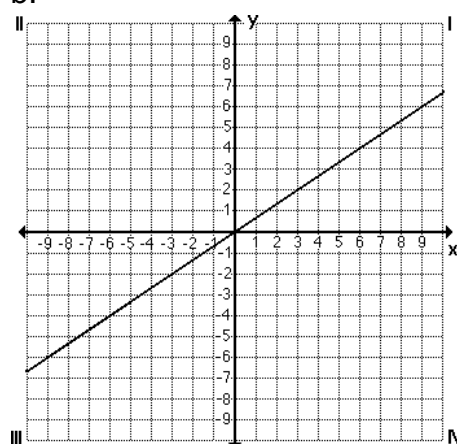
Lesson 12

1. a. -4 b. 1 c. $-\frac{1}{3}$ d. $\frac{2}{5}$
2. a. $y = -5x$ b. $y = -\frac{3}{7}x$ c. $y = \frac{1}{4}x$ d. $y = x$
3. a. $y = \frac{1}{3}x$ b. $y = -\frac{5}{4}x$

4. a.

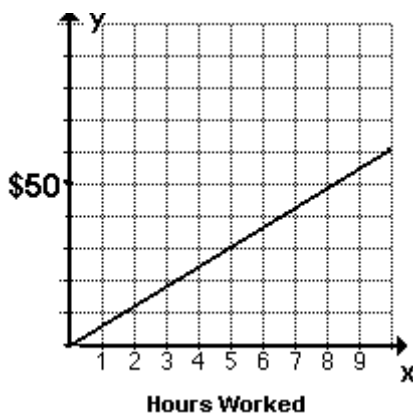


b.



5. $E = 6h$

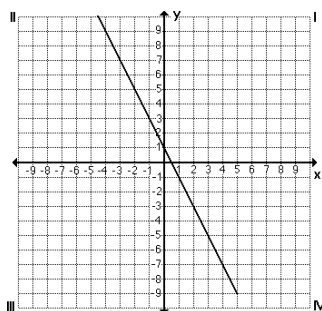
h	E
0	0
2	12
4	24
10	60



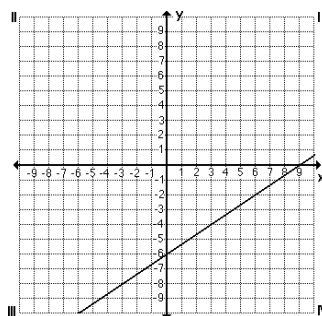
6. a. $y = -5x + 1$ b. $y = -\frac{3}{7}x - 4$ c. $y = \frac{1}{4}x + 6$

7. a. $y = \frac{7}{5}x + 5$ b. $y = -\frac{1}{3}x - 1$

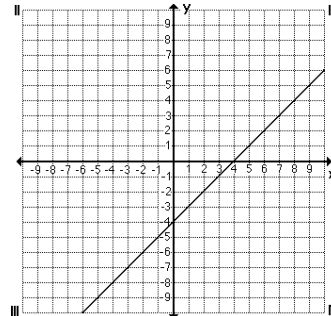
8. a.



b.

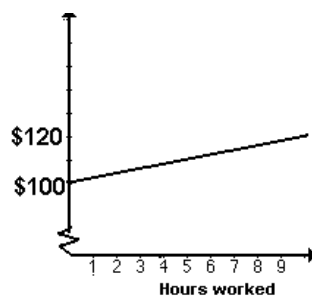


c.



