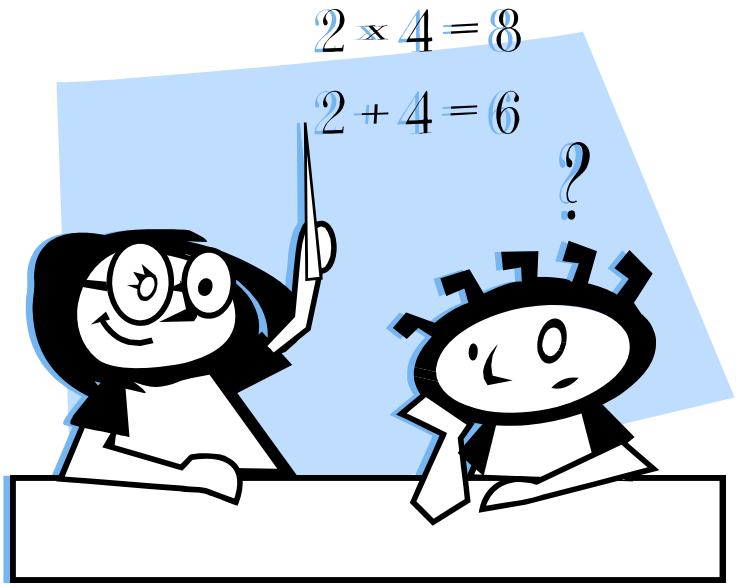
MFM1P

Foundations of Mathematics, Grade 9 - Applied



Version A

Grade 9 Mathematics (Applied)

Welcome to the Grade 9 Foundations of Mathematics, MFM1P. This full-credit course is part of the new Ontario Secondary School curriculum.

This course enables students to develop an understanding of mathematical concepts related to introductory algebra, proportional reasoning, and measurement and geometry through investigation, the effective use of technology, and hands-on activities. Students will investigate real-life examples to develop various representations of linear relations, and will determine the connections between the representations. They will also explore certain relationships that emerge from the measurement of three-dimensional figures and two-dimensional shapes. Students will consolidate their mathematical skills as they solve problems and communicate their thinking.

Material

This course is self-contained and does not require a textbook. You will require lined paper, graph paper, a ruler, a scientific calculator and a writing utensil.

Expectations

The overall expectations you will cover in the lesson are listed on the first page of each lesson.

Lesson Description

Each lesson contains one or more concepts with each being followed by support questions. At the end of the lesson the key questions covering all concepts in the lesson are assigned and will be submitted for evaluation.

Evaluation

In each lesson, there are support questions and key questions. You will be evaluated on your answers to the key questions in each lesson, the mid-term exam and the final exam.

Support Questions

These questions will help you understand the ideas and master the skills in each lesson. They will also help you improve the way you communicate your ideas. The support questions will prepare you for the key questions.

Write your answers to the support questions in your notebook. Do not submit these answers for evaluation. You can check your answers against the suggested answers that are given at the end of each unit.

Key Questions

The key questions evaluate your achievement of the expectations for the lesson. Your answers will show how well you have understood the ideas and mastered the skills. They will also show how well you communicate your ideas.

You must try all the key questions and complete most of them successfully in order to pass each unit. Write your answers to the key questions on your own paper and submit them for evaluation at the end of each unit. Make sure each lesson number and question is clearly labelled on your submitted work.

What You Must Do To Get a Credit

In order to be granted a credit in this course, you must

- ☑ Successfully complete the **Key Questions** for each unit and submit them for evaluation within the required time frame.
- ☑ Complete the **mid-term exam** after Unit 2.
- ☑ Complete and pass a **final examination**.

Your Final Mark

• Each Unit has 5 lessons each worth 2% (10% per Unit x 4 Units)	40%	} Term
• Midterm Test	30%	
• Final Examination	30%	

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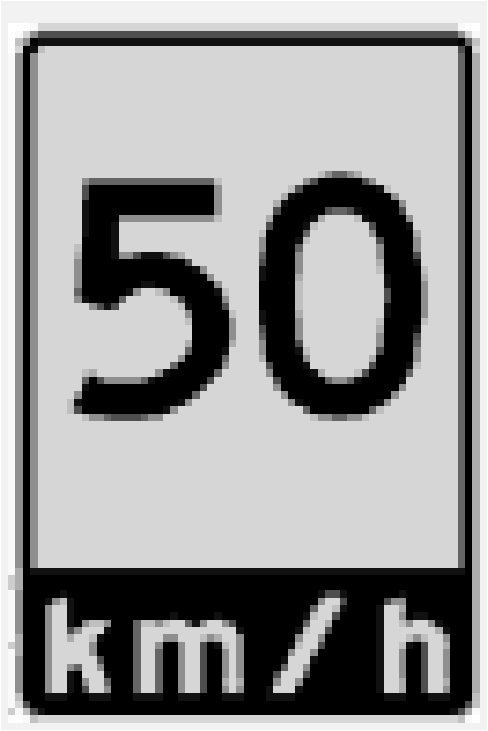
Ratios and Rate



Lesson 1

Lesson One Concepts

Overall Expectations

- Solve problems involving proportional reasoning.

Specific Expectations

- Illustrate equivalent ratios, using a variety of tools (e.g., concrete materials, diagrams, dynamic geometry software)
- Represent, using equivalent ratios and proportions, directly proportional relationships arising from realistic situations
- Solve for the unknown value in a proportion, using a variety of methods (e.g., concrete materials, algebraic reasoning, equivalent ratios, constant of proportionality)
- Make comparisons using unit rates
- Solve problems involving ratios, rates and directly proportional relationships in various contexts, using a variety of methods

Ratios

A **ratio** is a comparison of two or more numbers with the same units.

What is written first in the description matches with the first number in the ratio, and what is written second matches with the second and so on.

Example

♦♦♦♥♥♥♥ The ratio of ♦to ♥ is 3 to 4.

Ratios can written in any of the following three ways:

1. as a fraction, $\frac{3}{4}$
2. as a ratio, 3 : 4
3. with words, 3 to 4

Ratios in Lowest Terms

A ratio is in lowest terms (simplest form) when the greatest common factor of the terms is one. 6 : 4 in lowest terms is 3 : 2. Both terms are divided by two to produce a ratio in lowest terms.

Example

Give the ratio of A : B in lowest terms.

A A A A B B B B B B B B B A B A B B A B A A A B

Solution

The ratio of A : B is 10 : 15 or $\frac{10}{15}$.

In lowest terms both 10 and 15 are divisible by 5 so $\frac{10 \div 5}{15 \div 5} = \frac{2}{3}$ or 2 : 3.

Equivalent Ratios

Any two or more ratios that simplify to the same lowest terms are equivalent ratios.

Example

Show that the following ratios are equivalent:

12 : 16 and 60 : 80

Solution

12 : 16 has a greatest common factor of 4 reducing the ratio to 3 : 4 and
60 : 80 has a greatest common factor of 20 also reducing the ratio to 3 : 4

$\frac{12 \div 4 = 3}{16 \div 4 = 4}$ or 3 : 4 and $\frac{60 \div 20 = 3}{80 \div 20 = 4}$ or 3 : 4

Solving Proportions

Equivalent ratios are used when trying to solve questions involving proportion.

