

The Line



Lesson 11

Lesson Eleven Concepts

- Introduction to the line
- Standard form of an equation
- Y-intercept form of an equation
- Converting to and from standard to y-intercept form
- Parallel lines
- Perpendicular lines
- Point of intersection of two lines
- x and y intercepts

Equation of a Line

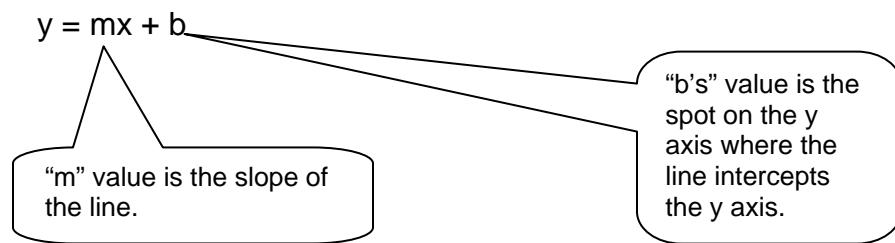
The equation of a line has 2 basic forms. One is y-intercept form and the other is called standard form.

y-intercept form takes the form $y = mx + b$

Standard form looks like $Ax + By + C = 0$

Y-intercept Form

It is called y-intercept form because we can use the y-intercept in the equation to help us understand and graph the equation.



Example

State the slope and y-intercept for the following equations.

a) $y = 3x + 6$

b) $y = -\frac{1}{2}x - 1$

Solution

a) $y = 3x + 6$

slope = $m = \frac{\text{rise}}{\text{run}} = \frac{3}{1} = 3$ y-intercept = $b = +6$

b) $y = -\frac{1}{2}x - 1$

slope = $m = \frac{\text{rise}}{\text{run}} = \frac{-1}{2}$ y-intercept = $b = -1$

Example

Graph the following equations using the slope and y-intercept.

