Area

Lesson 16

Lesson Sixteen Concepts

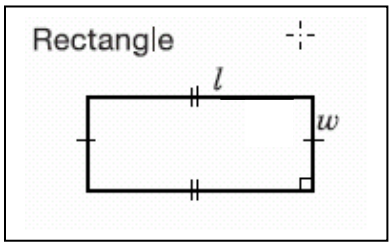
- Introduction to area and surface area
- Radius and diameter
- Calculations using pi (π)
- Solving area and surface area questions using formulas and substitution

Area and Surface Area

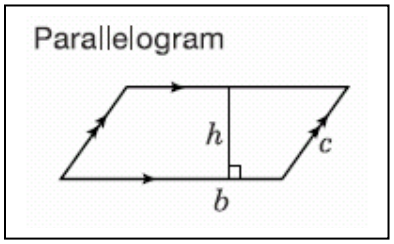
Area

Area is the number of square units needed to cover a region.

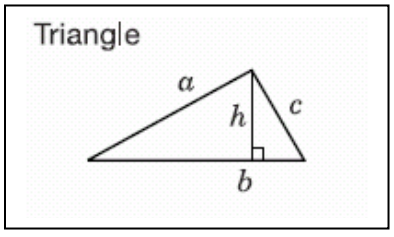
Formulas to be used to calculate area.



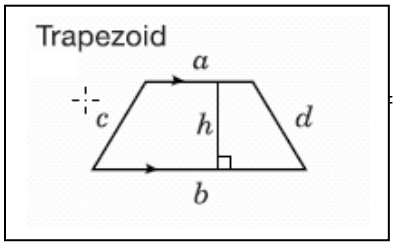
$A = lw$



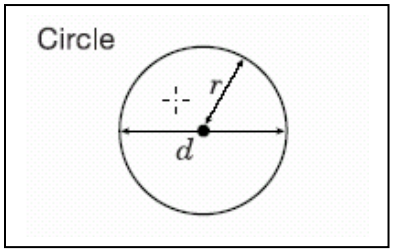
$A = bh$



$A = \frac{bh}{2}$



$\frac{(a+b)h}{2}$

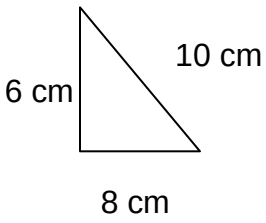


$A = \pi r^2$

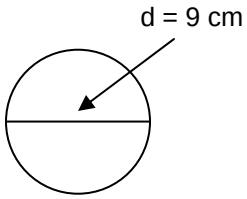
Example

Find the area of each shape.

a)



b)



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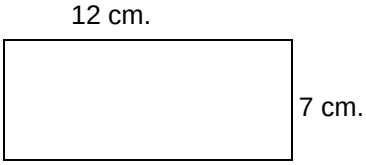
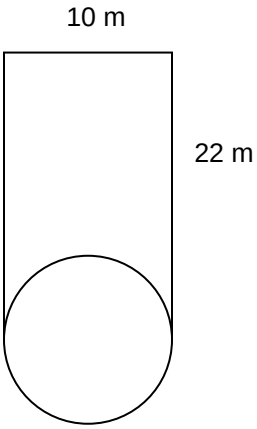
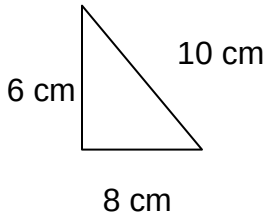


d)



Solution

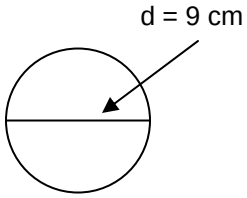
a)



$$A = \frac{bh}{2}$$
$$A = \frac{(6)(8)}{2}$$
$$A = \frac{48}{2}$$
$$A = 24cm^2$$

$8cm^1 \times 6cm^1 = 48cm^2$. The cm^2 comes from the multiplying of the cm together.

b)



$$r = \frac{d}{2}$$
$$r = \frac{9}{2}$$
$$r = 4.5$$

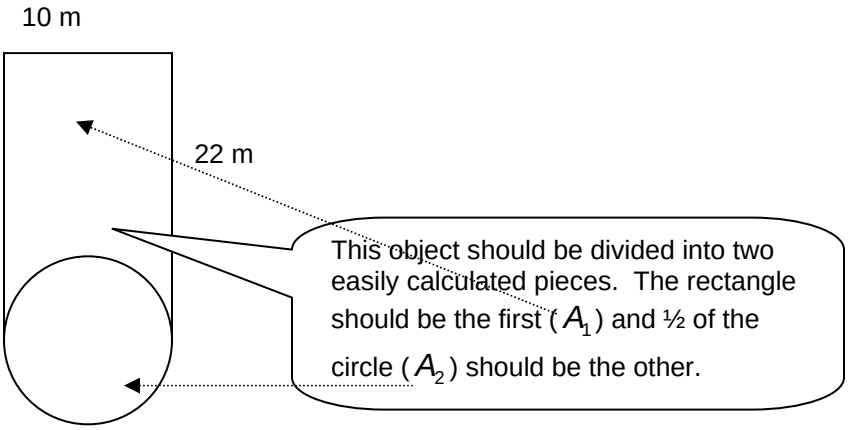
$$A = \pi r^2$$
$$A = (3.14)(4.5)^2$$
$$A \approx 63.59cm^2$$

Remember to use BEDMAS. Brackets before multiplication.

c)

$$A = lw$$
$$A = (12)(7)$$
$$A = 84cm^2$$

d)



Total area = area of rectangle + area of ½ circle.

$$A_t = A_1 + \frac{A_2}{2}$$

$$A_t = lw + \frac{\pi r^2}{2}$$

$$A_t = (10)(22) + \frac{(3.14)(5)^2}{2}$$

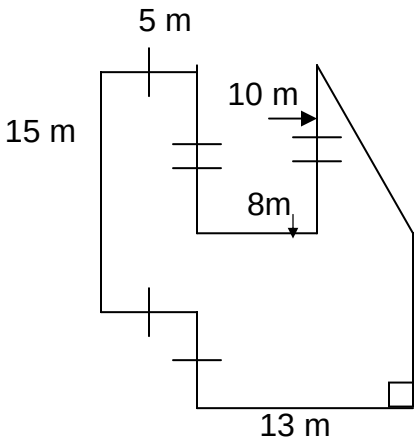
$$A_t = 220 + \frac{78.5}{2}$$

$$A_t = 220 + 39.25$$

$$A_t = 259.25m^2$$

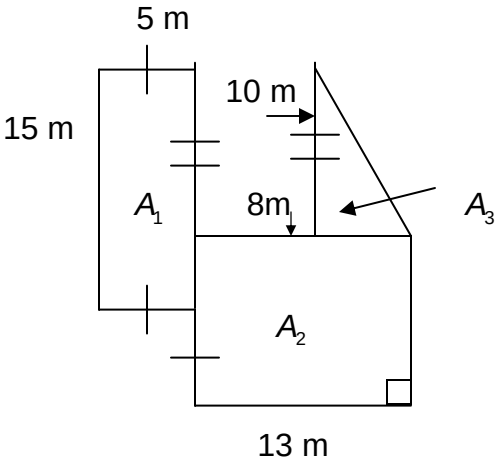
Example

Find the area.



Solution

Find the area.



Need to divide the object into easily calculated shapes.

A_t = Total Area

$A_t = A_1 + A_2 + A_3$

Area of Rectangle One

$A_1 = lw$

$A_1 = (5)(15)$

$A_1 = 75cm^2$

Area of Rectangle Two

$A_2 = lw$

$A_2 = (13)(10)$

$A_3 = 130cm^2$

By deductive reasoning the length of the side of the rectangle is 20–10 =10 cm.

Area of Triangle

$A_3 = \frac{bh}{2}$

$A_3 = \frac{(5)(10)}{2}$

$A_3 = \frac{50}{2}$

$A_3 = 25cm^2$

Total Area

$A_t = A_1 + A_2 + A_3$

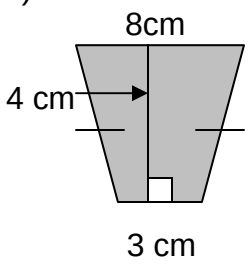
$A_t = 75 + 130 + 25$

$A_t = 230cm^2$

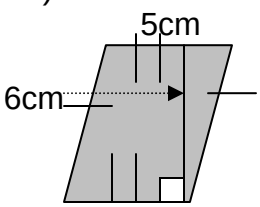
Support Questions

1. Calculate the area for each of the following objects.

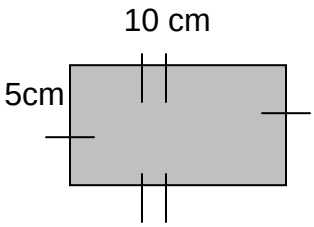
a)



b)



c)



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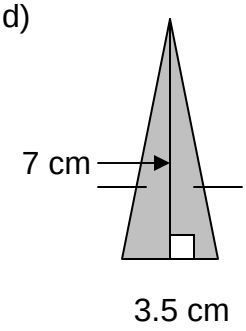

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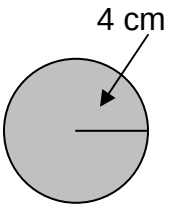
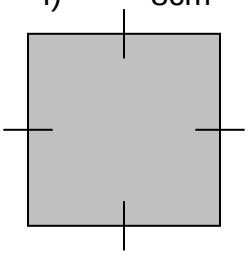
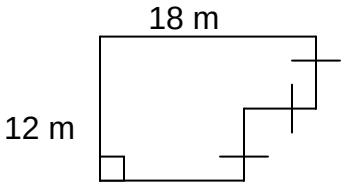


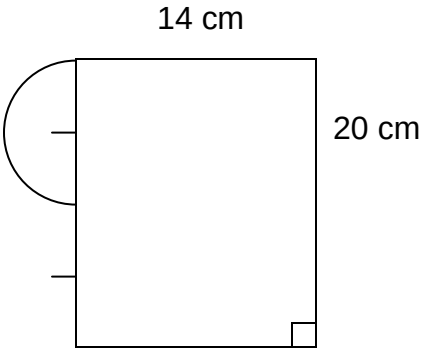
f)



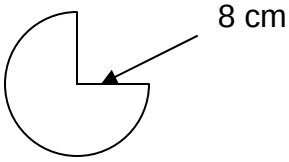
g)



h)



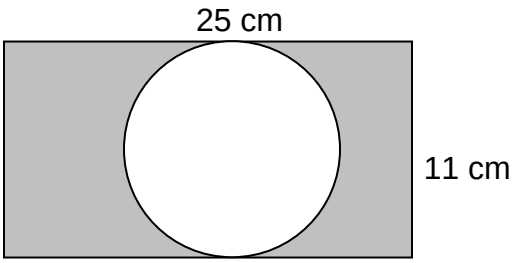
i)



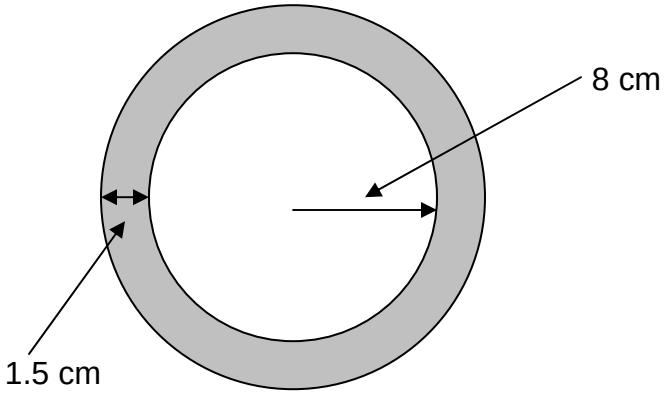
Support Questions

2. Calculate the shaded area for each of the following objects.

a)



b)

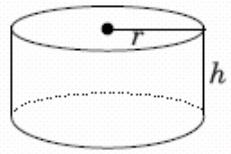


Surface Area

Surface Area is a measure of the area on the surface of a three-dimensional object.

Formulas to be used to calculate surface area.