Example

Solve each proportion.

a) $4 : 5 = x : 35$

b) $\frac{12}{20} = \frac{24}{x}$

Solution

a) To find x, cross multiplication is used then is followed by division.

$4 : 5 = x : 35$ is the same as $\frac{4}{5} = \frac{x}{35}$

(35)(4)=140

$140 = 5x$ (5)(x)=5x

$\frac{140}{5} = \frac{5x}{5}$

$28 = x$

b) $\frac{12}{20} = \frac{24}{x}$

$\frac{12}{20}$ $\frac{24}{x}$ cross multiply

$12(x) = (20)(24)$

$12x = 480$

$\frac{12x}{12} = \frac{480}{12}$

$x = 40$

Rate

Rates compare two numbers that are expressed in different units.

Unit rate is a comparison where the second term is always one.

Example

Write the following ratios as a unit rate.

a) 24 soft drinks for 12 people.

b) $475 earned in 40 hours worked.

Solution

a) $24 : 12 = \frac{24}{12}$ so $\frac{24}{12} = \frac{x}{1}$

$24 = 12x$

$\frac{24}{12} = \frac{12x}{12}$

$2 = x$

Therefore there are two soft drinks per person

b) $475 : 40 = \frac{475}{40}$ so $\frac{475}{40} = \frac{x}{1}$

$475 = 40x$

$\frac{475}{40} = \frac{40x}{40}$

$11.88 \approx x$

Therefore the unit rate is $11.88/hr.

Support Questions

a) Write the following ratios in simplest form.

a) 20 : 35
b) 75 : 125
c) 13 : 39

b) Write as a ratio in lowest terms.

a) 4 cups of water for every 3 cups of flour
b) $12 saved for every $75 spent
c) 8 people passed for every 10 writing a test

c) Write each ratio. ♣♣♣♣♦♦♦♦♥♥♥♥♥♥♠♠

a) clubs to diamonds
b) hearts to clubs
c) spades to diamonds
d) spades to hearts

Support Questions

d) Write each ratio in simplest terms.

a) 2 home runs to 3 home runs
b) $10 to $2
c) 32 days to 8 days

e) On a hockey team 6 of 19 players are defensemen. Write each ratio in simplest terms.

a) defensemen to team members
b) team members to defensemen
c) defensemen to non-defensemen

f) A gas tank in a car holds 50 litres. The cost of a fill up is $38.00.

a) What is the unit price per litre? (round to the nearest cent)
b) Using the answer in "a", how much will 35 litres cost?
c) How many litres can be purchase with $15.00?

g) Johnny had 15 hits in 45 at bats. How many at bats are needed to achieve 75 hits?

Key Question #1

1. Write the following ratios in simplest form. (3 marks)

a. 12 : 18
b. 25 : 155
c. 8 : 32

2. Write as a ratio in lowest terms (3 marks)

a) 4 teaspoons of sugar for every 5 cups of flour
b) 5 t-shirts for every $200 raised
c) 6 buckets of popcorn for every 3 tickets sold

Key Question #1 (continued)

3. Write each as a ratio. (3 marks)

   a) happy faces to airplanes
   b) stars to flowers
   c) flowers to airplanes
   d) stars to happy faces

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4. Write each ratio in simplest terms. (3 marks)

   a) 2 goals to 4 goals
   b) $15 to $5
   c) 32 people to 2 people

5. On a soccer team 3 of 11 players are defensemen. Write each ratio in simplest terms. (3 marks)

   a) defensemen to team members
   b) team members to defensemen
   c) defensemen to non-defensemen

Percent

Lesson 2

Lesson Two Concepts

Overall Expectations

- Solve problems involving proportional reasoning.

Specific Expectations

- Illustrate equivalent ratios, using a variety of tools (e.g., concrete materials, diagrams, dynamic geometry software)
- Represent, using equivalent ratios and proportions, directly proportional relationships arising from realistic situations
- Solve for the unknown value in a proportion, using a variety of methods (e.g., concrete materials, algebraic reasoning, equivalent ratios, constant of proportionality)
- Make comparisons using unit rates
- Solve problems involving ratios, rates and directly proportional relationships in various contexts, using a variety of methods
- Solve problems requiring the expression of percents, fractions, and decimals in their equivalent forms

Percent

Percent means the number of parts per hundred.

Expressing a fraction as a decimal then as a percent

Example

Express the following fraction as a percent.

$$\frac{4}{5}$$

Solution

$$\frac{4}{5} = 4 \div 5$$

1^{st} convert to decimal

$$4 \div 5 = 0.8$$

$$0.8 \times 100 = 80\%$$

Multiply the decimal by 100

Percent of a Number

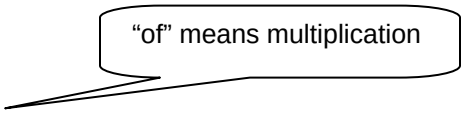
Example

a) Find 35% of 200.

b) 20% of a number is 15.

c) Calculate the total cost of a shirt costing $24.99 with 5% GST.

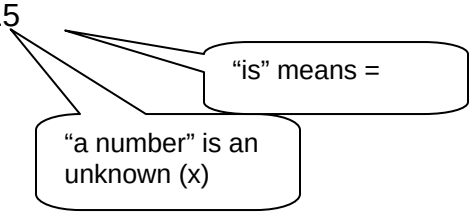
Solution



a) 35% of 200 = (0.35)(200)
= 70

Therefore 35% of 200 is 70

b) 20% of a number is 15



$(0.20)(x) = 15$

$0.20x = 15$

$x = \frac{15}{0.20}$

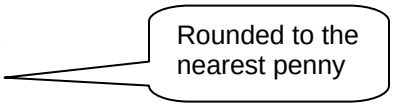
$x = 75$

Therefore 20% of 75 is 15

c) Calculate the total cost of a shirt costing $24.99 with 5% GST.

Total cost = cost of item + tax

5% of 24.99 = (0.05)(24.99)
= $1.25



Total cost = 24.99 + 1.25
= $26.24

Support Questions

1. Express each fraction as a percent.

   a) $\frac{2}{5}$ b) $\frac{7}{8}$ c) $\frac{9}{8}$

2. Express each percent as a decimal.

   a) 34% b) 89% c) 115% d) 23.5%

3. Express each percent as a fraction in lowest terms.

   a) 55% b) 78% c) 0.5% d) 125%

4. Find:

   a) 25% of 200 b) 70% of 350 c) 15% of 150
   d) 0.7% of 1000

5. Determine the number in each statement.

   a) 30% of a number is 90.
   b) 8% of a number is 24.
   c) 125% of a number is 50.

6. A pair of jeans that normally sells for $50.00 is on sale for 35% off. What is the sale price of the jeans before taxes?

7. A MP3 player costs $149.95. What is the total cost of the MP3 player including 5% GST and 8% PST.

8. The population in a town increased by 500 citizens. This represents an increase of 10% over last year. What was the population of the town last year?