a) $y = -2x + 3$

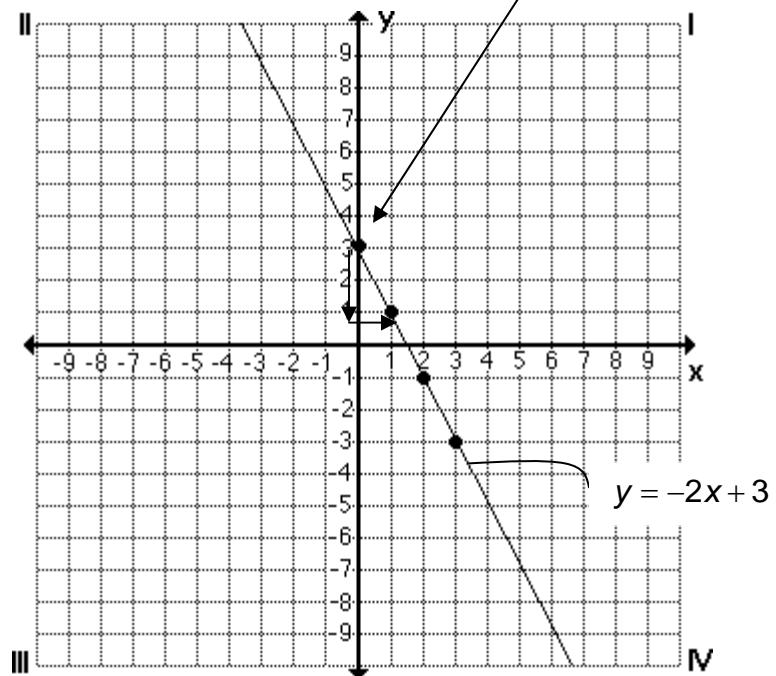
b) $y = \frac{2}{3}x - 1$

Solution

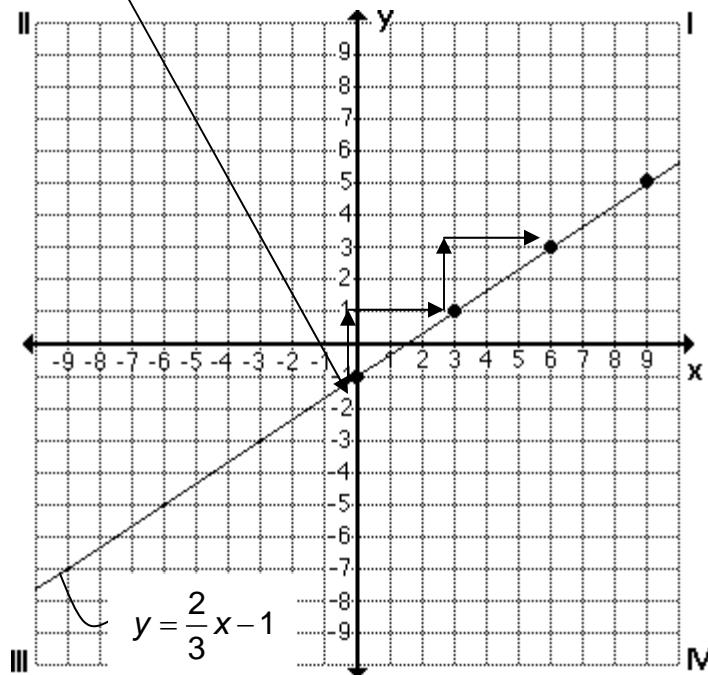
a) $y = -2x + 3$ $m = -2 = \frac{\text{rise}}{\text{run}} = \frac{-2}{1}$ and $b = +3$

Starting at the y-intercept go down 2 and right 1 then down 2 again and right one again and so on... Then draw a line through the points plotted to create your line.

First plot the y-intercept.



b) $y = \frac{2}{3}x - 1$

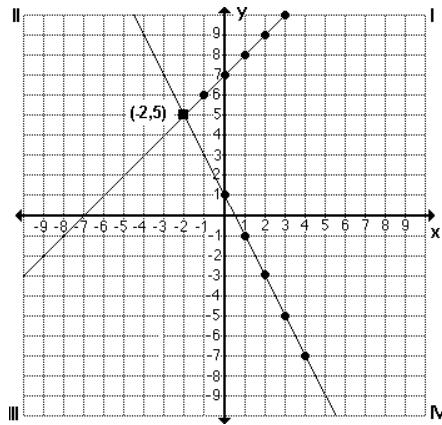


Example

Find the point of intersection of these two equations.
(Solve the system of equations graphically)

$$\begin{aligned}y &= -2x + 1 \\y &= x + 7\end{aligned}$$

Solution



The point of intersection is (-2, 5) which solves the system of equations.

(-2, 5) is the only ordered pair that will satisfy both equations.

$$y = -2x + 1$$

$$5 = -2(-2 + 1)$$

$$5 = 5$$



$$y = x + 7$$

$$5 = -2 + 7$$

$$5 = 5$$



Support Questions

1. State the slope and y-intercept of the equations below, then properly graph and label the equations using the slope and y- intercept.
 - a) $y = -4x + 3$
 - b) $y = x - 4$
 - c) $y = -\frac{2}{3}x + 7$
 - d) $y = \frac{3}{5}x - 1$
2. Show by substitution which of the order pairs satisfies that equation.
 - a) $y = -2x + 4$ (-3,-5), (4,-4), (-1,7), (3,-10)
 - b) $y = -\frac{3}{4}x + 1$ (9,5), (8,-5), (3,-7), (12, -8)
 - c) $y = -x - 2$ (3,-2), (-4,6), (4,-6), (-1,-1)
3. Solve for the system of equations by graphing. (Find the point of intersection)
 - a) $y = -3x + 4$
 - b) $y = \frac{2}{3}x + 6$
 - c) $y = x - 1$
 - a) $y = -2x + 5$
 - b) $y = x + 4$
 - c) $y = 4x + 2$

Converting to and from Y-intercept to Standard Form

Example

Convert into standard form.

a) $y = -\frac{1}{2}x + 3$

Solution

Needs to be in the form $Ax + By + C = 0$. Algebra is used to convert into standard form.

$$2(y) = 2\left(-\frac{1}{2}x + 3\right)$$

$$2y = -1x + 6$$

$$2y + x = -x + x + 6$$

$$2y + x = 6$$

$$2y + x - 6 = 6 - 6$$

$$x + 2y - 6 = 0$$

Multiply each term by 2 to eliminate the fraction.