Cylinder



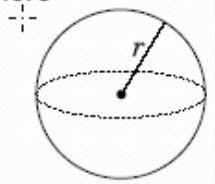
$S.A._{top} = \pi r^2$

$S.A._{base} = \pi r^2$

$S.A._{side} = 2\pi rh$

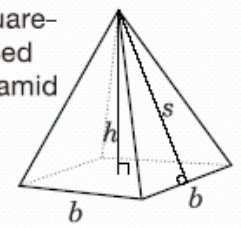
$S.A._{total} = 2\pi r^2 + 2\pi rh$

Sphere



$S.A. = 4\pi r^2$

Square-based pyramid



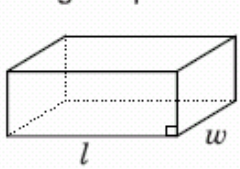
$S.A._{triangle} = \frac{bs}{2}$

$S.A._{base} = b^2$

$S.A._{total} = 4\left(\frac{bs}{2}\right) + b^2$

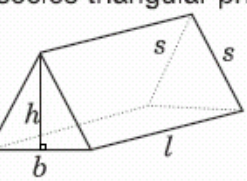
$S.A._{total} = 2bs + b^2$

Rectangular prism

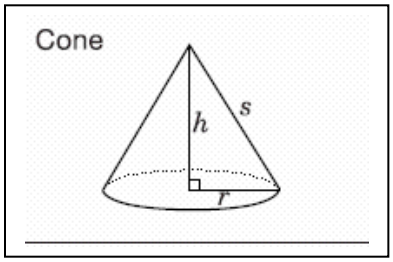


$S.A. = 2(wh + lw + lh)$

Isosceles triangular prism



$S.A. = 2\left(\frac{bh}{2}\right) + 2ls + lb$



$S.A._{cone} = \pi rs$

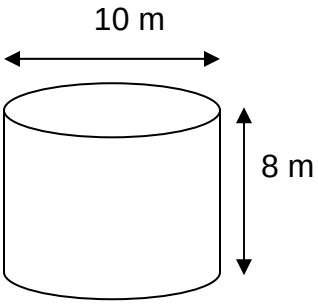
$S.A._{base} = \pi r^2$

$S.A._{total} = \pi r(s + r)$

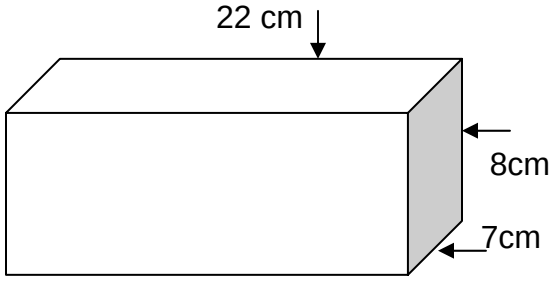
Example

Find the surface area of each figure.

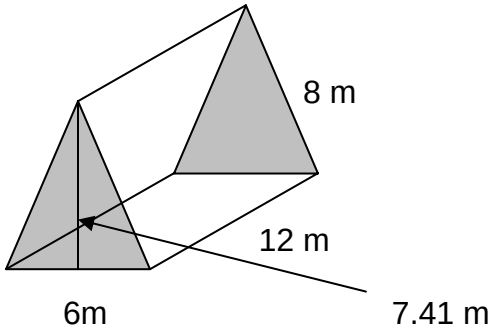
a)



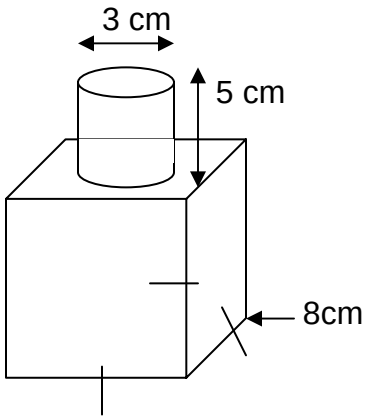
b)



c)



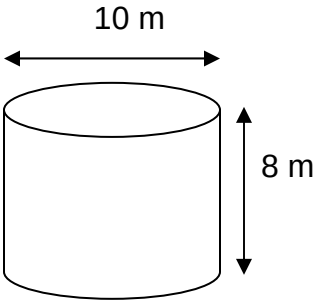
d)



Solution

Find the surface area of each figure.

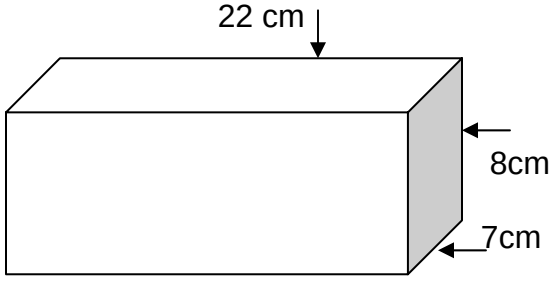
a)



$$S.A = 2\pi r^2 + 2\pi rh$$
$$S.A = 2(3.14)(5)^2 + 2\pi(5)(8)$$
$$S.A = 157 + 251.2$$
$$S.A = 408.2m^2$$

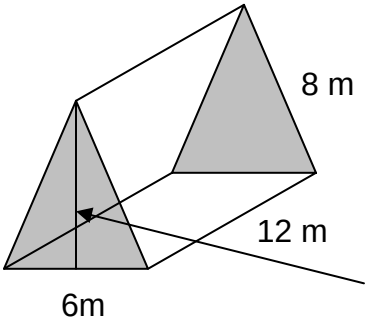
This is still area so the units are squared.

b)



$$S.A. = 2(wh + lw + lh)$$
$$S.A. = 2\left[(7)(8) + (22)(7) + (22)(8)\right]$$
$$S.A. = 2\left(56 + 154 + 176\right)$$
$$S.A. = 2(386)$$
$$S.A. = 772cm^2$$

c)



There are two $\frac{bh}{2}$'s because of the triangles on each end.

$$S.A. = 2\left(\frac{bh}{2}\right) + 2ls + lb$$

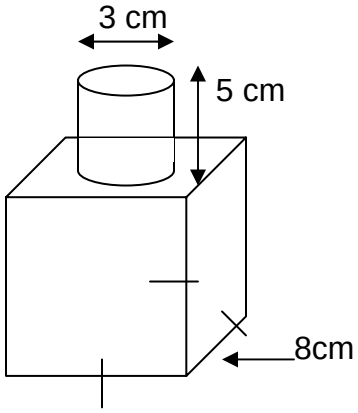
There are 2 ls's because two of the rectangles that make the sides of the prism have the same dimensions

$$S.A. = 2\left(\frac{(6)(7.41)}{2}\right) + 2(12)(8) + (12)(6)$$

$$S.A. = 44.46 + 192 + 72$$

$$S.A. = 308.46m^2$$

d)



The top of the cylinder would take the place of the circle missing on the top of the cube.

There is no bottom of the cylinder.

Therefore all that is needed to be calculated is the surface area of the cube and the side of the cylinder.

$$S.A. = 6(b^2) + 2\pi rh$$

$$S.A. = 6(8)^2 + 2(3.14)(1.5)(5)$$

$$S.A. = 384 + 47.1$$

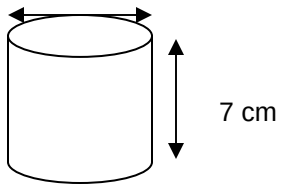
$$S.A. = 431.1\,\text{cm}^2$$

Since $l = b$ and $w = b$ and $h = b$ then S.A.=2(wh+lw+lh)
$= 2(bb+bb+bb)$
$= 2(3b^2)$
$= 6b^2$

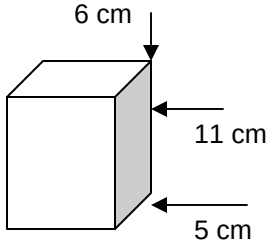
Support Questions

3. Calculate the surface area for each of the following objects.

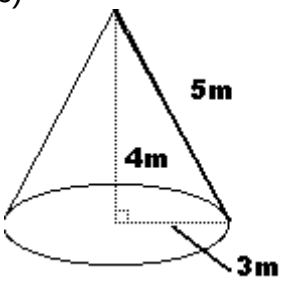
a)



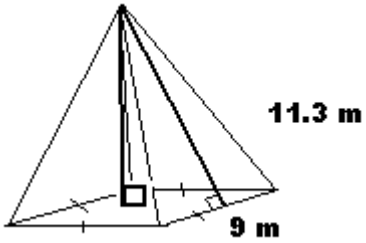
b)



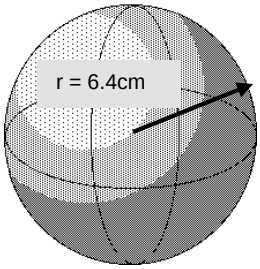
c)



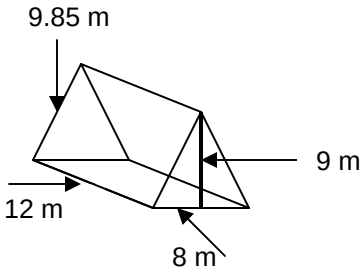
d)



e)

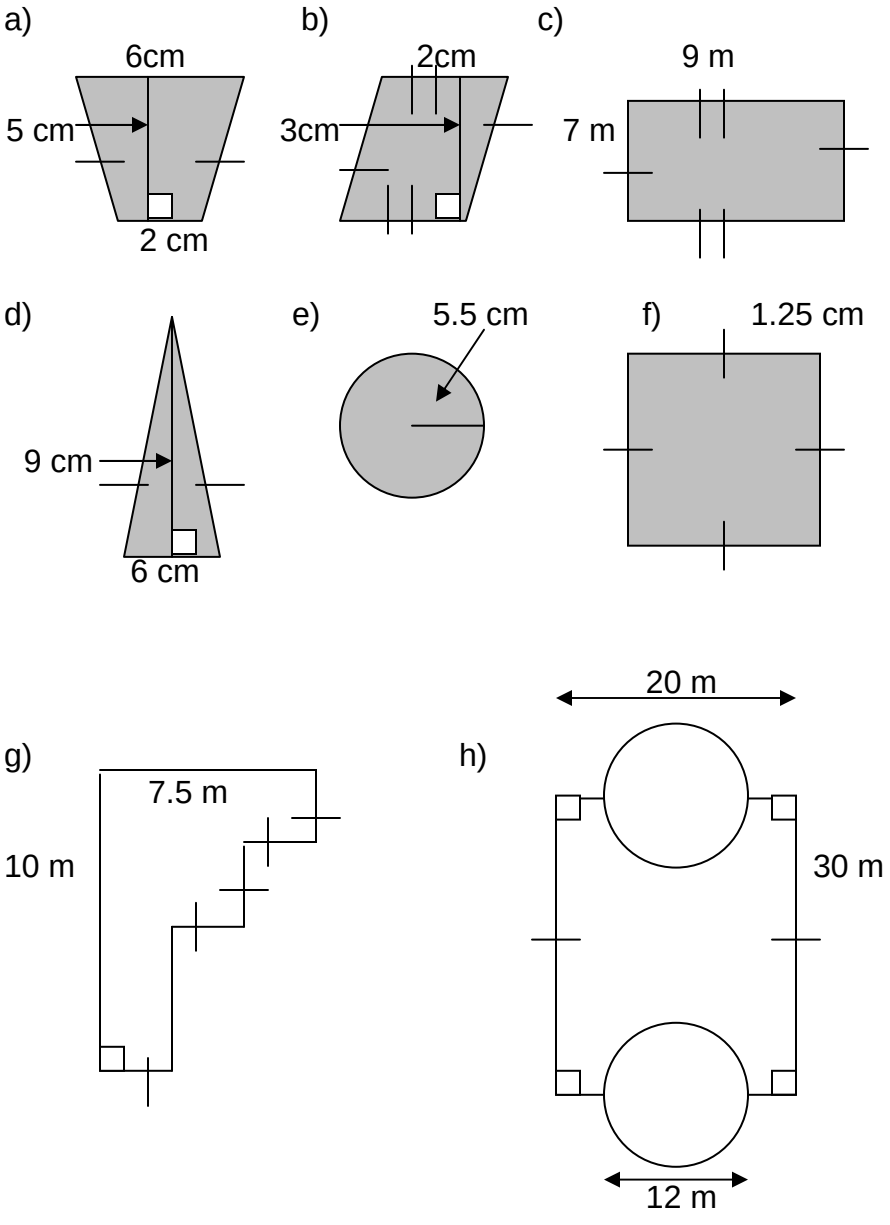


f)



Key Question #16

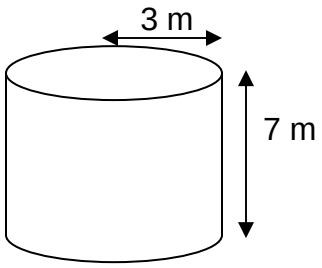
1. Calculate the area for each of the following objects. (10 marks)



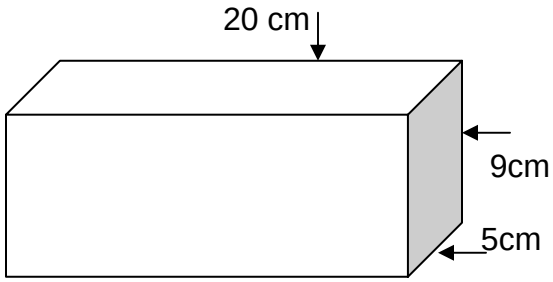
Key Question #16 (continued)

