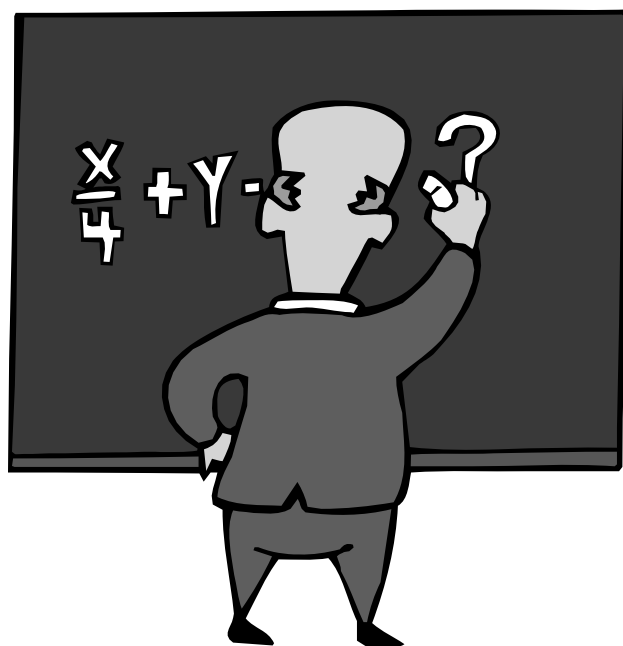


# ***Polynomials***



**Lesson 6**

## Lesson Six Concepts

### Overall Expectations

- Simplify numerical and polynomial expressions in one variable, and solve simple first-degree equations.

### Specific Expectations

- Add and subtract polynomials involving the same variable up to degree three;
- Multiply a polynomial by a monomial involving the same variable to give results up to degree three;
- Solve first-degree equations with non-fractional coefficients, using a variety of tools and strategies;
- Substitute into algebraic equations and solve for one variable in the first degree.

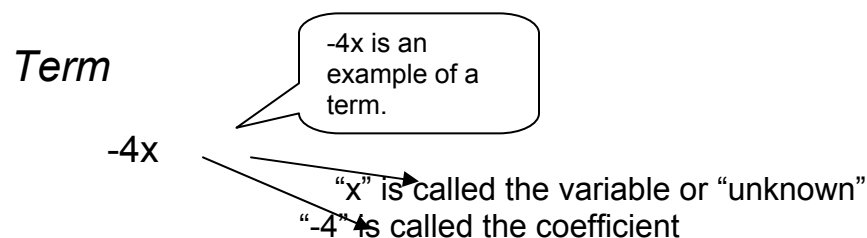
## Polynomials

**Polynomials** are a mathematical expression with one or more terms, in which the exponents are whole numbers and the coefficients are real numbers.

**Monomial** is a polynomial with one term; e.g.  $-8$  and  $3x^2$ .

**Binomial** is a polynomial with two terms; e.g.  $-4x + 7$  and  $y^3 - 12$ .

**Trinomial** is a polynomial with three terms; e.g.  $4x^2 - 3x + 9$  and  $-w^3 + 7w + 2$ .



## Adding and Subtracting Polynomials

To add and/or subtract polynomials the terms **must** be “like terms”.

**Like terms** are terms that have the same variables with the same value of exponent for each variable.

### Example

Underline the terms that are like terms.

a)  $-3x$ ,  $x$ ,  $7x^2$ ,  $2y$ ,  $12x$

b)  $7xy^3$ ,  $-5x^3y$ ,  $-y^3x$ ,  $5xy^2$

**Solution**

**Example**

Simplify.

a)  $(5x + 1) + (3x + 7)$

b)  $(-a + 4) - (2a - 3)$

c)  $(4w^2 + 6w - 8) - (9w^2 + 2w + 5)$

**Solutions**

a)  $(5x + 1) + (3x + 7)$

The first set of brackets can be removed if there is no value or negative sign in front of the brackets.

If this is a plus sign the second set of brackets can also be removed.