A=1, B=2 and C=-6

Example

Convert into y-intercept form.

a) $3x + 4y + 7 = 0$

Solution - Convert into y-intercept form.

$$3x + 4y + 7 = 0$$

$$3x - 3x + 4y + 7 = 0 + -3x$$

$$4y + 7 = -3x$$

$$4y + 7 - 7 = -3x - 7$$

$$4y = -3x - 7$$

$$\frac{4y}{4} = \frac{-3x - 7}{4}$$

$$y = -\frac{3}{4}x - \frac{7}{4}$$



Support Questions

4. Convert the following linear equations from y-intercept form to standard form.
- a) $y = x + 1$ b) $y = -\frac{3}{7}x - 2$
c) $y = 5x - \frac{1}{2}$ d) $y = -3x + 7$
5. Convert the following linear equations from standard form to y-intercept form.
- a) $3x - 2y + 6 = 0$ b) $2x + 5y - 10 = 0$ c) $5x - 2y + 7 = 0$

Parallel and Perpendicular Lines

Lines that are **parallel** to each other have the same slope.

Lines that are **perpendicular** to each other will have inverted slopes with opposite polarity.

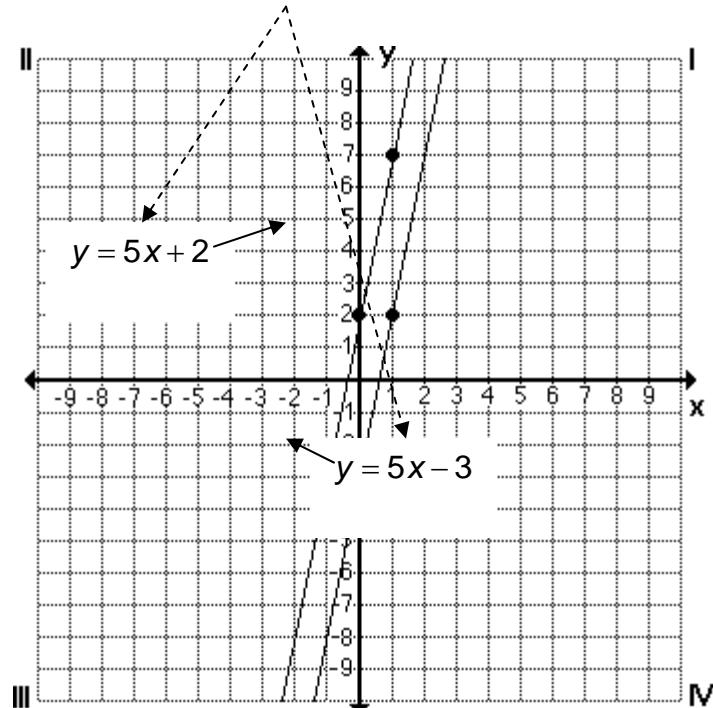
Example

Which of the following equations are parallel to each other?

$$y = 5x - 3 \quad y = -5x + 2 \quad y = 3x - 3 \quad y = 5x + 2$$

Solution

$y = 5x - 3$ and $y = 5x + 2$ are parallel to each other because they have the same slope of $5x$.