2. Calculate the surface area for each of the following objects. (8 marks)

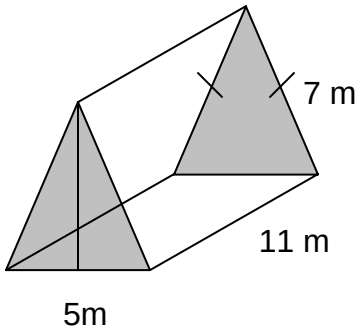
a)



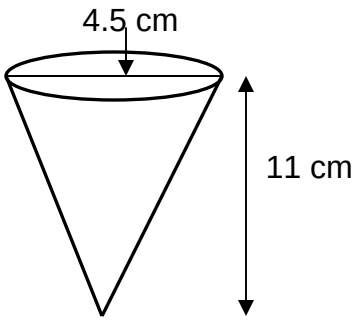
b)



c)



d)

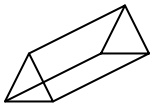


Key Question #16 (continued)

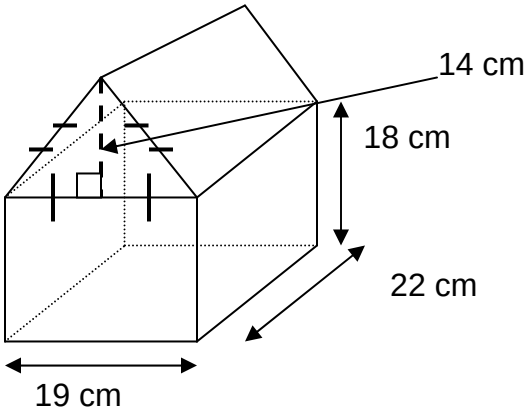
3. Determine the minimum amount of packaging needed to completely cover a triangular prism Toblerone bar with these dimensions:

   length 22.5 cm; triangular face has edges 4.5 cm and height 4.0.

   Express the surface area to the nearest square centimetre. (3 marks)



4. Calculate the surface area of the solid below. (4 marks)



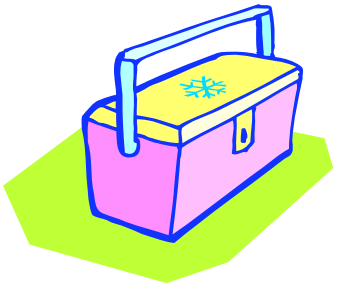
5. Look at the formula for the volume of a rectangular prism. How does the surface area change in each case? (3 marks)

   a) The length is doubled.
   b) Both the length and the width are doubled.
   c) All the length, width, and height are doubled.

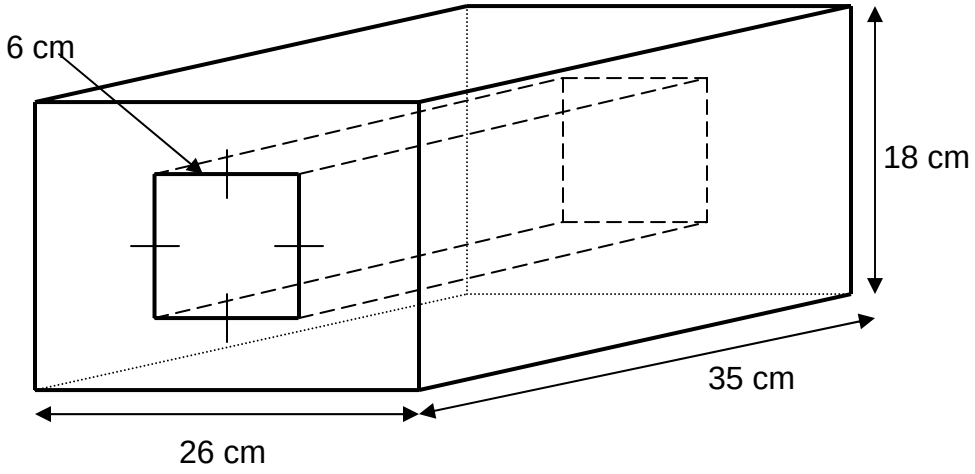
Key Question #16 (continued)

6. A cooler has a 60-L capacity. Its internal length is 60 cm and its internal width is 35 cm. Determine the internal height and the internal surface area of the cooler. (3 marks)

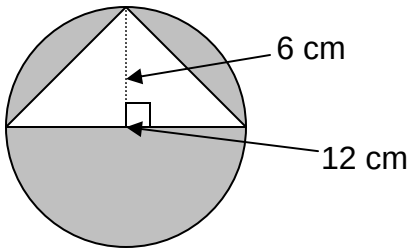
Hint 1 cm^3=1 ml



7. Calculate the surface area of the solid. (3 marks)



8. Calculate the shaded area. (3 marks)



Volume

Lesson 17

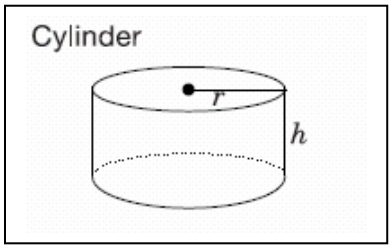
Lesson Seventeen Concepts

- Introduction to volume
- Radius and diameter
- Calculations using pi (π)
- Solving volume questions using formulas and substitution

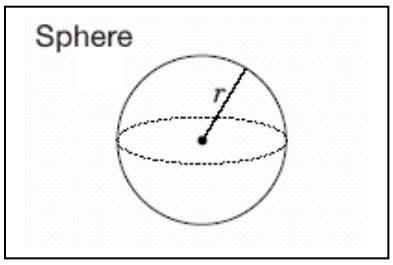
Volume

Volume is the amount of space occupied by a 3-dimensional object.

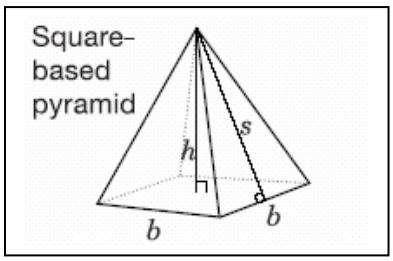
Formulas to be used to calculate volume.



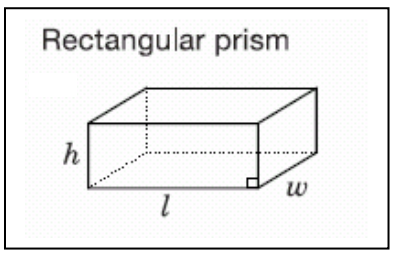
$V = \pi r^2 h$



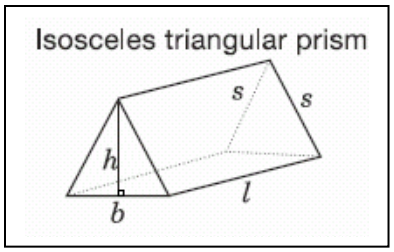
$V = \frac{4\pi r^3}{3}$



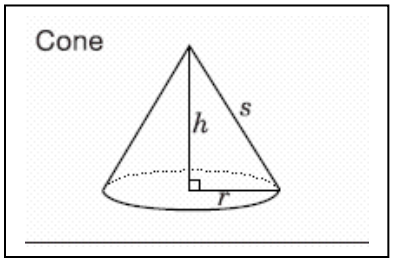
$V = \frac{b^2 h}{3}$



$V = lwh$



$$V = \frac{bhl}{2}$$

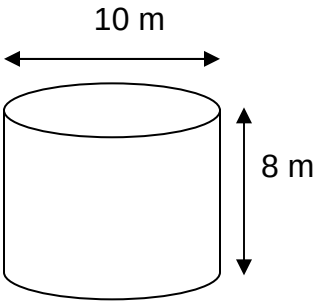


$$V = \frac{\pi r^2 h}{3}$$

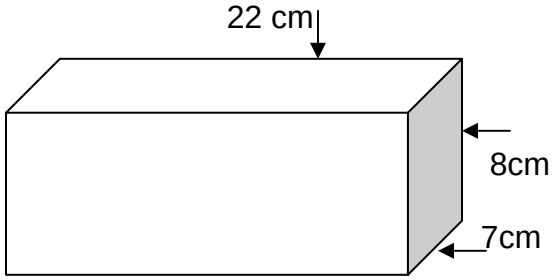
Example

Find the volume of each figure.

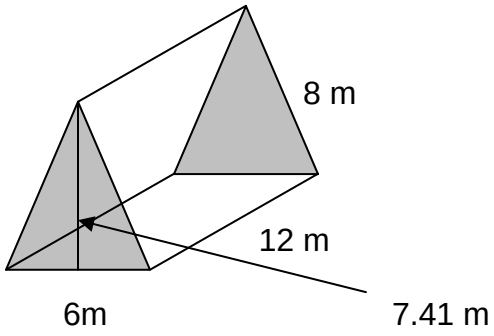
a)



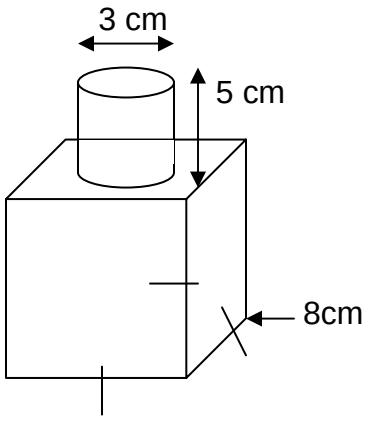
b)



c)



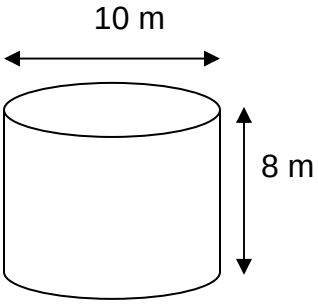
d)



Solution

Find the volume of each figure.

a)



$V = \pi r^2 h$

$V = (3.14)(5)^2 8$

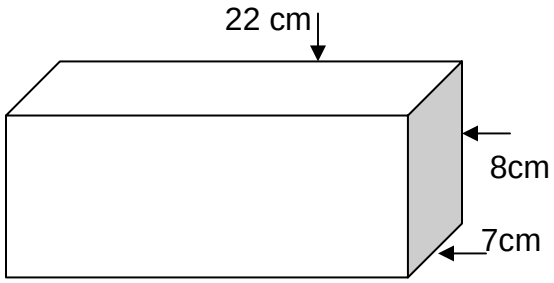
$V = 628m^3$

$5m^1 \times 5m^1 \times 8m^1 = 628m^{1+1+1}$

$= 628m^3$

Volume is always measured in

b)

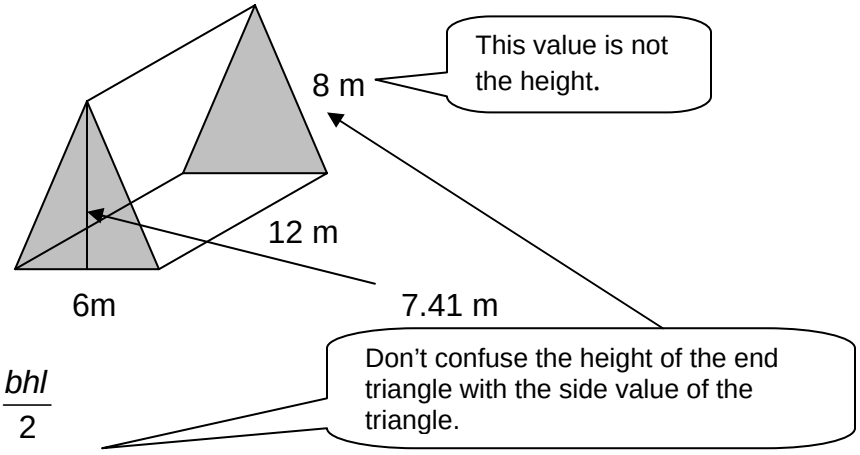


$V = lwh$

$V = (22)(7)(8)$

$V = 1232cm^3$

c)

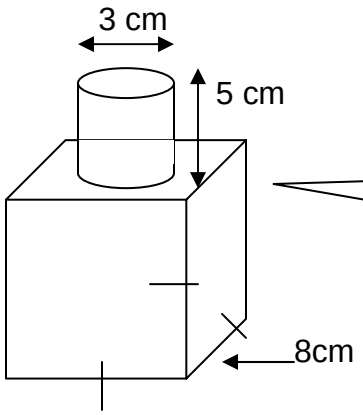


$V = \frac{bhl}{2}$

$V = \frac{(6)(7.41)(12)}{2}$

$V = 266.76m^3$

d)



Break this object into two pieces. One is the *cylinder* on top and the other is the *cube* on the bottom.