9. $E = 2h + 100$

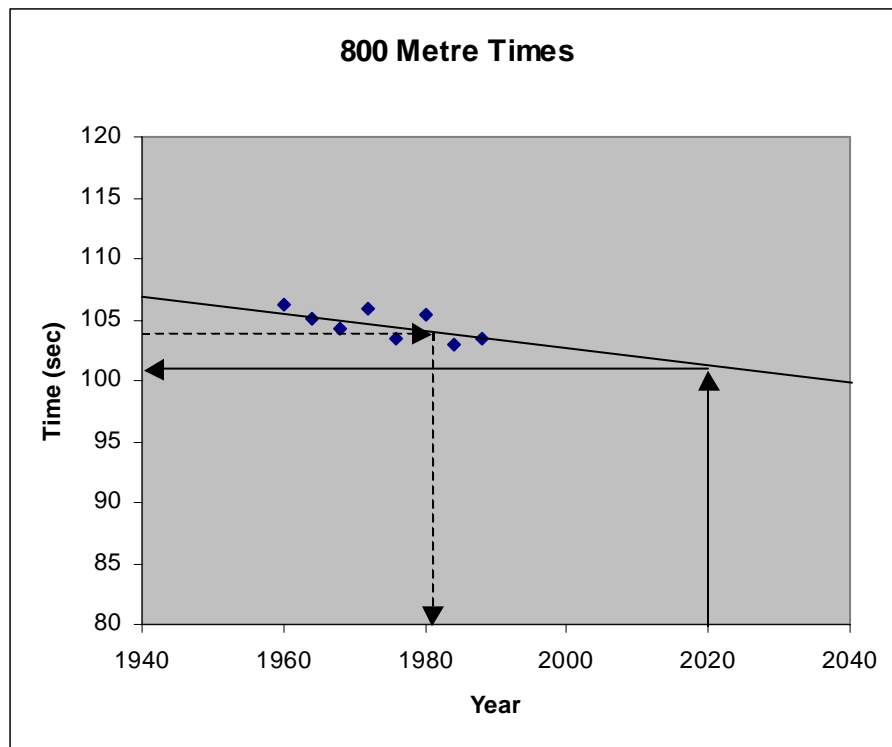
h	E
0	100
2	104
4	108
10	120



Lesson 13

1. a. no correlation b. positive correlation c. negative correlation

2. a.



2. b.

Equation of best fit

Use the coordinates (1964, 105.1) and (1988, 103.45)

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{103.45 - 105.1}{1988 - 1964} = \frac{-1.65}{24} = -0.06875$$

$$y - \text{int} = 107$$

$$y = -0.06875x + 107$$

c. ≈ 101 sec

d. ≈ 1981

Lesson 14

1. a. $\frac{3+6+2+7+3+5+8}{7} \approx 4.86$ b. $\frac{23+28+32+15+28+32+35+29+12}{9} = 26$
c. $\frac{75+43+57+69+84+88+94+97+51+43+45+62+61+57}{14} \approx 66.1$
2. $\left(\begin{array}{l} 2750000 + 2200000 + 2000000 + 2000000 + 2000000 + 1600000 + 1500000 + 1400000 + 950000 + 925000 \\ + 900000 + 850000 + 700000 + 650000 + 585640 + 575000 + 522500 + 500000 + 450000 + 400000 \end{array} \right) / 20$
 $= \$1\,172\,907$
3. a. $\{2,3,3, \underline{5}, 6,7,8\}$ median = 5
b. $\{12,15,23,28, \underline{28}, 29,32,32,35\}$ median = 28
c.
 $\{43,43,45,51,57,57, \underline{61,62}, 69,75,84,88,94,99\}$
 $\frac{61+62}{2} = \frac{123}{2} = 61.5$ median = 61.5
4. $\frac{925000 + 900000}{2} = \frac{1825000}{2} = 912500$ median = \$912 500
5. a. 3 (occurs 2 times) b. 28 (occurs three times) c. 43 (occurs 3 times)
6. \$850 000
7.
mean = $\left(\frac{63+74+77+68+71+74+70+65}{8} \right) = 70.25$
median = $\{63,65,70, \underline{71}, 74,74,77,\}$ = 71
mode = 74
8.
mean = $\frac{35600 \times 3 + 42750 \times 5 + 51000 \times 6 + 99000 + 150000}{16} \approx \54721.88
median = $\frac{42750 + 51000}{2} = \46875
mode = 51000

Lesson 15

1. a. $P = 7 + 3 + 2 + 3$
 $P = 15 \text{ cm}$
- b. $P = 2(b + c)$
 $P = 2(4 + 5)$
 $P = 2(9)$
 $P = 18$
- c. $P = 2(l + w)$
 $P = 2(8.25 + 3)$
 $P = 2(11.25)$
 $P = 22.5$
- d. $P = 6 + 6 + 2.5$
 $P = 14.5 \text{ cm}$
- e. $C = 2\pi r$
 $C = 2(3.14)(5)$
 $C = 31.4 \text{ cm}$
- f. $P = 4(l)$
 $P = 4(11)$
 $P = 44 \text{ cm}$
- g. $P = 12 + 18 + 6 + 6 + 6 + 12$
 $P = 60 \text{ m}$
- h. $P = 14 + 20 + 14 + 10 + \frac{2\pi r}{2}$
 $P = 58 + \frac{2(3.14)(5)}{2}$
 $P = 73.7 \text{ cm}$
- i. $P = \frac{3}{4}(2\pi r) + 12 + 12$
 $P = \frac{3}{4}(2)(3.14)(12) + 12 + 12$
 $P = 56.52 + 24$
 $P = 80.52 \text{ cm}$