Key Question #2

1. Express each fraction as a percent. (3 marks)

   a) $\frac{3}{4}$ b) $\frac{11}{8}$ c) $\frac{3}{8}$

2. Express each percent as a decimal. (3 marks)

   a) 40% b) 75% c) 0.65% d) 70.5%

3. Express each percent as a fraction in lowest terms. (4 marks)

   a) 34% b) 65% c) 0.05% d) 325%

4. Find: (4 marks)

   a) 29% of 300
   b) 62% of 100
   c) 150% of 2200
   d) 0.2% of 1000

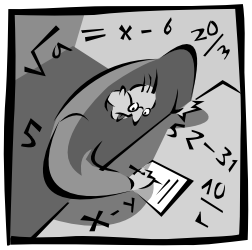
5. The regular price of a DVD player is $84.99. It is on sale for 30% off. What is the amount of the discount? What is the sale price? What is the total price after 13% sales tax is included? (3 marks)

6. The number of people in a town in 2003 was 2500. The population for 2004 increased by 5%. What was the population increase? What was the town's population in 2004? (4 marks)

Key Question #2

7. An instructor used the following equation to calculate a students' final mark. Final Mark = 75% of term + 25% of exam mark.

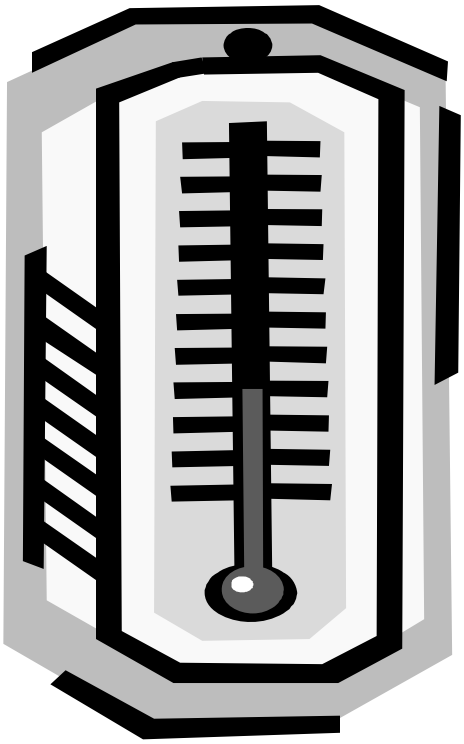
   Suppose your term mark is 77%. How will your mark change as your exam mark has been taken into account? Copy and complete the table below: (3 marks)



Term mark	*Exam mark*	*Final mark*
77%	100%	
77%	90%	
77%	80%	
77%	70%	
77%	60%	
77%	50%	
77%	40%	
77%	30%	
77%	20%	
77%	10%	
77%	0%	

8. Write a paragraph to explain how your final mark in question 7 changes as your examination mark goes from great to not so good. (3 marks)

Integers



Lesson 3

Lesson Three Concepts

Overall Expectations

- Solve problems involving proportional reasoning.

Specific Expectations

- Simplify numerical expressions involving integers and rational numbers with and without the use of technology

Integers

Integers are the set of numbers …,-3, -2, -1, 0, 1, 2, 3,…

Adding and Subtracting Integers

Example

a) (–3) + (–5)

b) (+5) – (–7)

Solution

If these two signs are opposite then the sign becomes negative

a) (–3) + (–5) = –3 –5
= –8

b) (+5) – (–7) = +5 +7
= +12

If these two signs are the same then the sign becomes positive

Support Questions

1. Add.

a) (+4) + (–5)	b) (–7) + (–3)	c) (–8) + (+8)
d) (0) + (–6)	e) (2) + (+8)	f) (–7) + (+2)
g) (+1) + (–7)	h) (+7) + (+1)	i) (+1) + (–5)

2. Subtract

a) (+1) – (–2)	b) (–4) – (–3)	c) (–9) – (+8)
d) (0) – (–2)	e) (+5) – (+6)	f) (–8) – (–2)
g) (–7) – (–7)	h) (–7) – (+3)	i) (+1) – (–5)

3. For each statement, write an expression then simplify.

   a) moves forward 3 steps and moves backwards 5 steps
   b) goes up an elevator 9 floors then down 6 floors
   c) a gain of $15 followed by a loss of $8
   d) a loss of $25 followed by a gain of $32
   e) a rise in temperature of 18°C then a drop of 20°C

4. For each statement, write an expression then finish the question.

   a) A plane goes up into the air 2000m then increases another 1500m and then decreases its elevation by 1000 m. Find the height of the plane.

   b) An elevator is on the 16th floor. The elevator descends 8 floors, and then ascends 14 floors to the top of the building. What floor is the elevator on?

   c) The opening value of a stock is $2. Later that day the stock increased by $8 only to fall back $4 by the end of the day. What is the closing value of the stock?

   d) At 8:00 am the temperature outside is 6°C. At noon the temperature had increased by 9°C. By 4:00 pm the temperature had increased another 3°C and by 9:00 pm the temperature had decreased 7°C. What was the temperature at 9:00 pm?

   e) The opening balance in a bank account is $4000. $2532 was spent on living expense. $3217 was deposited into the account from salary. What is the closing balance?

Multiplying and Dividing Integers

The product or quotient of an integer question with an even amount of negative values will always give a positive answer unless one of the values is zero.

The product or quotient of an integer question with an odd amount of negative values will always give a negative answer unless one of the values is zero.

Example

a) (+1)(–5)(–2)(–1)(+3)(–2)

b) (–3)(–2)(–1)(+1)

c) (-32) ÷ (-4)

d) $\frac{+28}{-7}$

Solution

a) (+1)(–5)(–2)(–1)(+3)(–2) = 60

or (+60)

4 negative signs in question gives positive answer

b) (–3)(–2)(–1)(+1) = – 6

3 negative signs in question gives negative answer

c) (-32) ÷ (-4) = +8

d) $\frac{+28}{-7} = -4$

Support Questions

5. Multiply;

a) (+4)(–5)(–2)(–3)
b) (–7)(–3)(+1)
c) (–8)(+9)
d) (0)(–6)(–2)(–5)
e) (2)(+8)
f) (–1)(+2)
g) (–1)(–7)
h) (+7)(–1)(+1)
i) (–1)(–5)(–1)(–4)

Support Questions

6. Divide;

a) $(+8) \div (-4)$ b) $(-9) \div (-3)$ c) $(-12) \div (+2)$ d) $(-15) \div (+5)$

e) $\frac{-36}{-9}$ f) $\frac{100}{-25}$ g) $\frac{(+8)}{(-2)}$ h) $\frac{(+42)}{(+7)}$

i) $(+120) \div (-10)$ j) $\frac{-72}{(+9)}$ k) $(-9) \div (+3)$ l) $\frac{(+81)}{(-3)}$

7. For each statement, write an expression then finish the question.

a) A person paid $2 a day for 6 days on lunch. How much did this person spend in total?

b) A person made $40 a day for 8 days worked. How much did this person make in total?

c) A car traveled 400 km in 5 hours. How many km did the car travel per hour?

d) An individual 3 times collected 8 stamps for his collection.