$V_{cylinder} = \pi r^2 h$

$V_{cylinder} = (3.14)(1.5)^2(8)$

$V_{cylinder} = 56.52cm^3$

$V_{cube} = lwh$

$V_{cube} = (8)(8)(8)$

$V_{cube} = 512cm^3$

Since l, w, and h are all the same value we could use the formula $V = s^3$; where s = side.

$V = (8)^3$

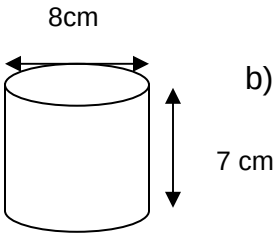
$V = 512cm^3$

Volume Total = Volume of Cylinder + Volume of Cube
= 56.52 cm^3 + 512 cm^3
= 568.52 cm^3

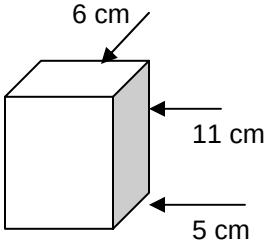
Support Questions

1. Calculate the volume for each of the following objects.

a)

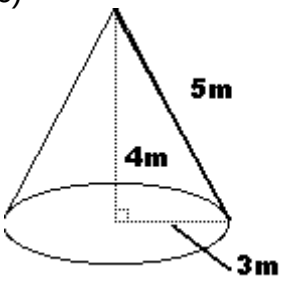


b)

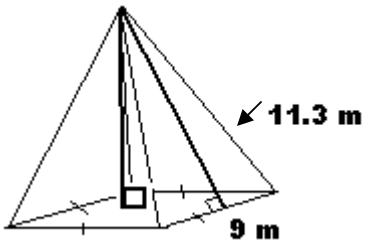


Support Questions (continued)

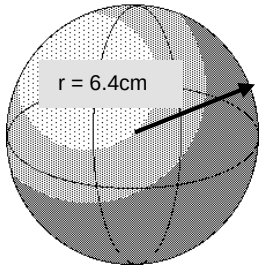
c)



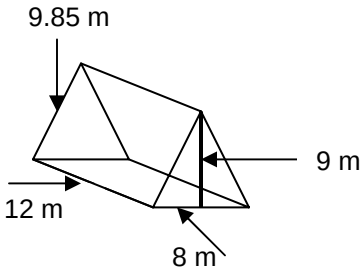
d)



e)



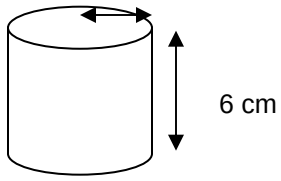
f)



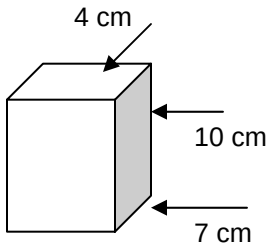
Key Question #17

1. Calculate the volume for each of the following objects. (6 marks)

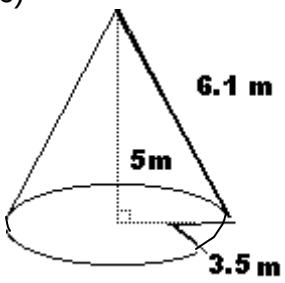
a)



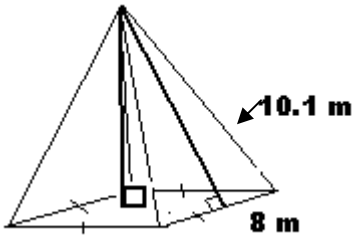
b)



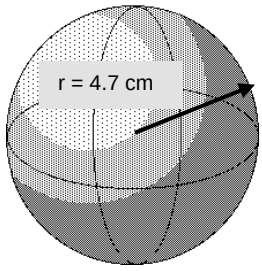
c)



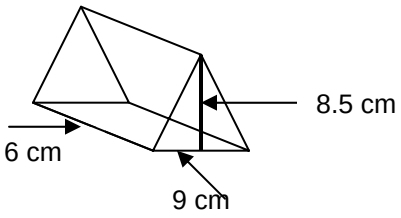
d)



e)



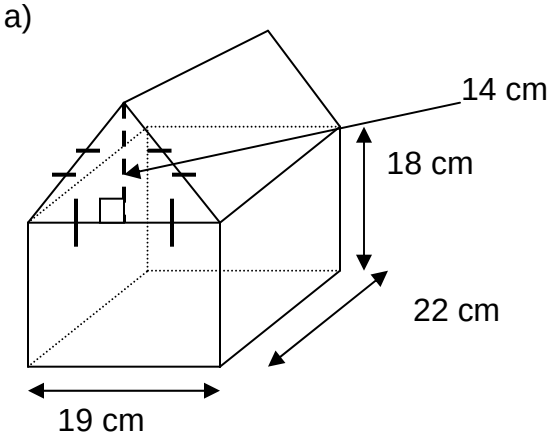
f)



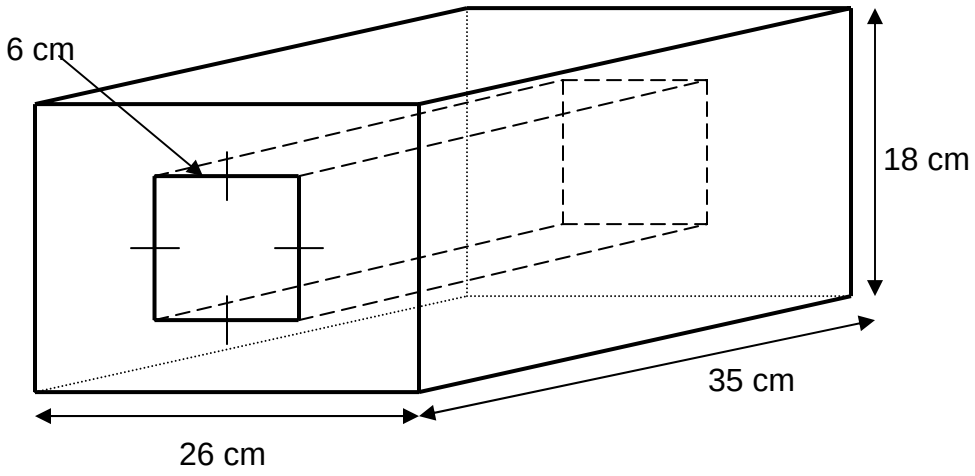
Key Question #17 (continued)

2. Calculate the volume of each solid. (8 marks)

a)



b)



Key Question #17 (continued)

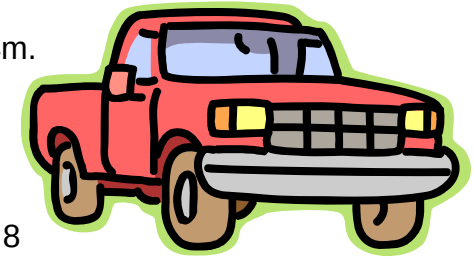
3. A cone has a height of 10 cm and a volume of $350\,cm^3$. What is the radius of the cone? (3 marks)

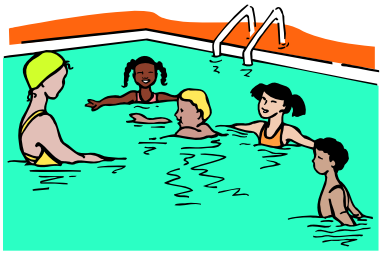
4. Look at the formula for the volume of a rectangular prism. How does the volume of a rectangular prism change in each case? (3 marks)

   a) The length is doubled.
   b) Both the length and width are doubled.
   c) All the length, width, and height are doubled.

5. A storage bin is a rectangular prism. Its volume is $300\ cm^3$. The width of the prism is one-third its length. Its height is two-thirds its length. Determine the dimension of the bin. (4 marks)

6. The box of a truck has dimensions 1 m by 2 m by 4m. Explain how this truck was able to carry $9\ m^3$ of sand. (3 marks)





7. A circular swimming pool has a diameter of 8 m and a depth of 2 m. What is the volume of the swimming pool? (3 marks)

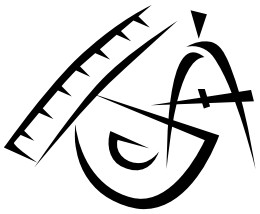
Angle Geometry

Lesson 18

Lesson Eighteen Concepts

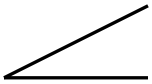
- Introduction to angle geometry
- Angle types
- Angle properties
- Angle properties involving parallel lines and transversals
- Finding unknown angles with justification

Angle Geometry

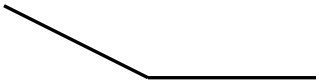


Angle Properties

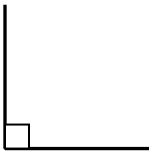
Acute Angle is an angle that is more than 0° and less than 90°.



Obtuse Angle is an angle greater than 90° and less than 180°.

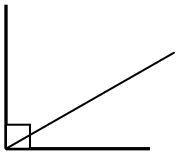


Right Angle is an angle that is 90°.

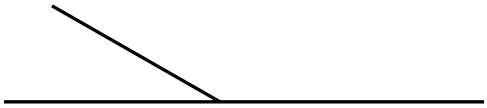


Straight Angle is an angle that is 180°.

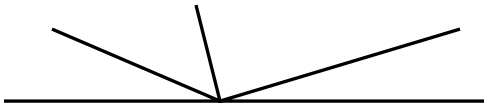
Complementary Angles are angles that add to 90°.



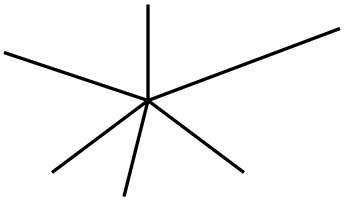
Supplementary Angles are angles that add to 180°.



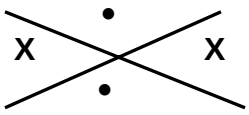
Angles on a line add to 180°.



Angles at a point add to 360°.



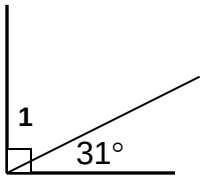
Vertically opposite angles are equal.



Example

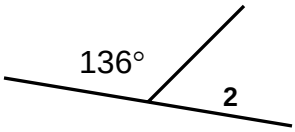
Find the measure of each required angle and give a reason for that answer.

a)



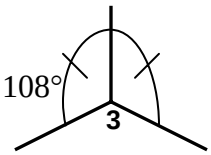
∠1 = _____ Reason_____

b)



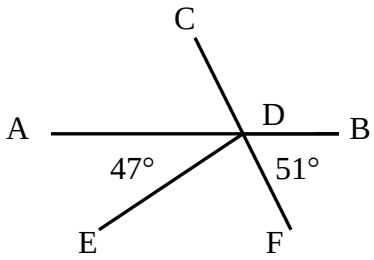
∠2 = _____ Reason_____

c)



∠3 = _____ Reason_____

d)



∠EDF = _____ Reason_____

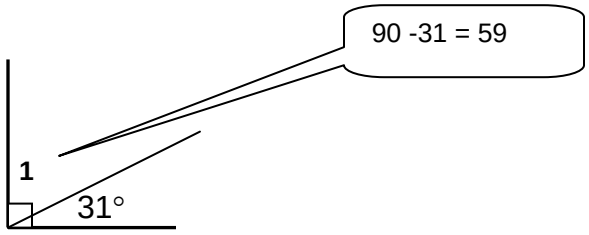
∠CDB = _____ Reason_____

∠ADC = _____ Reason_____

Solution

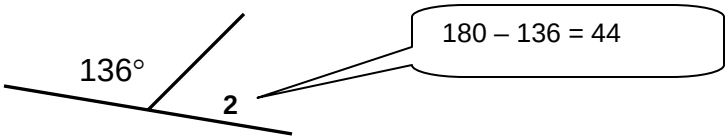
Find the measure of each required angle and give a reason for that answer.

a)



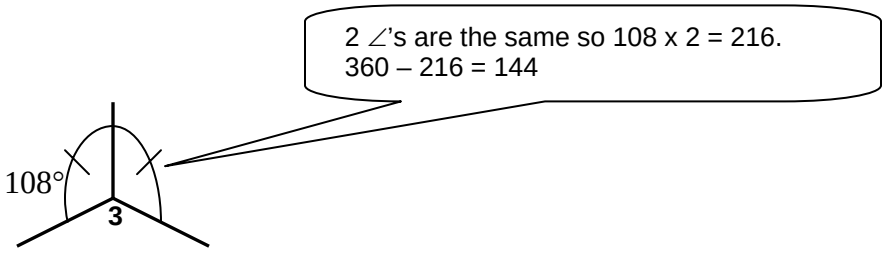
∠1 = _*59°*_ Reason *Complementary Angles*

b)



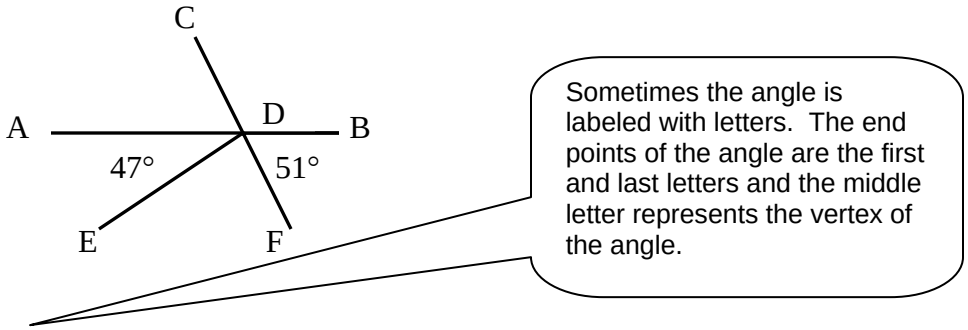
∠2 = *44°* Reason *Supplementary Angles*

c)



∠3 = *144°* Reason *Angles at a point add to 360°*

d)



∠EDF = *82°* Reason *Angles on a line add to 180°*

∠CDB = *129°* Reason *Supplementary Angles*

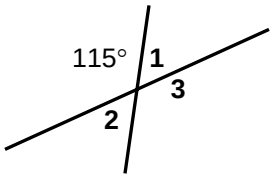
Vertically opposite ∠FDB.