**Distributive Law:**

$$a(b+c) = ab + ac$$

$a(b-c) = ab - ac$   
where a, b, and c can be any real number

$$\begin{aligned}(5x + 1) + (3x + 7) &= 5x + 1 + 3x + 7 \\ &= 5x + 3x + 1 + 7 \\ &= 8x + 8\end{aligned}$$

Group like terms and add the like terms together.

b)  $(-a + 4) - (2a - 3) = -a + 4 - 2a + 3$

$$\begin{aligned}&= -a - 2a + 4 + 3 \\ &= -3a + 7\end{aligned}$$

Because this sign is negative the distributive law is used.

$$\begin{aligned}\text{c) } (4w^2 + 6w - 8) - (9w^2 + 2w + 5) &= 4w^2 + 6w - 8 - 9w^2 - 2w - 5 \\ &= 4w^2 - 9w^2 + 6w - 2w - 8 - 5 \\ &= -5w^2 + 4w - 13\end{aligned}$$

**Support Questions**

1. State the like terms in each group.

a)  $3x, 5y, 5z, x^2, -x, 3w, 3v$

b)  $4x, 3y^2, 4z, 2y, y^2, 4w$

2. Simplify.

a)  $-3x + 3 + 4x + 2$

b)  $3 - 5n - 6n - 2$

c)  $(8a^2 + 2a - 3) + (-5a^2 + 4a + 7)$

d)  $(-6x^2 + 5x + 2) - (4x^2 + 5 - 2x)$

e)  $(3 - 2m - n^2) + (7 - 6m + n^2)$

f)  $(2 + 6x^2) - (7 - 3x^2)$

g)  $(5 - 6w^2) - (3 - w^2)$

h)  $(5x^2 - 3x) + (-4x + 5x^2)$

3. Simplify. Then determine the value of the polynomial when  $n = 2$  and when  $n = -1$ .

a)  $(-3n + 4) + (-2n - 3)$

b)  $(3n^2 - 7n + 2) + (-2n^2 + 6n + 3)$

## Multiplying and Dividing Polynomials

To multiply and/or divide polynomials the terms **do not have to** be “like terms”.

### Example

Simplify.

a)  $(4x^2)(-3x)$

b)  $\frac{16x^3y^3}{-8xy}$

c)  $(-2w^2y)(5wy)$

d)  $100x^2yz^3 \div 25xyz^2$

### Solutions

a)  $(4x^2)(-3x) = -3 \cdot 4 \cdot x \cdot x \cdot x$   
 $= -12x^3$

or

$$= (4)(-3)(x^2)(x^1)$$

$$= -12x^{2+1}$$

$$= -12x^3$$

Use the multiplication of exponents rule

b)  $\frac{16x^3y^3}{-8xy} = \frac{16 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y}{-8 \cdot x \cdot y}$

$$= \frac{16 \cdot \overset{-2}{x \cdot x \cdot x} \cdot \cancel{y \cdot y \cdot y}}{-8 \cdot \cancel{x} \cdot \cancel{y}}$$

Divide common factors

$$= \frac{-2 \cdot x \cdot x \cdot y \cdot y}{1}$$

$$= -2x^2y^2$$

or

$$= \frac{16}{-8} x^{3-1} y^{3-1}$$

$$= -2x^2y^2$$

c)  $(-2w^2y)(5wy) = -10w \cdot w \cdot w \cdot y \cdot y$

$$= -10w^3y^2$$

$$\begin{aligned} \text{d) } 100x^2yz^3 \div 25xyz^2 &= 4x^{2-1}y^{1-1}z^{3-2} \\ &= 4xz \end{aligned}$$

Any base to the zero exponent equals 1



## Support Questions

4. Determine the product or quotient.

a)  $(3m^2)(-7m)$

b)  $(12n^2) \div (-6n)$

c)  $\frac{7ab^2}{-3ba}$

d)  $(-3x)(4xy)(-2y)$

e)  $\frac{-28a^3}{-7a}$

f)  $(8z^3)(7)$

g)  $(-4a^3b) \div (2ab)$

h)  $(5x)(4x)$

i)  $\left(-\frac{3}{5}ab^2\right)\left(-\frac{10}{9}a^2\right)$

5. Simplify. Then determine the value of the polynomial when  $a = 2$  and when  $b = -1$ .

a)  $(2ab)^2$

b)  $(-3ab)(-5ab^2)$

## Multiplying Polynomials with a Monomial

This process requires the use of the distributive law.