Key Question #3

1. Add, subtract, multiply or divide as required. (1/2 mark each = 13 marks)

a) $(+4) + (-5)$ b) $(-7) + (-3)$ c) $(-8) + (+8)$

d) $(0) + (-6)$ e) $(2) + (+8)$ f) $(-7) + (+2)$

g) $(0) - (-2)$ h) $(+5) - (+6)$ i) $(-8) - (-2)$

j) $(-7) - (-7)$ k) $(-7) - (+3)$ l) $(+1) - (-5)$

m) $(+4)(-5)(-2)(-3)$ n) $(-7)(-3)(+1)$ o) $(-8)(+9)$

p) $(0)(-6)(-2)(-5)$ q) $(2)(+8)$ r) $(-1)(+2)$

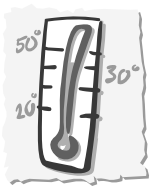
s) $(+8) \div (-4)$ t) $(-9) \div (-3)$ u) $\frac{-36}{-9}$

v) $\frac{100}{-25}$ w) $(+120) \div (-10)$ x) $\frac{-72}{(+9)}$

y) $(-9) \div (+8)$ z) $\frac{(+81)}{(-3)}$

Key Question #3 (continued)

2. For each statement, write an expression then simplify. (4 marks)



   a) moves backwards 3 steps and moves forward 7 steps
   b) goes up 14 stairs then down 8 stairs
   c) a gain of $11 followed by a loss of $18
   d) a rise in temperature of 1°C then a drop of 9°C

3. A hockey player was given $5 for each of 6 goals she scored. Find her total earnings. (2 marks)

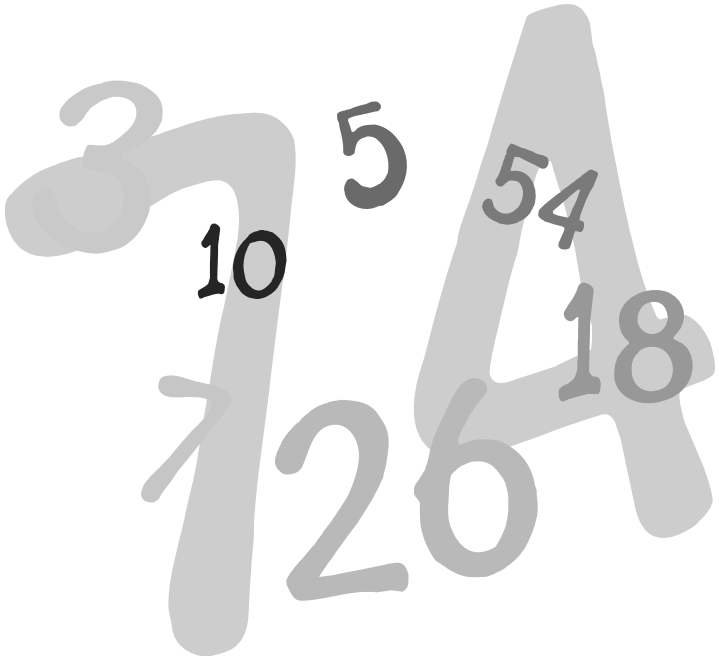
4. An overall loss of $16 000 occurred over an 8 day period. Find the mean loss per day. (2 marks)

5. For each statement, write an expression then write its quotient or a product. (4 marks)

   a) A company twice suffers a loss of 5 trucks.
   b) On 5 occasions, a baseball player lost 6 baseballs.
   c) The mean (average) daily gain when $8 000 is gained over 6 days.
   d) A chicken place sells 5 buckets of 20 pieces of chicken.

6. What does it mean when it is said that “two negatives make a positive?” Is this always true? Prove with examples. (4 marks)

Rational Numbers



Lesson 4

Lesson Four Concepts

Overall Expectations

- Solve problems involving proportional reasoning.

Specific Expectations

- Solve problems requiring the expression of percents, fractions, and decimals in their equivalent forms
- Simplify numerical expressions involving integers and rational numbers with and without the use of technology

Rational Numbers

Rational numbers are the set of numbers that can be written in the form $\frac{m}{n}$, where *m* and *n* are integers and $n \neq 0$.

Rational numbers can always be represented on a number line.

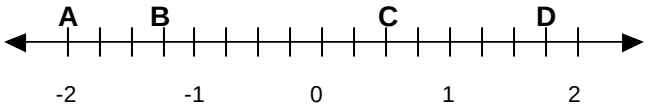
Rational numbers can be written as a decimal by dividing the numerator by the denominator.

Examples of rational numbers;

$$\text{-4, 5.7, 1.23415, 125.67, } \frac{3}{4}, -\frac{32}{5}, -4\frac{1}{8}, \frac{2}{9}$$

Example

a) State the rational number represented by the letter on the number line.



b) Which rational number is greater?

.7 or 0.69

c) Which rational number is greater? $\frac{3}{5}$ or $\frac{16}{25}$

Solution

a) A = –2; B = –1.25; C = 0.5; D = 1.75

b) Since 0.7 means 70 parts per one hundred and 0.69 mean 69 parts per one hundred then $0.7 > 0.69$

"$>$" means greater than and "$<$" means less than

c) $\frac{3}{5} = 0.60$ and $\frac{16}{25} = 0.64$,therefore, $0.64 > 0.60$ or $\frac{16}{25} > \frac{3}{5}$

Support Questions

1. Which rational number is greater?

   a) $\frac{2}{4}, -\frac{3}{4}$ b) $\frac{1}{2}, \frac{3}{7}$ c) $-\frac{2}{5}, 0.41$ d) $\frac{1}{3}, \frac{5}{18}$

   e) $0.3, -0.33$ f) $5.9, 5.99$ g) $-\frac{7}{12}, -\frac{8}{18}$ h) $0.1, -0.11$

   i) $\frac{2}{3}, \frac{66}{90}$ j) $-\frac{3}{8}, -\frac{4}{9}$

2. List in order from least to greatest.

   a) $0.6, \frac{1}{9}, -\frac{6}{4}, -\frac{1}{2}, 0, 1, 0.1$

   b) $-\frac{5}{12}, -0.75, \frac{1}{5}, -1.7, -\frac{1}{6}, 0.9$

   c) $\frac{2}{3}, 0.67, \frac{3}{4}, -0.71, \frac{1}{8}, 0.13$

3. Represent all of the following rational numbers on the same number line.

   a) 0.43 b) 0.85 c) 0.62 d) 0.08 e) 0.21

Adding and Subtracting Rational Numbers (Fractions)

To add and subtract fractions a common denominator must be present. A lowest common denominator is best.

Example

Add or subtract as indicated.

a) $\frac{1}{6}+\frac{4}{6}$

b) $\frac{3}{8}+\frac{5}{6}$

c) $1\frac{2}{3}-2\frac{2}{5}$

Solution

a) $\frac{1}{6}+\frac{4}{6}=\frac{5}{6}$

Only the numerators are added. 4 + 1 = 5

The denominator is the same, as required

b) $\frac{3}{8}+\frac{5}{6}=\frac{9}{24}+\frac{20}{24}$

3 x 3 = 9

8 x 3 = 24

24 is the lowest common denominator. In other words, 24 is the lowest number that both 8 and 6 divide evenly into.