∠ADC = *51°* Reason *Vertically Opposite Angles*

Support Questions

Find the measure of each required angle and give a reason for that answer.

1.

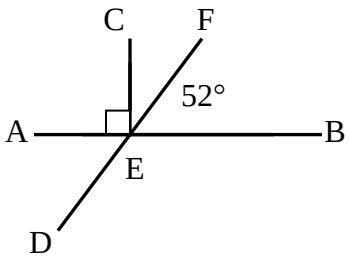


∠1 = _____ Reason_____

∠2 = _____ Reason_____

∠3 = _____ Reason_____

2.



∠AEC = _____ Reason_____

∠AED = _____ Reason_____

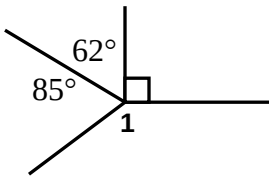
∠DEB = _____ Reason_____

∠CEF = _____ Reason_____

Support Questions (continued)

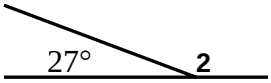
Find the measure of each required angle and give a reason for that answer.

3.



∠1 = _____ Reason_____

4.



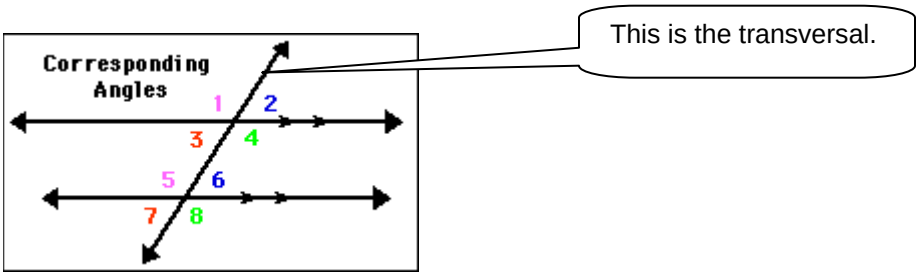
∠2 = _____ Reason_____

Parallel Lines and Transversal

Parallel lines are lines in the same plane that do not intersect.

A Transversal is a line crossing two or more lines.

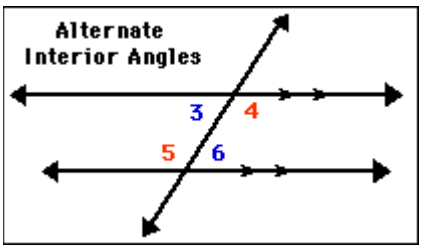
Corresponding Angles



If two parallel lines are cut by a transversal, then each pair of corresponding angles are *congruent*. ∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8

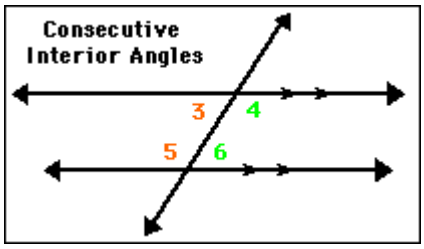
Means the angles are the same degrees.

Alternate Interior Angles



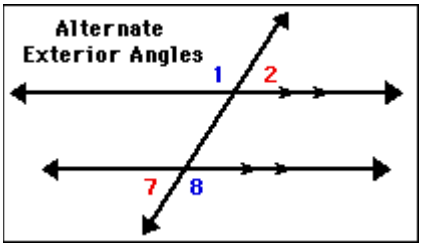
If two parallel lines are cut by a transversal, then each pair of alternate interior angles are congruent. ∠3 and ∠6, ∠4 and ∠5

Same-Side Interior Angles



If two parallel lines are cut by a transversal, then each pair of consecutive interior angles are supplementary. ∠3 and ∠5, ∠4 and ∠6

Alternate Exterior Angle

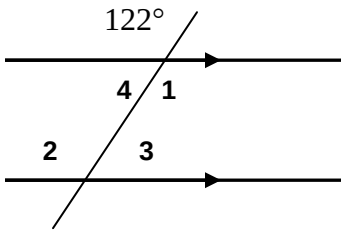


If two parallel lines are cut by a transversal, then each pair of alternate exterior angles are congruent. ∠1 and ∠8, ∠2 and ∠7

Example

Find the measure of each required angle and give a reason for that answer.

a)



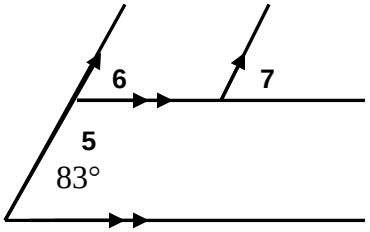
∠1 = _____ Reason_____

∠2 = _____ Reason_____

∠3 = _____ Reason_____

∠4 = _____ Reason_____

b)



∠5 = _____ Reason_____

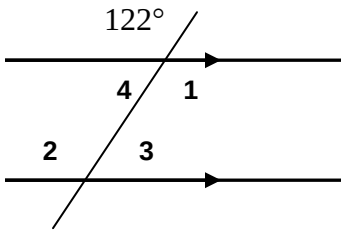
∠6 = _____ Reason_____

∠7 = _____ Reason_____

Solution

Find the measure of each required angle and give a reason for that answer.

a)



∠1 and ∠2

∠1 = 122° Reason *Vertically Opposite*

∠2 = 122° Reason *Alternate Interior Angles*

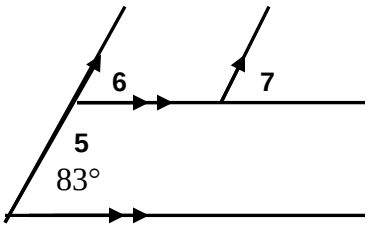
∠3 = 58° Reason *Supplementary Angles*

∠4 = 58° Reason *Alternate Exterior Angle*

or *Supplementary Angle* — Supplementary to ∠1.

or *Vertically Opposite* — Vertically opposite to ∠3.

b)



∠5 and 83° add to 180°.

∠5 = 97° Reason *Consecutive Interior Angle*

∠5 and ∠6 add to 180°.

∠6 = 83° Reason *Supplementary Angles*

or *Corresponding Angles* — ∠6 and 83° correspond.

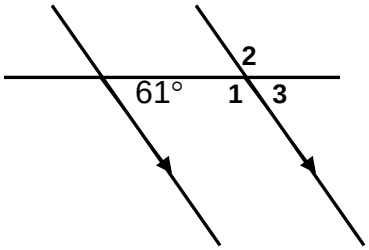
∠7 = 83° Reason *Corresponding Angles*

∠6 and ∠7 correspond.

Support Questions

Find the measure of each required angle and give a reason for that answer.

5.

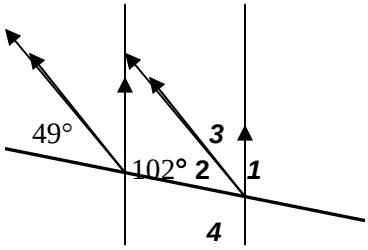


∠1 = _____ Reason_____

∠2 = _____ Reason_____

∠3 = _____ Reason_____

6.



∠1 = _____ Reason_____

∠2 = _____ Reason_____

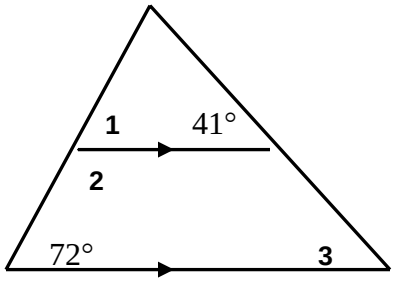
∠3 = _____ Reason_____

∠4 = _____ Reason_____

Support Questions (continued)

Find the measure of each required angle and give a reason for that answer.

7.

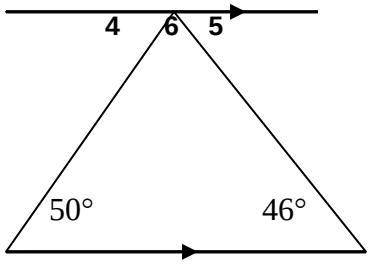


∠1 = _____ Reason_____

∠2 = _____ Reason_____

∠3 = _____ Reason_____

8.



∠4 = _____ Reason_____

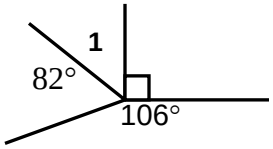
∠5 = _____ Reason_____

∠6 = _____ Reason_____

Key Question #18

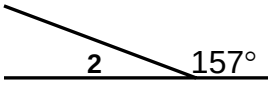
Find the measure of each required angle and give a reason for that answer.

1. (2 marks)



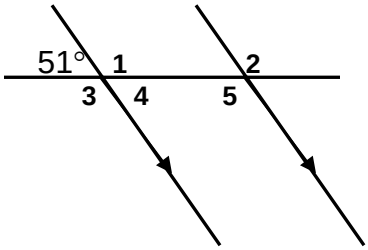
∠1 = _____ Reason_____

2. (2 marks)



∠2 = _____ Reason_____

3. (10 marks)



∠1 = _____ Reason_____

∠2 = _____ Reason_____

∠3 = _____ Reason_____

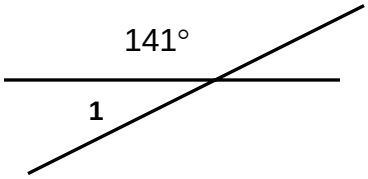
∠4 = _____ Reason_____

∠5 = _____ Reason_____

Key Question #18 (continued)

Find the measure of each required angle and give a reason for that answer.

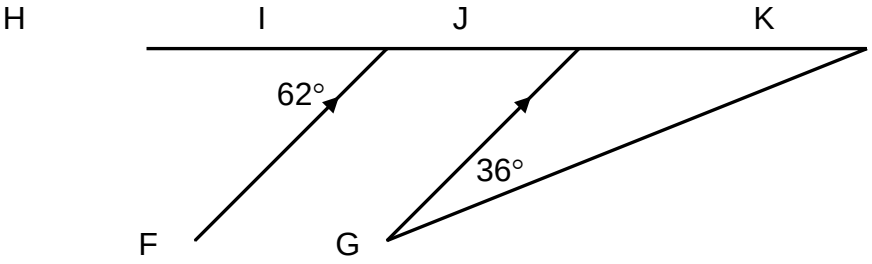
4. (2 marks)



∠1 = _____ Reason_____

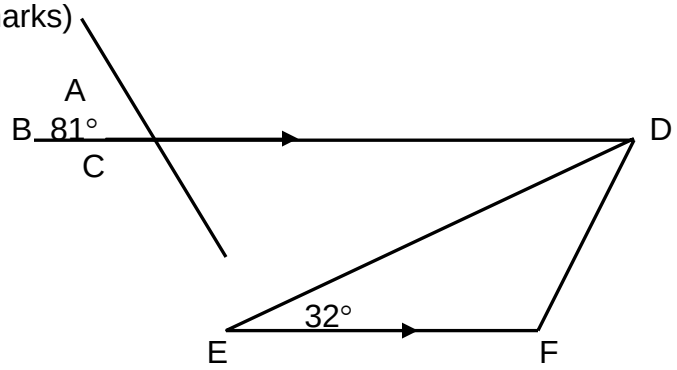
For questions 5 – 8, find the measure of the required angle. List in sequence with reasons the angles you had to find to determine the required angle.