Example

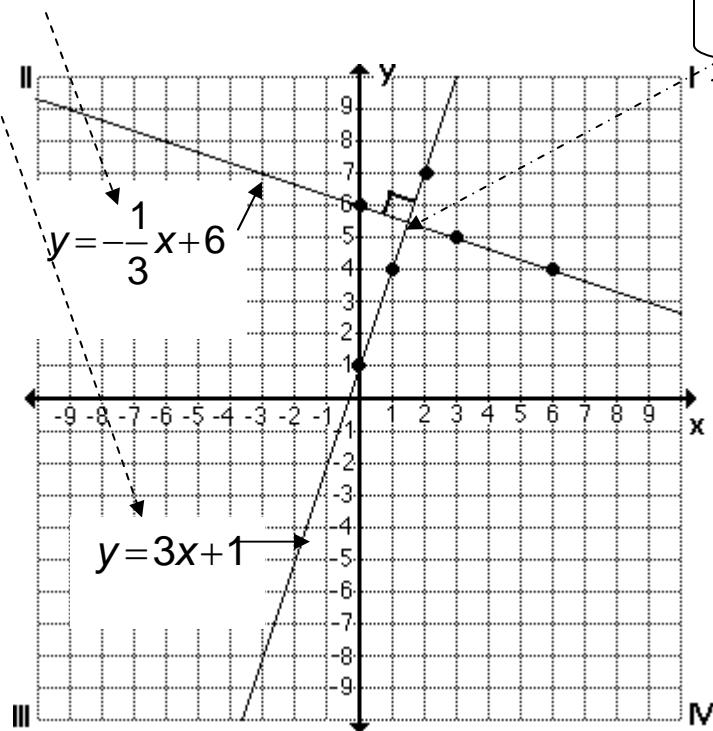
Which of the following equations are perpendicular to each other.

$$y = 3x + 1 \quad y = -2x + 2 \quad y = -\frac{1}{3}x + 6 \quad y = -\frac{1}{2}x - 1$$

Solution

$y = 3x + 1$ and $y = -\frac{1}{3}x + 6$ are perpendicular to each other because they have slope inverted of each other and opposite polarity.

3 and $-\frac{1}{3}$

**Support Questions**

6. Identify then graph the pair of equations that are parallel.

$$y = -x + 2 \quad y = \frac{2}{3}x + 2 \quad y = \frac{2}{3}x - 5 \quad y = x - 6 \quad y = -\frac{2}{3}x - 7$$

7. Identify then graph the pair of equations that are perpendicular.

$$y = 3x + 2 \quad y = 2x + 7 \quad y = \frac{1}{2}x - 5 \quad y = -\frac{1}{2}x - 6 \quad y = \frac{1}{3}x + 2$$

Finding the x intercept and the y intercepts

The x intercept is where a line intersects or “hits” the x axis and the y intercept is where that same line intersects the y axis.

Example

Find the x and y intercepts for the equation $3x + 2y - 5 = 0$.

Solution

To find the x intercept:

Set $y = 0$

$2y = 2(0) = 0$. Since this equals zero we can just ignore the y part of the equation.

Therefore we ignore the “y” portion of the equation and solve for x.

$$3x - 5 = 0$$

$$3x - 5 + 5 = 0 + 5$$

$$3x = 5$$

$$\frac{3x}{3} = \frac{5}{3}$$

$$x = \frac{5}{3}$$

This is the x intercept. Where the line $3x + 2y - 5 = 0$ “hit” the x-axis.

To find the y intercept:

Set $x = 0$

Therefore we ignore the “x” portion of the equation and solve for y.

$$2y - 5 = 0$$

$$2y - 5 + 5 = 0 + 5$$

$$2y = 5$$

$$\frac{2y}{2} = \frac{5}{2}$$

$$y = \frac{5}{2}$$

$3x = 3(0) = 0$. Since this equals zero we can just ignore the x part of the equation.

This is the y intercept. Where the line $3x + 2y - 5 = 0$ “hit” the y-axis.



Support Questions

8. Find the x and y intercepts for each of the following equations.
- a) $3x - 2y + 6 = 0$ b) $2x + 5y - 10 = 0$ c) $5x - 2y + 7 = 0$
9. Find the x and y intercepts for each of the following equations.
- a) $y = x + 1$ b) $y = -\frac{3}{7}x - 2$



Key Question #11

1. State the slope and y-intercept of the equations below, then properly graph and label the equations using the slope and y - intercept. (8 marks)
- a) $y = 3x - 2$ b) $y = -x + 4$
 c) $y = -\frac{1}{2}x + 1$ d) $y = \frac{2}{3}x - 3$
2. Show by substitution which of the order pairs satisfies that equation. (3 marks)
- a) $y = -5x - 2$ (-3,-5), (4,-4), (-1,3), (3,-10)
 b) $y = \frac{1}{3}x + 1$ (9,5), (8,-5), (3,-7), (12, 5)
 c) $y = x + 7$ (3,-2), (-4,3), (3,-6), (-1,-1)
3. Solve for the system of equations by graphing. (Find the point of intersection) (6 marks)
- a) $y = x + 4$
 $y = -x + 5$
- b) $y = \frac{1}{2}x - 1$
 $y = x + 4$
- c) $y = x + 1$
 $y = 2x - 5$