$=\frac{29}{24}$ or $1\frac{5}{24}$

c) $1\frac{2}{3}-2\frac{2}{5}=\frac{5}{3}-\frac{12}{5}$

3 x 1 + 2 = 5

5 x 2 + 2 = 12

$=\frac{25}{15}-\frac{36}{15}$

$=\frac{-11}{15}$ or $-\frac{11}{15}$

Support Questions

4. Add or subtract as indicated.

a) $\frac{4}{7}-\frac{3}{7}$ b) $\frac{11}{14}+\frac{2}{14}$ c) $-\frac{3}{2}+\frac{1}{6}$ d) $\frac{5}{6}-\frac{3}{8}$

e) (+3.2) – (-3.9) f) (-1.8) – (-6.7) g) 3.9 + (-4.5) h) (-1.1) – (-1.1)

i) $\left(+\frac{3}{5}\right)-\left(\frac{-4}{6}\right)$ j) $\left(-\frac{7}{8}\right)+\left(\frac{-5}{6}\right)$ k) $\left(-\frac{2}{3}\right)+\left(\frac{4}{5}\right)$ l) $\left(-\frac{5}{14}\right)-\left(\frac{7}{8}\right)$

Multiplying and Dividing Rational Numbers (Fractions)

You should not use common denominators when multiplying or dividing fractions.

Example

Multiply or divide as indicated.

a) $\frac{2}{7}\times\frac{8}{9}$

b) $2\frac{1}{6}\times 3\frac{7}{8}$

c) $\frac{4}{5}\div\frac{8}{11}$

Solution

a) $\frac{2}{7}\times\frac{8}{9} = \frac{16}{63}$

Both 14 and 6 can be divided by 2 so it is usually best to simplify before multiplying.

b) $2\frac{2}{6}\times 3\frac{1}{3} = \frac{14}{6}\times\frac{10}{3} = \frac{7}{3}\times\frac{10}{3} = \frac{70}{9}$

Simply multiply by the inverse

c) $\frac{4}{5}\div\frac{8}{11} = \frac{4}{5}\times\frac{11}{8} = \frac{\cancel{4}^{1}}{5}\times\frac{11}{\cancel{8}^{2}} = \frac{11}{10}$ or $1\frac{1}{10}$

Support Questions

5. Multiply or divide as indicated. Simplify all fraction answers.

a) $\left(-\frac{4}{7}\right)\left(-\frac{3}{7}\right)$ b) $\frac{\left(\frac{11}{7}\right)}{+\left(\frac{2}{5}\right)}$ c) $\frac{-3}{2}\times\frac{1}{6}$ d) $\frac{5}{6}\div\frac{3}{8}$

e) (+3.2) ÷ (-3.9) f) (-1.8) × (-6.7) g) 3.9 ÷ (-4.5) h) (-1.1) × (-1.1)

i) $\left(+\frac{3}{5}\right)\left(\frac{-4}{6}\right)$ j) $\left(-\frac{7}{8}\right)\div\left(\frac{3}{-4}\right)$ k) $\left(-\frac{2}{3}\right)\times\left(\frac{6}{5}\right)$ l) $\left(-\frac{5}{14}\right)\div\left(\frac{-7}{-10}\right)$

Key Question #4

1. Which rational number is greater? (1/2 mark each = 5 marks)

a) $\frac{1}{3},\frac{4}{9}$ b) $\frac{22}{25},\frac{21}{24}$ c) $-\frac{2}{5},-.41$ d) $\frac{4}{3},\frac{23}{18}$

e) $0.01,\frac{3}{200}$ f) 4.7,4.71 g) $-\frac{6}{11},-\frac{5}{9}$ h) 0.2,–0.22

i) $\frac{2}{7},\frac{22}{70}$ j) $-\frac{5}{8},-\frac{6}{9}$

2. List in order from least to greatest. (3 marks)

a) $0.51,\frac{1}{2},-\frac{5}{4},1.1,\frac{1}{3},0$ b) $\frac{4}{13},0.9,\frac{1}{5},-1.27,-\frac{7}{5},0.93$

c) $\frac{1}{4},0.76,\frac{2}{5},-0.17,\frac{2}{7},0.31$

3. Represent all of the following rational numbers on the same number line. (3 marks)

a) 0.72 b) 0.81 c) $\frac{23}{100}$ d) .06 e) $\frac{3}{5}$

Key Question #4

4. Add, subtract, multiply or divide as indicated and simplify if needed. (13 marks)

a) $-\frac{2}{5}-\frac{4}{5}$ b) $\frac{2}{8}+\frac{4}{8}$ c) $-\frac{4}{3}+\frac{2}{5}$ d) $\frac{4}{5}-\frac{4}{9}$

e) (+5.1) + (-3.1) f) (1.6) – (-2.4) g) 8.1 + (-5.7) h) (-2.2) – (-2.2)

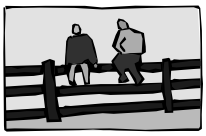
i) $\left(+\frac{3}{7}\right)-\left(\frac{-4}{7}\right)$ j) $\left(-\frac{5}{6}\right)+\left(\frac{7}{-8}\right)$ k) $\left(-\frac{1}{3}\right)+\left(\frac{3}{7}\right)$ l) $\left(-\frac{5}{6}\right)-\left(\frac{7}{4}\right)$

m) $\left(-\frac{4}{3}\right)\left(-\frac{3}{8}\right)$ n) $\frac{\left(\frac{10}{6}\right)}{\left(\frac{3}{12}\right)}$ o) $-\frac{4}{5}\times\frac{1}{8}$ p) $\frac{4}{7}\div\frac{3}{8}$

q) (+7.1) ÷(-4.9) r) (-0.8) × (-6.3) s) 8.9 ÷ (-9.5) t) (-2.2) × (-2.2)

u) $\left(+\frac{3}{4}\right)\left(\frac{-1}{6}\right)$ v) $\left(\frac{3}{8}\right)\div\left(\frac{1}{-6}\right)$ w) $\left(-\frac{1}{2}\right)\times\left(\frac{4}{9}\right)$

5. Cliff fenced his yard using lengths of fencing that were 2.4 m, 5.7 m and 4.3 m respectively. The fencing cost $20.00/m. How much did the fence cost? (3 marks)

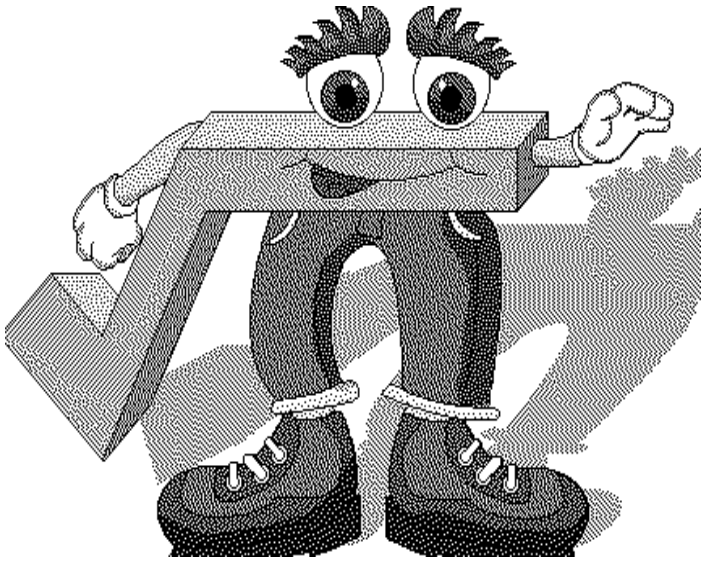


6. Add 1 to the numerator of the fraction $\frac{1}{2}$. How does the size of the fraction change? (2 marks)

7. Add 1 to the denominator of the fraction $\frac{1}{2}$. How does the size of the fraction change? (2 marks)

8. How does the size of the fraction $\frac{1}{2}$ change when both the numerator and the denominator are increased by 1? (2 marks)

Square Root



Lesson 5

Lesson Five Concepts

Overall Expectations

- Solve problems involving proportional reasoning.
- Simplify numerical and polynomial expressions in one variable, and solve simple first-degree equations.