5. (4 marks)



∠JKG = _____

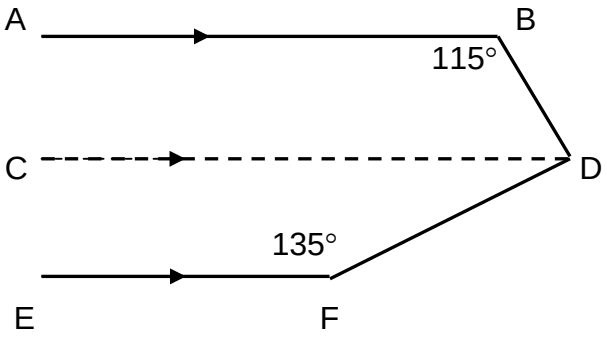
6. (4 marks)



∠AED = _____

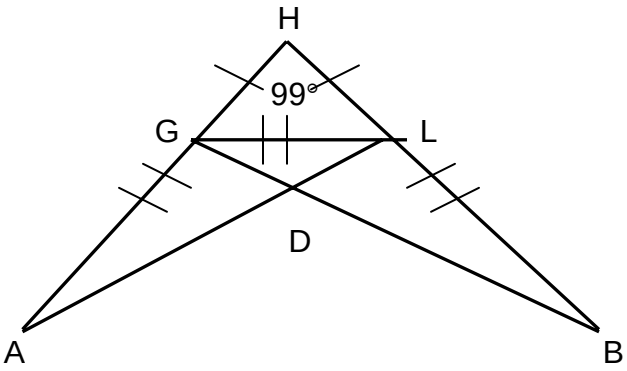
Key Question #18 (continued)

7. (4 marks)



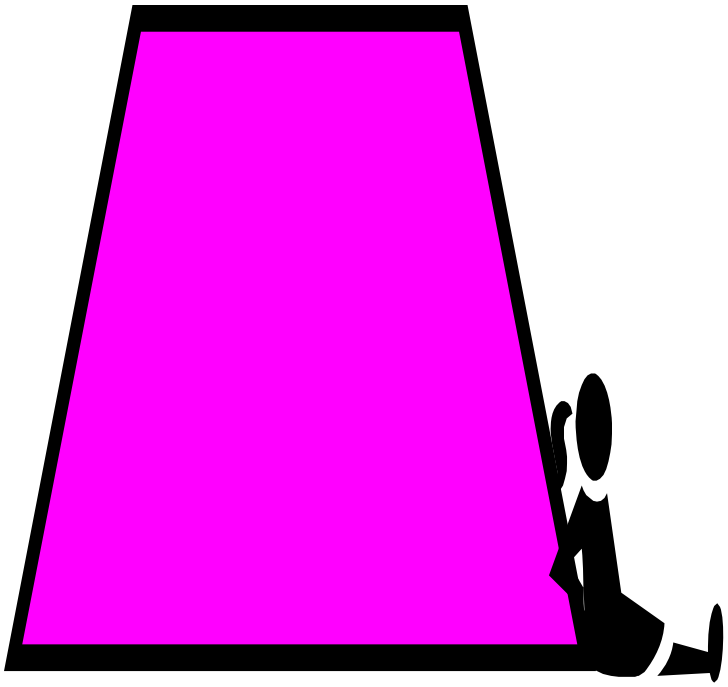
∠BDF = _____

8. (4 marks)



∠ADB = _____

Triangles and Quadrilaterals



Lesson 19

Lesson Nineteen Concepts

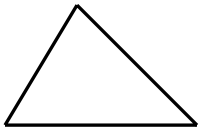
- Introduction to Triangles and Quadrilaterals
- Triangle types
- Triangle properties
- Quadrilateral types
- Quadrilateral properties
- Finding unknown angles with justification

Triangles and Quadrilaterals

Triangle Types and Properties

∠sum of a triangle is 180°

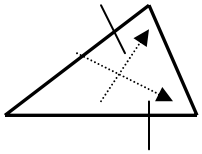
Scalene Triangle



Properties

1) no sides equal
2) no angles equal

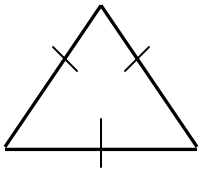
Isosceles Triangle



Properties

1) two sides equal
2) angles opposite equal sides are equal

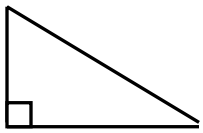
Equilateral Triangle



Properties

1) all sides equal
2) all angles equal (each is 60°)

Right Triangle



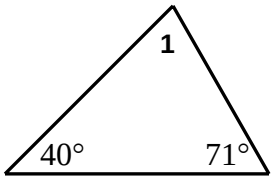
Properties

1) one right angle (90°)
2) hypotenuse is opposite the right angle
3) Property of Pythagoras $a^2 + b^2 = c^2$

Example

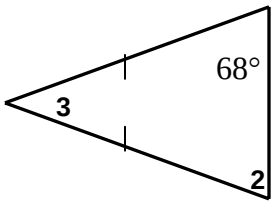
Find the measure of each required angle and give a reason for that answer.

a)



∠1 = _____ Reason_____

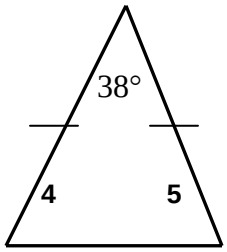
b)



∠2 = _____ Reason_____

∠3 = _____ Reason_____

c)



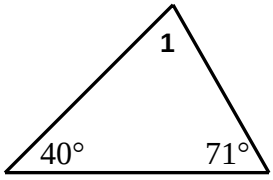
∠4 = _____ Reason_____

∠5 = _____ Reason_____

Solution

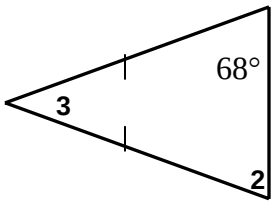
Find the measure of each required angle and give a reason for that answer.

a)



∠1 = 69° Reason *∠ sum of a Δ = 180°*

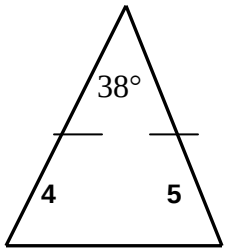
b)



∠2 = 68° Reason Isos Δ

∠3 = 44° Reason *∠ sum of a Δ = 180°*

c)



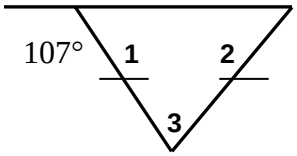
∠4 = 71° Reason Isos Δ , ∠ *sum of a* Δ *= 180°*

∠5 = 71° Reason Isos Δ

Support Questions

Find the measure of each required angle and give a reason for that answer.

1.



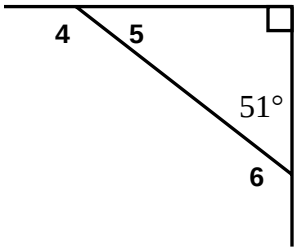
∠1 = _____ Reason_____

∠2 = _____ Reason_____

∠3 = _____ Reason_____

Support Questions (continued)

2.

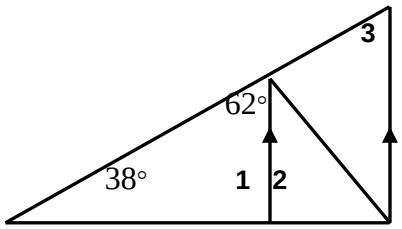


∠4 = _____ Reason_____

∠5 = _____ Reason_____

∠6 = _____ Reason_____

3.



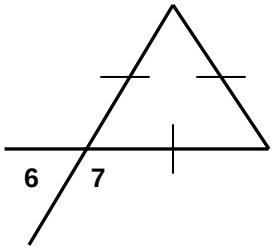
∠1 = _____ Reason_____

∠2 = _____ Reason_____

∠3 = _____ Reason_____

Support Questions (continued)

4.



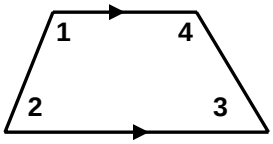
∠6 = _____ Reason_____

∠7 = _____ Reason_____

Quadrilateral Types and Properties

∠sum of a quadrilateral is 360°

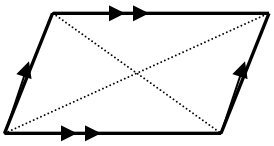
Trapezoid



Properties

1) one pair of parallel sides
2) ∠1and ∠2 = 180°, ∠3 and ∠4 = 360° (interior ∠'s on same side of transversal)

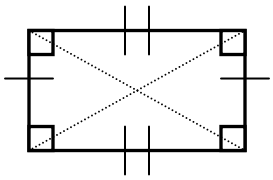
Parallelogram



Properties

1) opposite sides equal and parallel
2) opposite ∠'s are equal
3) consecutive ∠'s add to 180°
4) diagonals bisect each other

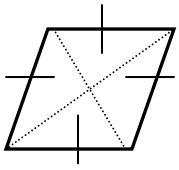
Rectangle



Properties

1) opposite sides equal and parallel
2) each ∠ is 90°
3) diagonals are equal and bisect each other

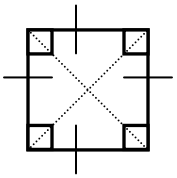
Rhombus



Properties

1) parallelogram with 4 equal sides
2) diagonals bisect at right angles
3) diagonals bisect the ∠'s of the rhombus

Square



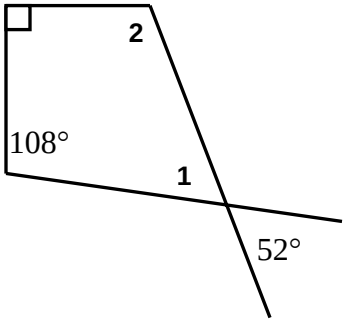
Properties

1) rhombus with 4 right ∠'s, or
2) rectangle with 4 equal sides

Example

Find the measure of each required angle and give a reason for that answer.

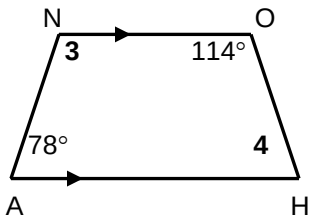
a)



∠1 = _____ Reason_____

∠2 = _____ Reason_____

b)



NOAH is a _____

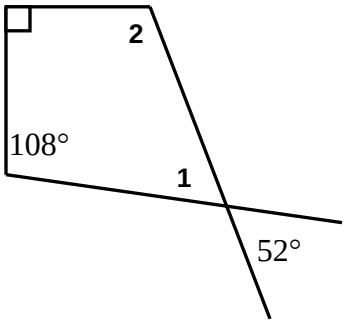
∠3 = _____ Reason_____

∠4 = _____ Reason_____

Solution

Find the measure of each required angle and give a reason for that answer.

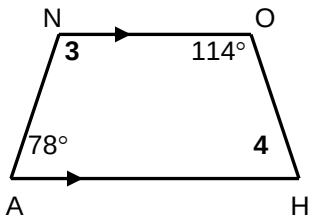
a)



∠1 = _52°_ Reason *Vertically Opposite Angles*

∠2 = 110°_ Reason *∠ sum of a quad = 360°*

b)



NOAH is a *Trapezoid*

∠3 = 102° Reason *Consecutive Interior Angles = 180°*

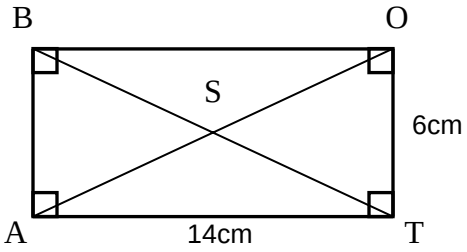
∠4 = 66° Reason *∠ sum of a quad = 360°*

or *Consecutive Interior Angles = 180°*

Support Questions

Complete the following questions by naming the quadrilateral, finding the measures of angles and lengths, and giving reasons for the answers.

5.



BOAT is a _____

AB = TO Reason_____

AB = ___cm Reason_____

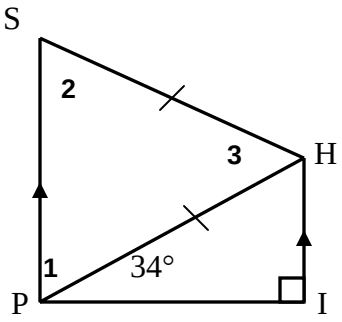
BS = ___cm Reason_____

ΔBSO is _____

ΔOBA is _____

Support Questions (continued)

6.



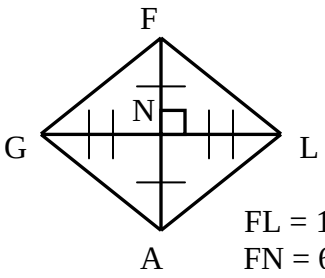
SHIP is a _____

∠1 = _____ Reason_____

∠2 = _____ Reason_____

∠3 = _____ Reason_____

7.