Key Question #11 (continued)

4. Convert the following linear equations from y-intercept form to standard form. (4 marks)

a) $y = 2x - 3$

b) $y = \frac{2}{3}x - 7$

c) $y = -3x + \frac{1}{2}$

d) $y = 2x + 7$

5. Convert the following linear equations from standard form to y - intercept form. (3 marks)

a) $5x - y + 3 = 0$

b) $x + 7y - 14 = 0$

c) $6x - 3y + 9 = 0$

6. Identify then graph the pair of equations that are parallel. (4 marks)

$$y = -x + 2 \quad y = \frac{2}{3}x + 2 \quad y = -\frac{2}{3}x - 5 \quad y = -x - 6 \quad y = -\frac{2}{3}x - 7$$

7. Identify then graph the pair of equations that are perpendicular. (4 marks)

$$y = -3x + 2 \quad y = 2x + 7 \quad y = \frac{1}{2}x - 5 \quad y = \frac{1}{2}x - 6 \quad y = \frac{1}{3}x + 2$$

8. Find the x and y intercepts for the following equations. (5 marks)

a) $5x - y + 3 = 0$

b) $x + 7y - 14 = 0$

c) $6x - 3y + 9 = 0$

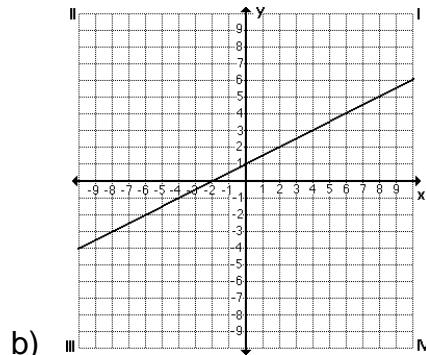
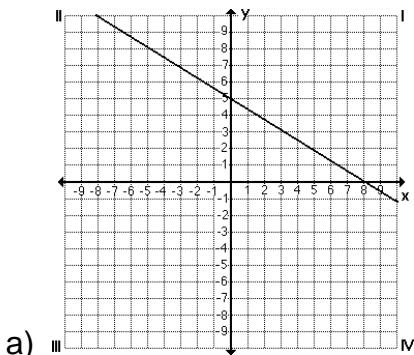
d) $y = 2x - 3$

e) $y = \frac{2}{3}x - 7$



Key Question #11 (continued)

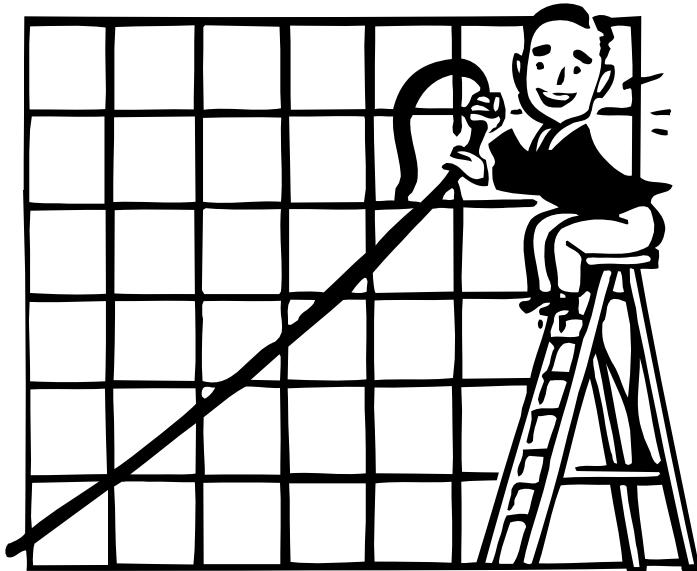
9. State the x-intercept, y-intercept and slope for each line. (4 marks)



10. Is it easier to graph an equation in y-intercept form or standard form? Explain. (3 marks)
11. An equation has an x-intercept of 5 and a y-intercept of -3. Graph this equation. (3 marks)



Direct and Partial Variation



Lesson 12

Lesson Twelve Concepts

- Introduction to direct and partial variation
- The origin on a Cartesian plane
- “m” in the equation $y=mx$
- graphing equation of the form $y=mx$
- Introduction to partial variation
- “b” in the equation $y=mx + b$
- graphing equation of the form $y=mx + b$

Direct and Partial Variation

Direct Variation

Direct Variation is a relation that is of the form $y=mx$.