Specific Expectations

- Relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations;

Square Root

Square root is a number, when multiplied by itself, results in a given number.

Example

Find the square root.

a) $\sqrt{25}$ b) $\sqrt{235}$

Solution

a) $\sqrt{25} = \pm 5$ — -5 and +5, when multiplied to itself, result in 25

b) $\sqrt{235} \approx \pm 15.33$ — square roots questions always give 2 answers

Support Questions

1. What are the square roots of each number?

a) 9	b) 64	c) 1	d) 121
e) 23	f) 58	g) 2314	h) -16
i) .25	j) 1.25	k) $\frac{1}{16}$	l) $\frac{4}{9}$

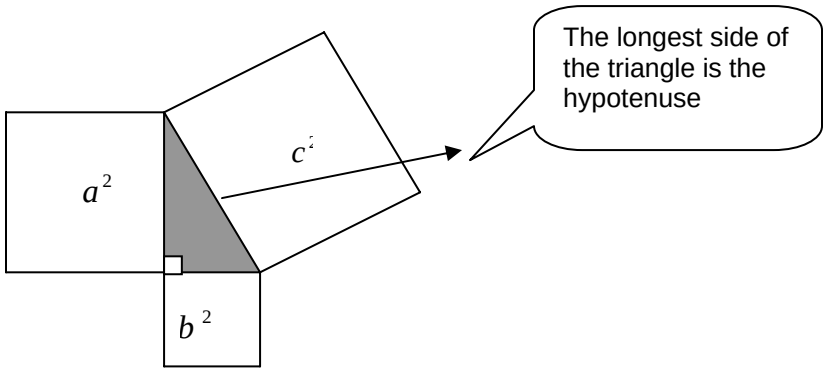
2. Determine the square roots of each number.

a) .09	b) 9	c) 900
d) 90 000	e) 9 000 000	f) 900 000 000

Pythagorean Theorem

The Pythagorean Theorem is one method used to calculate an unknown side of a right angle triangle if the other two sides are known.

Pythagorean Theorem states $a^2 + b^2 = c^2$. Where the area of square "a", plus the area of square "b", is equal to the area of square "c".



Since the formula for the area of a square is $A = s^2$ then to find the length of the side of a square we square root the value of the area.

Example

Find the length of a side of a square that has an area of $100\,cm^2$.

Solution

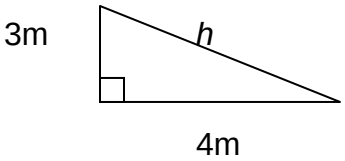
The area of square was 100 then the side would be 10.

We only use the positive answer since you cannot have negative length.

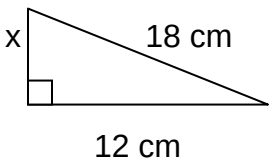
$\sqrt{100} = \pm 10$

Example

a) Find the length of the hypotenuse in the given triangle using Pythagorean Theorem.



b) Find the length of the missing side in the given triangle using Pythagorean Theorem.



Solution

a) Using Pythagorean Theorem $a^2 + b^2 = c^2$

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$
$$25 = c^2$$
$$5 = c$$

The area of square created by the hypotenuse is 25 so the length of its side is the square root of 25.

Therefore, the length of the hypotenuse is 5 m.

b) Using Pythagorean Theorem $a^2 + b^2 = c^2$

To get x^2 by itself 12^2 needs to be subtracted from both sides.

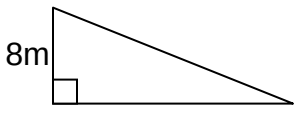
$$a^2 + b^2 = c^2$$
$$x^2 + 12^2 = 18^2$$
$$x^2 + 12^2 - 12^2 = 18^2 - 12^2$$
$$x^2 = 18^2 - 12^2$$
$$x^2 = 180$$
$$x \approx 13.4$$

Therefore, the length of the missing side is 13.4 cm.

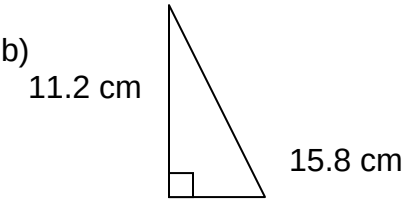
Support Questions

3. Calculate the length of the third side of each triangle. Round to one decimal place.

a)

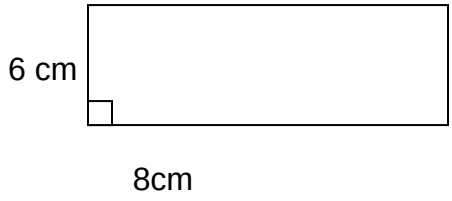


b)

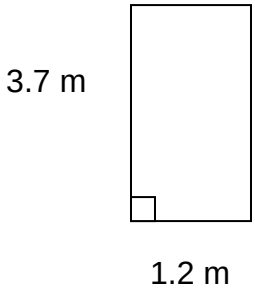


4. Calculate the diagonal of each rectangle. Round to one decimal place.

a)



b)



Key Question #5

1. What are the square roots of each number? Round to two decimals. (1/2 mark each = 4 marks)

   a) 14 b) 81 c) 100 d) 1

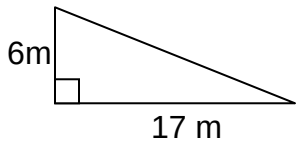
   e) 0.625 f) 1.73 g) $\frac{1}{25}$ h) $\frac{36}{64}$

2. Simplify each expression. (2 marks)

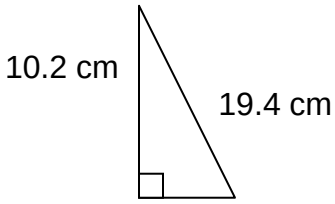
   a) $\sqrt{25}+\sqrt{36}$ b) $\sqrt{25+36}$

3. Calculate the length of the third side of each triangle. Round to one decimal place. (4 marks)

   a)

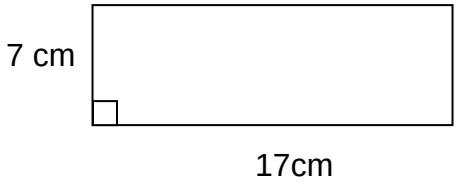


   b)

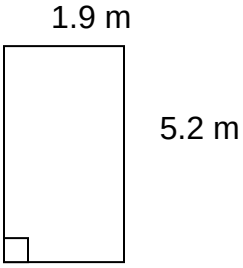


4. Calculate the diagonal of each rectangle. Round to one decimal place. (4 marks)

   a)