FL = 10 cm
FN = 6 cm

FLAG is a _____

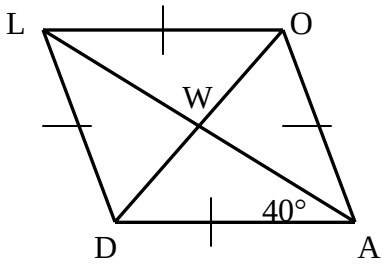
FG= ___cm Reason_____

NL = ___cm Reason_____

FA = ___cm Reason_____

Support Questions (continued)

8.



LOAD is a _____

ΔDOL is _____

$\angle$AWD = ___ Reason_____

ΔDWL is _____

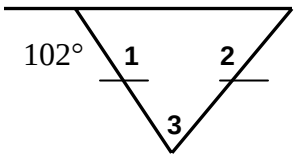
$\angle$LDA = ___ Reason_____

$\angle$DLA = ___ Reason_____

Key Question #19

Find the measure of each required angle and give a reason for that answer.

1. (6 marks)



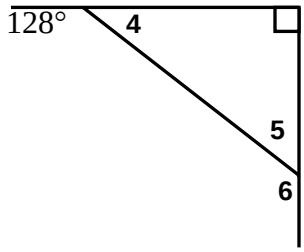
$\angle$1 = _____ Reason_____

$\angle$2 = _____ Reason_____

$\angle$3 = _____ Reason_____

Key Question #19 (continued)

2. (6 marks)

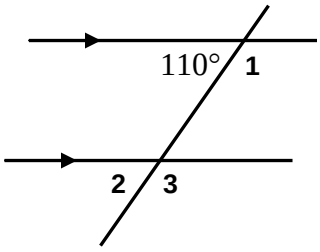


∠4 = _____ Reason_____

∠5 = _____ Reason_____

∠6 = _____ Reason_____

3. (6 marks)



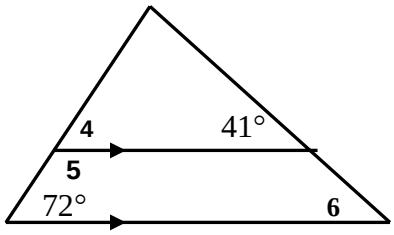
∠1 = _____ Reason_____

∠2 = _____ Reason_____

∠3 = _____ Reason_____

Key Question #19 (continued)

4. (6 marks)

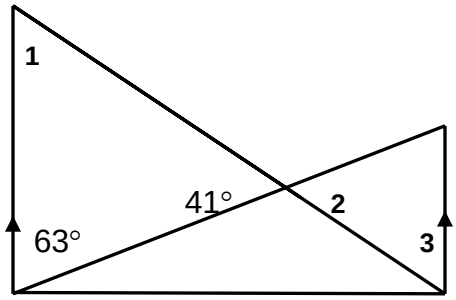


∠4 = _____ Reason_____

∠5 = _____ Reason_____

∠6 = _____ Reason_____

5. **(6 marks)**



∠1 = _____ Reason_____

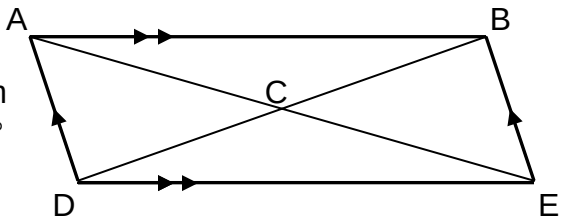
∠2 = _____ Reason_____

∠3 = _____ Reason_____

Key Question #19 (continued)

6. (9 marks)

AC = 2.7 cm
DB = 6.16 cm
∠ADE = 104°



ABDE is a _____

CE = ___ cm Reason_____

DC = ___ cm Reason_____

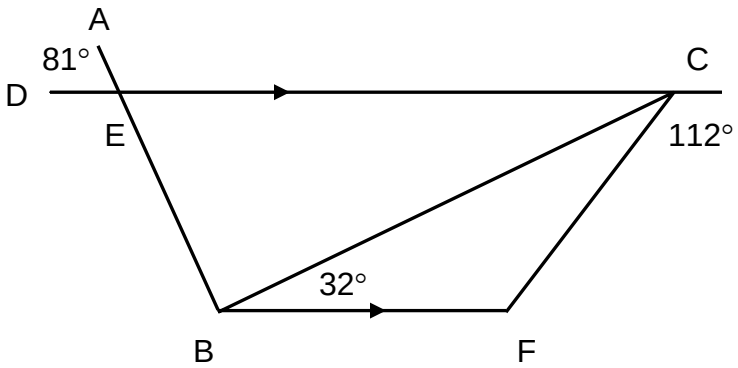
∠ABE = ___ Reason_____

∠DEB = ___ Reason_____

For the following questions find the measure of the required angle. List in sequence with reasons the angles you had to find to determine the required angle.

7. (3 marks)

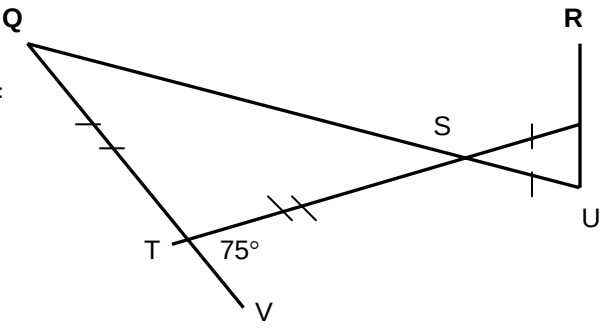
∠FCB=



Key Question #19 (continued)

8. (3 marks)

$\angle SUR =$



Triangle Medians and Altitudes

Lesson 20

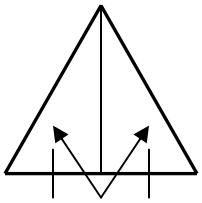
Lesson Twenty Concepts

- Introduction to Triangle medians and altitudes
- Median properties
- Centroid
- Altitude properties
- Orthocentre

Triangle Medians and Altitudes

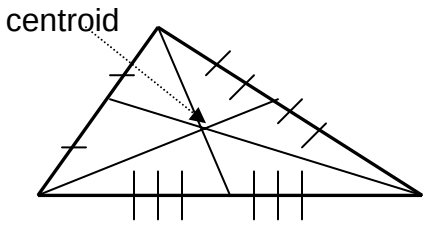
Medians Properties

1. A median divides the area of a triangle in half.

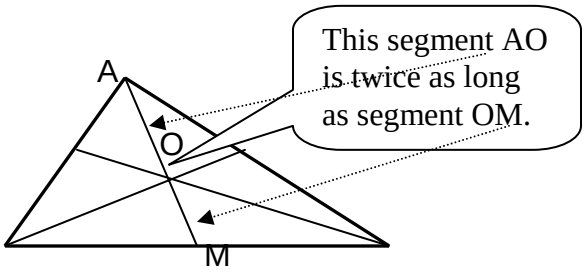


Equal areas

2. The three medians of a triangle meet at one point. This point is called the centroid.



3. On each median, the centroid is twice as far from the vertex as it is from the midpoint of the opposite side.

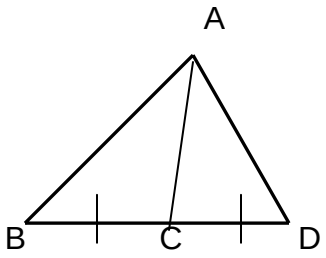


AO = 2OM

Example

The area of ΔABC is 56 cm^2. Calculate the area of ΔABD.

a)



Solution

a) Since AC is the median of ΔABD and ΔABC is 56 cm^2 then ΔABC is half of the area of ΔABD. Therefore, ΔABD 's area is twice the area of ΔABC.

$\Delta ABD = 2 \times 56$

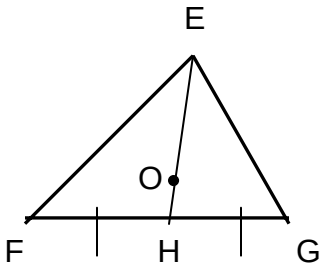
$\Delta ABD = 112 cm^2$

Property 1

Example

In ΔEFG, O is the centroid; OH = 12 cm. Calculate the length of HE.

b)



Solution

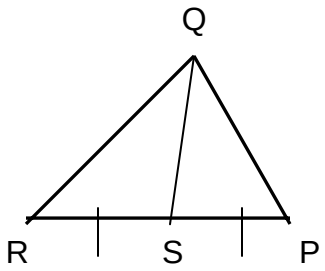
b) Since OH is 1/3 of the length HE then HE is three times that length.

HE = 3(OH)
= 3(12)
= 36 cm

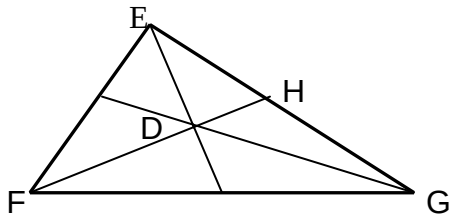
Property 3

Support Questions

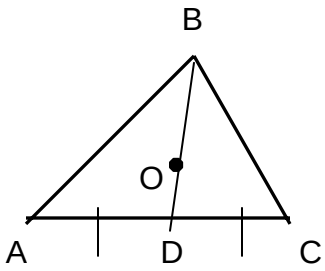
1. The area of ΔPQR is $64\,cm^2$. Calculate the area of ΔPQS.



2. In ΔEFG, D is the centroid; DF = 9 cm. Calculate the length of DH.

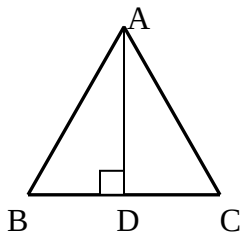


3. In ΔABC, O is the centroid; DO = 12 cm. Calculate the length of OB.

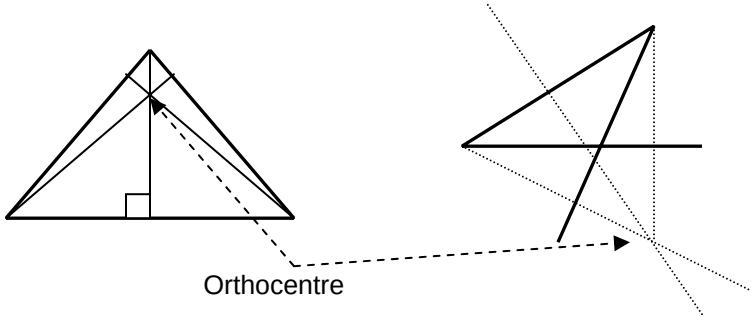


Altitude Properties of a Triangle

1. The length of an altitude is a height of the triangle. It is used to calculate the area of a triangle



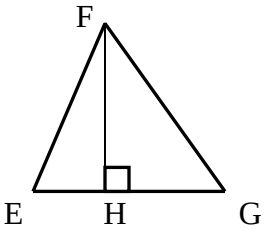
2. The three altitudes of a triangle meet at the orthocentre, O. When the triangle is obtuse, the orthocentre is outside the triangle.



Example

In ΔEFG, the altitude FH = 10 cm and the base EG = 12cm. Calculate the area of ΔEFG.

a)



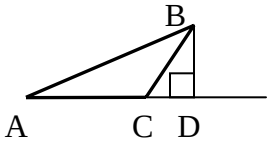
Solution

$$A = \frac{bh}{2}$$
$$A = \frac{(12)(10)}{2}$$
$$A = 60cm^2$$

Example

The area of ΔABC is 16 cm^2. Altitude BD = 2 cm. What is the length of AC?

b)



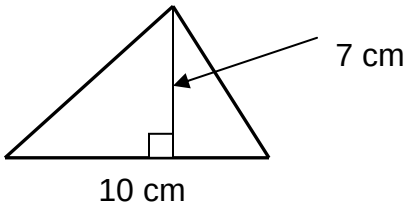
Solution

$$A = \frac{bh}{2}$$
$$16 = \frac{(b)(2)}{2}$$
$$16 = \frac{2b}{2}$$
$$16 = b$$

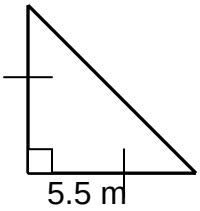
Therefore the length of AC is 16 cm.

Support Questions

4. Calculate the area of the triangle below.



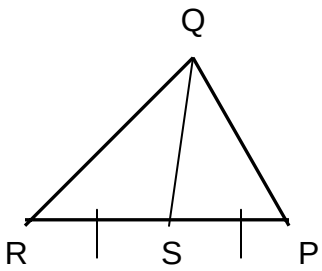
5. Calculate the area of the triangle below.