The graph of $y=mx$ is a straight line with the slope of m .

The line $y=mx$ always passes through the order pair $(0,0)$. $(0,0)$ is called the origin.

The relation $y=mx$ represents direct variation because there is a direct relationship.

Example

Graph each equation which models direct variation.

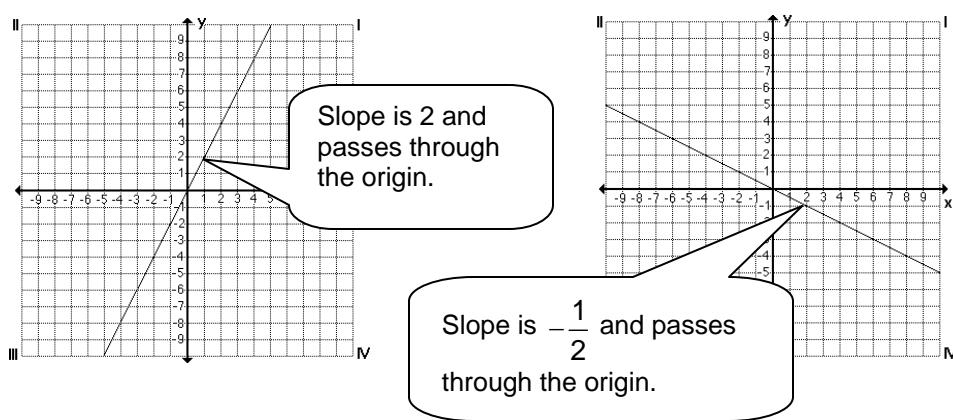
a) $y = 2x$

b) $y = -\frac{1}{2}x$

Solution

a) $y = 2x$

b) $y = -\frac{1}{2}x$





Support Questions

1. State the value of m in each equation of the form $y=mx$.

a) $y = -4x$

b) $y = x$

c) $y = -\frac{1}{3}x$

d) $y = \frac{2}{5}x$

2. Write an equation of the line through the origin with each slope.

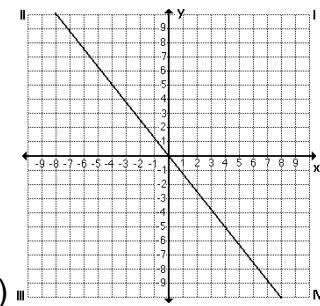
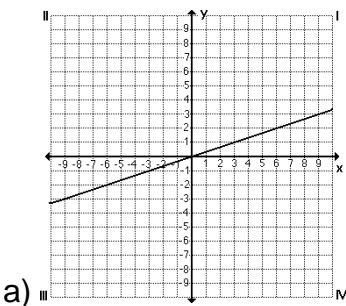
a) $m = -5$

b) $m = -\frac{3}{7}$

c) $m = \frac{1}{4}$

d) $m = 1$

3. Find the slope of each line, then write its equation.



4. Graph each line.

a) $y = -\frac{1}{5}x$

b) $y = \frac{2}{3}x$

5. Johnny earns \$6 for every hour worked. Write an equation for this statement, then create a table of values and graph.



Partial Variation

Partial Variation is a relation that is of the form $y=mx+b$.

The graph of $y=mx+b$ is a straight line with the slope of m and a y -intercept of b .

The line $y=mx+b$ does not pass through the origin.

The relation $y=mx+b$ represents partial variation because the value of y varies partially with the value of x .