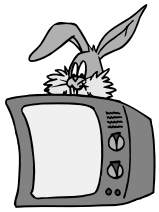


   b)



Key Question #5 (continued)

5. Televisions are sold by the size of their screen's diagonal. What is the diagonal size of a television that has a monitor screen height of 22 inches and a monitor screen width of 38 inches? (2 marks)



6. For a sloping ladder to be safe, the distance from the wall to the base of the ladder must be $\frac{1}{4}$ of the vertical distance from the ground to the top of the ladder. A 16 m ladder is placed against a wall. If the base of the ladder is 3.88 m from the wall use the Pythagorean Theorem to determine how far up the wall will the ladder reach? (4 marks)

7. Does Pythagorean Theorem work for triangles that don't have a 90° angle? Prove your answer with examples. (4 marks)

Support Question Answers

Lesson 1

1. a. 4:7 b. 3:5 c. 1:3

2. a. 4:3 b. 4:25 c. 4:5

3. a. 4:4 b. 6:4 c. 2:4 d. 2:6

4. a. 2:3 b. 5:1 c. 4:1

5. a. 6:19 b. 19:6 c. 6:13

6. a. $\frac{50L}{38.00} = \frac{1L}{x}$

$50x = 38.00$

$\frac{50x}{50} = \frac{38.00}{50}$

$x = \$0.76 / L$

b. $35\text{L} \times .71 = \$24.85$

c. $\$15.00 \div .71 \approx 21\text{L}$

7. a. $\frac{15}{45} = \frac{90}{x}$ $15x = 4050$ $x = 270$ at bats

Lesson 2

1. a. $2 \div 5 = 0.4$

$0.40 \times 100 = 40\%$

b. $7 \div 8 = 0.875$

$0.875 \times 100 = 87.5\%$

c. $9 \div 8 = 1.125$

$1.125 \times 100 = 112.5\%$

2. a. $34 \div 100 = 0.34$ b. $112 \div 100 = 1.12$

c. $0.9 \div 100 = 0.009$ d. $23.5 \div 100 = 0.235$

3. a. $55 \div 100 = 0.55$

$0.55 = \frac{55}{100} = \frac{11}{20}$

b. $78 \div 100 = 0.78$

$0.78 = \frac{78}{100} = \frac{39}{50}$

c. $0.5 \div 100 = 0.005$

$$0.005 = \frac{5}{1000} = \frac{1}{200}$$

d. $125 \div 100 = 1.25$

$$1.25 = \frac{125}{100} = \frac{5}{4}$$

4. a. $25 \div 100 = 0.25$

$0.25 \times 200 = 50$

b. $70 \div 100 = 0.7$

$0.7 \times 350 = 245$

c. $15 \div 100 = 0.15$

$0.15 \times 150 = 22.5$

d. $0.7 \div 100 = 0.007$

$0.007 \times 1000 = 7$

5. a. $0.30 \times n = 90$

$0.30n = 90$

$$\frac{0.30}{0.30}n = \frac{90}{0.30}$$

$n = 300$

b. $0.08 \times n = 24$

$0.08n = 24$

$$\frac{0.08}{0.08}n = \frac{24}{0.08}$$

$n = 300$

c. $1.25 \times n = 50$

$1.25n = 50$

$$\frac{1.25}{1.25}n = \frac{50}{1.25}$$

$n = 40$

6. $0.35 \times 50.00 = 17.50$

$50.00 - 17.50 = \$32.50$

$GST = 149.95 \times .05$

$GST = 7.50$

7. $PST = 149.95 \times .08$

$PST = 12.00$

$Sales\ Tax = 7.50 + 12.00$

$Sales\ Tax = \$19.50$

8. $0.10 \times n = 500$

$0.10n = 500$

$$\frac{0.10}{0.10}n = \frac{500}{0.10}$$

$n = 5000$

Lesson 3

1. a. –1 b. –10 c. 0 d. –6
   e. 10 f. –5 g. –6 h. 8 i. –4
2. a. 3 b. –1 c. –17 d. 2
   e. –1 f. –6 g. 0 h. –10
   i. 6
3. a. +3 – 5 = -2 b. +9 – 6 = 3 c. +15 – 8 = 7 d. –25 + 32 = 7
   e. +18 – 20 = -2
4. a. +2000 +1500 – 1000 = 2500 b. 16 – 8 + 14 = 22
   c. 2 + 8 – 4 = 6 d. 6 + 9 + 3 – 7 = 11
   e. 4000 – 2532 + 3217 = 4685
5. a. –120 b. 21 c. –72 d. 0
   e. 16 f. –2 g. 7 h. –7
   i. 20
6. a. –2 b. 3 c. –6 d. –3
   e. 4 f. –4 g. –4 h. 6
   i. –12 j. –8 k. –3 l. –27
7. a. $2 \times 6 = 12$ b. $40 \times 8 = 320$ c. $400 \div 5 = 80$ d. $3 \times 8 = 24$

Lesson 4

1. a. $\frac{2}{4} = 0.5$, $-\frac{3}{4} = -0.75$ b. $\frac{1}{2} = 0.5$, $\frac{3}{7} \approx 0.43$

   $0.5 > -0.75$ therefore $\frac{2}{4} > -\frac{3}{4}$ $0.5 > 0.43$ therefore $\frac{1}{2} > \frac{3}{7}$

c. $-\frac{2}{5} = -0.40$

$0.41 > -0.40$ therefore $.41 > -.40$

d. $\frac{1}{3} = 0.3\overline{3}$, $\frac{5}{18} \approx 0.2\overline{7}$

$0.33 > 0.27$ therefore $\frac{1}{3} > \frac{5}{18}$

e. $0.3 = 0.30$
$0.3 > -0.33$

f. $5.9 = 5.90$
$5.99 > 5.90$ therefore $5.99 > 5.9$

g. $-\frac{7}{12} = -0.58\overline{3}$, $\frac{8}{18} \approx -0.4\overline{4}$

$-0.4\overline{4} > -0.58\overline{3}$ therefore $-\frac{8}{18} > -\frac{7}{12}$

h. $0.1 = 0.10$
$0.10 > -0.11$

i. $\frac{2}{3} = 0.6\overline{6}$, $\frac{66}{90} \approx 0.7\overline{3}$

$0.7\overline{3} > 0.6\overline{6}$ therefore $\frac{66}{90} > \frac{2}{3}$

j. $-\frac{3}{8} = -0.375$, $-\frac{4}{9} \approx -0.44\overline{4}$

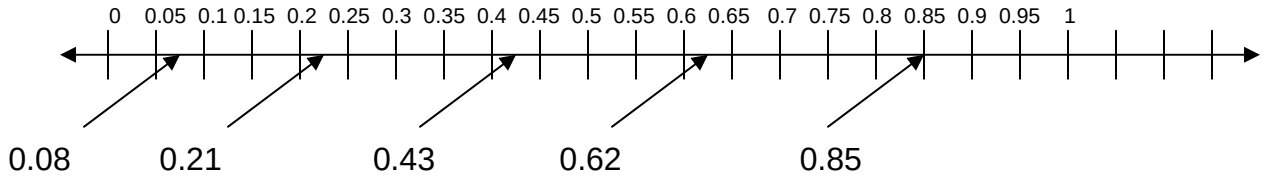
$-0.375 > -0.444$ therefore $-\frac{3}{8} > -\frac{4}{9}$

2. a. $-\frac{6}{4}, -\frac{1}{2}, 0, .1, \frac{1}{9}, 0.6, 1$

b. $-1.7, -0.75, -\frac{5}{12}, -\frac{1}{6}, \frac{1}{5}, 0.9$

c. $-0.71, \frac{1}{8}, 0.13, \frac{2}{3}, 0.67, \frac{3}{4}$

3.