6. Brianna is planning to paint a triangle on a rectangular piece of paper that is 30 cm by 44 cm. Calculate the area of the largest triangle she can paint.

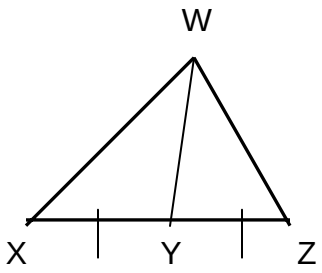
Key Question #20

1. The area of ΔQSP is $80\,cm^2$. Calculate the area of ΔQRP. (2 marks)

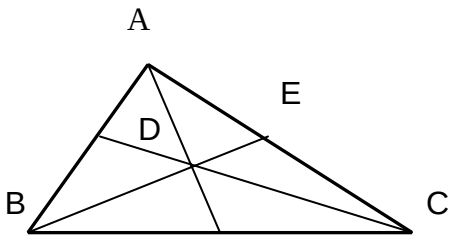


Key Question #20 (continued)

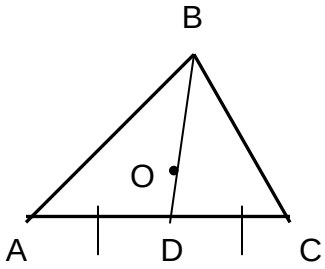
2. The area of ΔWXZ is $34\,cm^2$. Calculate the area of ΔWXY. (2 marks)



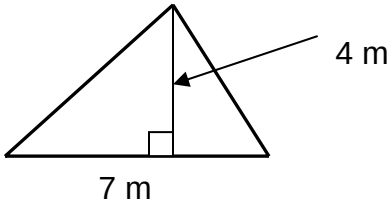
3. In ΔABC, D is the centroid; DB = 12 cm. Calculate the length of DE. (2 marks)



4. In ΔABC, O is the centroid; DO = 9 cm. Calculate the length of OB. (2 marks)

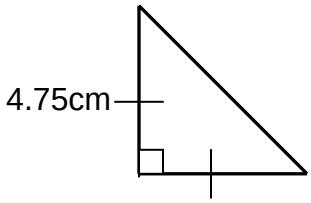


5. Calculate the area of the triangle below. (2 marks)

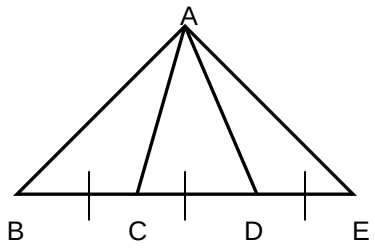


Key Question #20 (continued)

6. Calculate the area of the triangle below. (2 marks)



7. The area of ΔABC is $36\,cm^2$. What is the area of ΔABE? (3 marks)



8. Draw the triangle with vertices B(3,1), C(5,7) and D(8,1). Draw the altitude from C to join BD at E. (6 marks)

   a) State the coordinates of point E.
   b) Determine the equation of the altitude.

9. Plot ΔDEF with vertices D(0,3), E(8,5) and F(4,1). (6 marks)

   a) Draw the medians. Find the coordinates of the centroid.
   b) Find the equations of the three medians of the triangle.

10. The side of XY of ΔXYZ has slope 3. Find the slope of the altitude from Z to XY. (4 marks)

11. Explain what the medians of a triangle are. The word “median” is an appropriate word. Why do you think this is so? (4 marks)

Lesson 16

1. a.

$$A = \frac{(a+b)h}{2}$$
$$= \frac{(8+3)4}{2}$$
$$= \frac{44}{2}$$
$$= 22\ cm^2$$

b.

$$A = bh$$
$$= (5)(6)$$
$$= 30cm^2$$

c.

$$A = lw$$
$$= (10)(5)$$
$$= 50cm^2$$

d.

$$A = \frac{bh}{2}$$
$$= \frac{(3.5)(7)}{2}$$
$$= \frac{24.5}{2}$$
$$= 12.25\ cm^2$$

e.

$$A = \pi r^2$$
$$= (3.14)(4)^2$$
$$= 3.14(16)$$
$$= 50.24\ cm^2$$

f.

$$A = 4l$$
$$= 4(8)$$
$$= 64\ cm^2$$

g.

$$A_{total} = A_1 + A_2$$
$$A_1 = lw$$
$$= (12)(12)$$
$$= 144\ m^2$$
$$A_2 = lw$$
$$= (6)(6)$$
$$= 36\ m^2$$
$$A_{total} = 144 + 36$$
$$= 180\ m^2$$

h.

$$A_{total} = A_1 + A_2$$
$$A_1 = \frac{\pi r^2}{2}$$
$$= \frac{(3.14)(5)^2}{2}$$
$$= \frac{78.5}{2}$$
$$= 39.25\ cm^2$$
$$A_2 = lw$$
$$= (14)(20)$$
$$= 280cm^2$$
$$A_{total} = 39.25 + 280$$
$$= 319.25cm^2$$

i.

$$A = .75(\pi r^2)$$
$$= (.75)(3.14)(8)^2$$
$$= 150.72\ cm^2$$

2. a.

$$A_{Total} = A_1 - A_2$$

$$\begin{aligned} A_1 &= lw \\ &= (25)(11) \\ &= 275 \end{aligned}$$

$$\begin{aligned} A_2 &= \pi r^2 \\ &= (3.14)(5.5)^2 \\ &= 94.985 \end{aligned}$$

$$\begin{aligned} A_{Total} &= A_1 - A_2 \\ &= 275 - 94.985 \\ &\approx 180 \text{ cm}^2 \end{aligned}$$

b.

$$A_{Total} = A_1 - A_2$$

$$\begin{aligned} A_1 &= \pi r^2 \\ &= (3.14)(9.5)^2 \\ &= 283.385 \end{aligned}$$

$$\begin{aligned} A_2 &= \pi r^2 \\ &= (3.14)(8)^2 \\ &= 200.96 \end{aligned}$$

$$\begin{aligned} A_{Total} &= A_1 - A_2 \\ &= 283.385 - 200.96 \\ &= 82.43 \text{ cm}^2 \end{aligned}$$

3. a.

$$\begin{aligned} S.A. &= 2\pi r^2 + 2\pi rh \\ &= (2)(3.14)(4)^2 + (2)(3.14)(4)(7) \\ &= 100.48 + 175.84 \\ &= 276.32\ cm^2 \end{aligned}$$

b.

$$\begin{aligned} S.A. &= 2[wh + lw + lh)] \\ &= 2[(5)(11) + (6)(5) + (6)(11)] \\ &= 2(55 + 30 + 66) \\ &= 2(151) \\ &= 302\ cm^2 \end{aligned}$$

c.

$$\begin{aligned} S.A. &= \pi r(s + r) \\ &= (3.14)(3)(5 + 3) \\ &= (3.14)(3)(8) \\ &= 75.36\ m^2 \end{aligned}$$

d.

$$\begin{aligned} S.A. &= 2bs + b^2 \\ &= 2(9)(11.3) + (9)^2 \\ &= 203.4 + 81 \\ &= 284.4\ m^2 \end{aligned}$$

e.

$$\begin{aligned} S.A. &= 4\pi r^2 \\ &= (4)(3.14)(6.4)^2 \\ &\approx 514.46 cm^2 \end{aligned}$$

f.

$$S.A. = 2\left(\frac{bh}{2}\right) + 2ls + lb$$
$$= 2\left(\frac{(8)(9)}{2}\right) + 2(12)(9.85) + (12)(8)$$
$$= 72 + 236.4 + 96$$
$$= 404.4\ m^2$$

Lesson 17

1. a.

$$V = \pi r^2 h$$
$$= (3.14)(4)^2(7)$$
$$= 351.68\ cm^3$$

b.

$$V = lwh$$
$$= (6)(5)(11)$$
$$= 330\ cm^3$$

c.

$$V = \frac{\pi r^2 h}{3}$$
$$= \frac{(3.14)(3)^2(4)}{3}$$
$$= 37.68\ m^3$$

d.

$$V = \frac{b^2 h}{3}$$
$$= \frac{(9)^2(10.37)}{3}$$
$$= 278.1\ m^3$$

e.

$$V = \frac{4\pi r^3}{3}$$
$$= \frac{(4)(3.14)(6.4)^3}{3}$$
$$\approx 1097.5\ cm^3$$

f.

$$V = \frac{bhl}{2}$$
$$= \frac{(8)(9)(12)}{2}$$
$$= 432\ m^3$$

Lesson 18

1. $\angle 1 = 65°$; supplementary to 115°
   $\angle 2 = 65°$; vertically opposite $\angle 1$
   $\angle 3 = 115°$; supplementary to $\angle 2$ or vertically opposite to 115°

2. $\angle AEC = 90°$; right angle given
   $\angle AED = 52°$; vertically opposite $\angle FEB$
   $\angle DEB = 128°$; supplementary to $\angle FEB$
   $\angle CEF = 38°$; complementary to $\angle FEB$

3. $\angle 1 = 123°$; angles at a point equal 360°

4. $\angle 2 = 153°$; angles on a line equal 180°

5. $\angle 1 = 119°$; consecutive interior angles are supplementary
   $\angle 2 = 119°$; vertically opposite $\angle 1$
   $\angle 3 = 61°$; supplementary to $\angle 2$ or corresponding to 61°

6. $\angle 1 = 102°$; corresponding to $102°$
   $\angle 2 = 49°$; corresponding to $49°$
   $\angle 3 = 29°$; angles on a line equal $180°$
   $\angle 4 = 102°$; angles on a line equal $180°$ or alternate interior angle with $102°$

7. $\angle 1 = 72°$; corresponding to $72°$
   $\angle 2 = 108°$; supplementary to $\angle 1$ or consecutive interior angles are supplementary
   $\angle 3 = 41°$; corresponding to $41°$

8. $\angle 4 = 50°$; alternate interior angle with $50°$
   $\angle 5 = 46°$; alternate interior angle with $46°$
   $\angle 6 = 84°$; angles on a line equal $180°$ or interior $\angle$'s of triangle equals $180°$

Lesson 19

1. $\angle 1 = 73°$; supplementary to $107°$
   $\angle 2 = 73°$; base $\angle$'s of IsosΔ are equal
   $\angle 3 = 34°$; interior $\angle$'s of triangle equals $180°$

2. $\angle 4 = 141°$; supplementary to $\angle 5$
   $\angle 5 = 39°$; interior $\angle$'s of triangle equals $180°$
   $\angle 6 = 129°$; supplementary to $51°$

3. $\angle 1 = 80°$; interior $\angle$'s of triangle equals $180°$
   $\angle 2 = 100°$: supplementary to $\angle 1$
   $\angle 3 = 62°$; corresponding to $62°$

4. $\angle 6 = 60°$; $\angle$'s of equilateral Δ are equal ($180°$)
   $\angle 7 = 120°$: supplementary to $\angle 6$

5. BOAT is a rectangle
   AB =TO; opposites sides of rectangle are equal lengths
   AB = 6 cm' TO is 6 cm.
   BS = 7.62 cm BS is half of BT because AO bisects BT Note: Pythagorean theorem needed to find length BT
   ΔBSO is an Isos Δ because SO and SB are equal lengths.
   ΔOBA is a right angle Δ.

6. SHIP is a Trapezoid
   $\angle 1 = 56°$; complementary to $34°$
   $\angle 2 = 56°$: base $\angle$'s of IsosΔ are equal
   $\angle 3 = 68°$; interior $\angle$'s of triangle equals $180°$

7. FLAG is a Rhombus
FG = 10 cm; FG = FL
NL = 8 cm; Pythagorean Theorem
FA= 12 cm; FA bisected by GL

8. LOAD is a Rhombus
ΔDOL is an Isos Δ because LO and LD are equal lengths
$\angle$AWD= 90° ; diagonals bisect at right angles
ΔDWL is a right angle Δ. because $\angle$DWL is 90°
$\angle$LDA= 100° ; consecutive angles of rhombus equal 180°
$\angle$DLA= 40° ; base $\angle$'s of IsosΔ are equal

Lesson 20

1.

$$\Delta PQS = \frac{64}{2}$$
$$= 32cm^2$$

2.

$$DH = \frac{DF}{2}$$
$$= \frac{9}{2}$$
$$= 4.5\ cm$$

3.

$$OB = 2OD$$
$$= 2(12)$$
$$= 24\ cm$$

4.

$$A = \frac{bh}{2}$$
$$= \frac{(10)(7)}{2}$$
$$= 35cm^2$$

5.

$$A = \frac{bh}{2}$$
$$= \frac{(5.5)(5.5)}{2}$$
$$= \frac{30.25}{2}$$
$$= 15.125m^2$$

6.

$$A = \frac{bh}{2}$$
$$= \frac{(30)(44)}{2}$$
$$= \frac{1320}{2}$$
$$= 660cm^2$$