### Example

Expand.

a)  $2(3x - 4)$

b)  $2x(-3x - 5)$

c)  $(-3p^2)(3 - 5p)$

### Solutions

a)  $2(3x - 4) = 6x - 8$

$(2)(3x) = 6x$  and  $(2)(-4) = -8$

$$\begin{aligned} \text{b) } 2x(-3x - 5) &= 2x(-3x) + 2x(-5) \\ &= -6x^2 - 10x \end{aligned}$$

$$\begin{aligned} \text{c) } (-3p^2)(3 - 5p) &= -3p^2(3) - 3p^2(-5p) \\ &= -9p^2 + 15p^3 \end{aligned}$$



### Support Questions

6. Expand.

$$\text{a) } x(3x - 9)$$

$$\text{b) } (-4n)(2n - 3)$$

$$\text{c) } b(2b^2 - 3b + 1)$$

$$\text{d) } (-x)(x - 2)$$

$$\text{e) } (-4m)(m^2 - m)$$



### Key Question #6



1. State the like terms in each group. (2 marks)

$$\text{a) } 4w, 5w^2, 5z, x^2, -x, 3w, 3v$$

$$\text{b) } 4x^2, -3x^2, 4z, 2y, y^2, 4w$$

2. Simplify. (5 marks)

$$\text{a) } -12t + 2 + 7t + 5$$

$$\text{b) } 6 - 4r - 5r - 1$$

$$\text{c) } 4n^2 + 4n + 1 - 7n^2 - 2n - 6$$

$$\text{d) } -4x^2 + 3x + 1 - 3x^2 - 7 + x$$

$$\text{e) } 2 + 5m + n^2 - 5 + 1m - 4n^2$$

3. Simplify. (3 marks)

$$\text{a) } (3 + 7x^2) + (4 - 6x^2)$$

$$\text{b) } (1 - 7w^2) - (-4 + 8w^2)$$

$$\text{c) } (6x^2 - 7x) + (-2x + 9x^2)$$

4. Simplify. Then determine the value of the polynomial when  $n = -3$  and when  $n = 2$ . (4 marks)

$$\text{a) } (5n^2 + 3n - 4) + (-3n^2 + 4n - 1)$$

$$\text{b) } (7n^2 - 5n - 2) - (-n^2 + 6n + 8)$$

5. Expand. (5 marks)

$$\text{a) } 2w(3w + 4)$$

$$\text{b) } -4n(5n - 9)$$

$$\text{c) } c(7c^2 - 5c - 6)$$

$$\text{d) } (-h)(h + 6)$$

$$\text{e) } (-6x)(-x^2 + x)$$



### Key Question #6 (continued)

6. Determine the product or quotient. (9 marks)

a)  $(2x^2)(-4x)$

b)  $(36m^2) \div (-9m)$

c)  $\frac{-11a^3b^3}{5b^3a}$

d)  $(8x^3)(2y)(-y)$

e)  $\frac{-75w^3}{-15w^2}$

f)  $(3d^2)(9d)$

g)  $(-12a^2b^3) \div (3ab^3)$

h)  $(7k)(-k)$

i)  $\left(\frac{4}{6}a^2b^2\right)\left(-\frac{12}{8}ab\right)$



7. Simplify. Then determine the value of the polynomial when  $a = -1$  and when  $b = 3$ . (4 marks)

a)  $(3ab)^3$

b)  $(-4ab)^2$

8. When the terms of a polynomial in  $x$  are arranged from the highest to the lowest powers of  $x$ , the polynomial is in descending order. Simplify the following polynomial in descending order then evaluate for  $n = 1$ . (3 marks)

$$6 - (2n^2 + n) - (5n + n^2 - 6) - (4n + 2n^2 - 11)$$



9. When are the sum, difference, product and quotient of two monomials also a monomial? (3 marks)