4. a. $\frac{4}{7} - \frac{3}{7} = \frac{1}{7}$

b. $\frac{11}{14} + \frac{2}{14} = \frac{13}{14}$

c. $-\frac{3}{2} + \frac{1}{6} = \frac{-3^{x3}}{2^{x3}} + \frac{1}{6} = \frac{-9}{6} + \frac{1}{6} = \frac{-8}{6} = \frac{-4}{3}$ or $-1\frac{1}{3}$

d. $\frac{5}{6} - \frac{3}{8} = \frac{5^{x4}}{6^{x4}} - \frac{3^{x3}}{8^{x3}} = \frac{20}{24} - \frac{9}{24} = \frac{11}{24}$

e. 7.1 f. 4.9 g. –0.6 h. 0

i. $\left(+\frac{3}{5}\right)-\left(\frac{-4}{6}\right)=\frac{3}{5}+\frac{4}{6}=\frac{3^{\times 6}}{5^{\times 6}}+\frac{4^{\times 5}}{6^{\times 5}}=\frac{18}{30}+\frac{20}{30}=\frac{38}{30}=\frac{19}{15}\text{ or }1\frac{4}{15}$

j. $\left(-\frac{7}{8}\right)+\left(\frac{5}{-6}\right)=\frac{-7}{8}+\frac{-5}{6}=\frac{-7^{\times 3}}{8^{\times 3}}+\frac{-5^{\times 4}}{6^{\times 4}}=\frac{-21}{24}+\frac{-20}{24}=\frac{-41}{24}\text{ or }-1\frac{17}{24}$

k. $\left(-\frac{2}{3}\right)+\left(\frac{4}{5}\right)=\frac{-2}{3}+\frac{4}{5}=\frac{-2^{\times 5}}{3^{\times 5}}+\frac{4^{\times 3}}{5^{\times 3}}=\frac{-10}{15}+\frac{12}{15}=\frac{2}{15}$

l. $\left(-\frac{5}{14}\right)-\left(\frac{-7}{-8}\right)=\frac{-5}{14}-\frac{7}{8}=\frac{-5^{\times 4}}{14^{\times 4}}-\frac{7^{\times 7}}{8^{\times 7}}=\frac{-20}{56}-\frac{49}{56}=\frac{-69}{56}\text{ or }-1\frac{13}{56}$

5. a. $\left(-\frac{4}{7}\right)\left(-\frac{3}{7}\right)=\frac{-4}{7}\times\frac{-3}{7}=\frac{12}{49}$

b. $\frac{\left(\frac{11}{7}\right)}{+\left(\frac{2}{5}\right)}=\frac{11}{7}\div\frac{2}{5}=\frac{11}{7}\times\frac{5}{2}=\frac{55}{14}\text{ or }3\frac{13}{14}$

c. $\frac{-\cancel{3}^{1}}{2}\times\frac{1}{\cancel{6}^{2}}=\frac{-1}{2}\times\frac{1}{2}=\frac{-1}{4}\text{or}-\frac{1}{4}$

d. $\frac{5}{6}\div\frac{3}{8}=\frac{5}{6}\times\frac{8}{3}=\frac{5}{\cancel{6}^{3}}\times\frac{\cancel{8}^{4}}{3}=\frac{20}{9}\text{or}2\frac{2}{9}$

e. ≈ -0.82 f. ≈ 12.06 g. ≈ -0.87 h. 1.21

i. $\left(+\frac{3}{5}\right)\left(\frac{-4}{6}\right)=\frac{-12}{30}=\frac{-2}{5}\text{or}-\frac{2}{5}$

j. $\left(-\frac{7}{8}\right)\div\left(\frac{3}{-4}\right)=\frac{-7}{8}\times\frac{-4}{3}=\frac{-7}{\cancel{8}^{2}}\times\frac{-\cancel{4}^{1}}{3}=\frac{7}{6}$

k. $\frac{-2}{3}\times\frac{6}{5}=\frac{-2}{\cancel{3}^{1}}\times\frac{\cancel{6}^{2}}{5}=\frac{-4}{5}\text{or}-\frac{4}{5}$

l. $\left(-\frac{5}{14}\right)\div\left(\frac{-7}{-10}\right)=\frac{-5}{14}\times\frac{10}{7}=\frac{-5}{\cancel{14}^{7}}\times\frac{\cancel{10}^{5}}{7}=\frac{-25}{49}\text{or}-\frac{25}{49}$

Lesson 5

1. a. $\sqrt{9} = \pm 3$ b. $\sqrt{64} = \pm 8$ c. $\sqrt{1} = \pm 1$

   d. $\sqrt{121} = \pm 11$ e. $\sqrt{23} \approx \pm 4.8$ f. $\sqrt{58} \approx \pm 7.62$

   g. $\sqrt{2314} \approx \pm 48.1$ h. $\sqrt{-16} =$ undefined i. $\sqrt{.25} = \pm 0.5$

   j. $\sqrt{1.25} \approx \pm 1.12$ k. $\sqrt{\frac{1}{16}} = \frac{\sqrt{1}}{\sqrt{16}} = \pm\frac{1}{4}$ l. $\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \pm\frac{2}{3}$

2. a. $\sqrt{.09} = \pm 0.3$ b. $\sqrt{9} = \pm 3$ c. $\sqrt{900} = \pm 30$

   d. $\sqrt{90000} = \pm 300$ e. $\sqrt{9000000} = \pm 3000$ f. $\sqrt{900000000} = \pm 30000$

3. a.

$$a^2 + b^2 = c^2$$
$$(8)^2 + (13)^2 = c^2$$
$$64 + 169 = c^2$$
$$233 = c^2$$
$$\sqrt{233} = \sqrt{c^2}$$
$$15.3 \approx c$$

   b.

$$a^2 + b^2 = c^2$$
$$(11.2)^2 + b^2 = (15.8)^2$$
$$125.44 + b^2 = 249.64$$
$$125.44 - 125.44 + b^2 = 249.64 - 125.44$$
$$b^2 = 124.2$$
$$\sqrt{b^2} = \sqrt{124.2}$$
$$b \approx 11.1$$

4. a.

$$a^2 + b^2 = c^2$$
$$(6)^2 + (8)^2 = c^2$$
$$36 + 64 = c^2$$
$$100 = c^2$$
$$\sqrt{100} = \sqrt{c^2}$$
$$10 = c$$

   b.

$$a^2 + b^2 = c^2$$
$$(1.2)^2 + (3.7)^2 = c^2$$
$$1.44 + 13.69 = c^2$$
$$15.13 = c^2$$
$$\sqrt{15.13} = \sqrt{c^2}$$
$$3.9 \approx c$$