Example

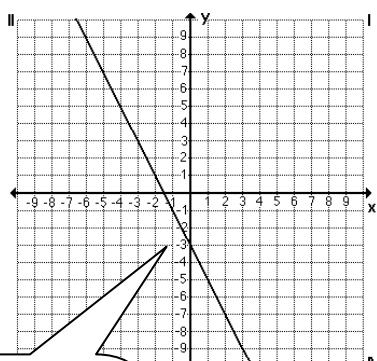
Graph each equation which models partial variation.

a) $y = -2x - 3$

b) $y = \frac{4}{5}x + 1$

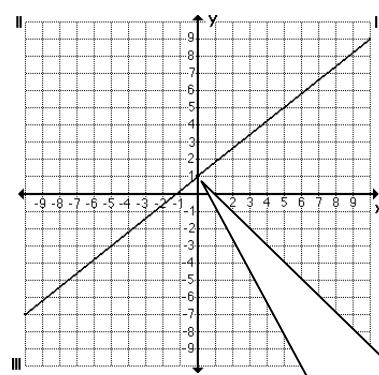
Solution

a) $y = -2x - 3$



Slope is -2 and intersects the y -axis at -3 .

b) $y = \frac{4}{5}x + 1$

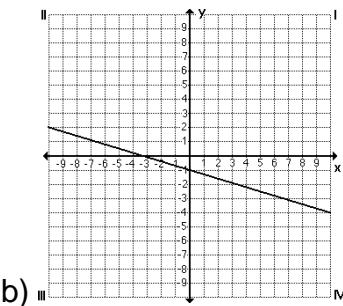
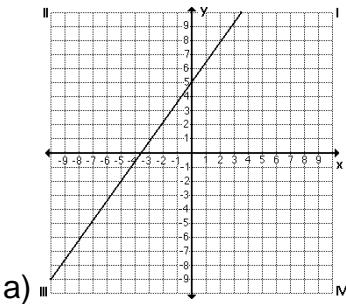


Slope is $\frac{4}{5}$ and intersects the y -axis at $+1$.



Support Questions

6. Write an equation of the line with each slope and y-intercept.
- a) $m = -5, b = 1$ b) $m = -\frac{3}{7}, b = -4$ c) $m = \frac{1}{4}, b = +6$
7. Find the slope and y-intercept of each line, and then write its equation.



8. Graph each line.
- a) $y = -2x + 1$ b) $y = \frac{2}{3}x - 6$ c) $y = x - 4$
9. Noah earns a \$100 a week and \$2 for every hour worked. Write an equation for this statement, then create a table of values and graph.



Key Question #12

1. State the value of m in each equation of the form $y = mx$. (4 marks)
- a) $y = 7x$ b) $y = -x$
- c) $y = \frac{2}{7}x$ d) $y = -\frac{3}{8}x$



Key Question #12 (continued)

2. Write an equation of the line through the origin with each slope. (4 marks)

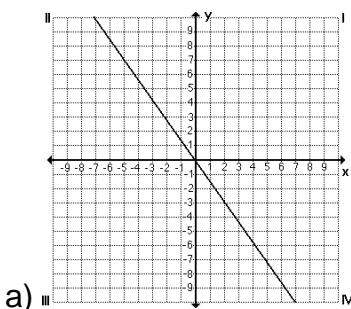
a) $m = 7$

b) $m = \frac{1}{2}$

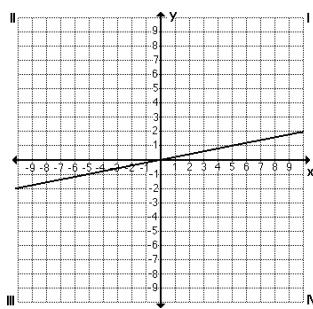
c) $m = -3$

d) $m = -\frac{3}{8}$

3. Find the slope of each line, and then write its equation. (4 marks)



a)



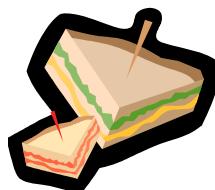
b)

4. Graph each line. (6 marks)

a) $y = -3x$

b) $y = \frac{3}{5}x$

c) $y = -x$



5. Brook spends \$4.00 each day for lunch on work days. Write an equation for this statement, then create a table of values and graph. (4 marks)

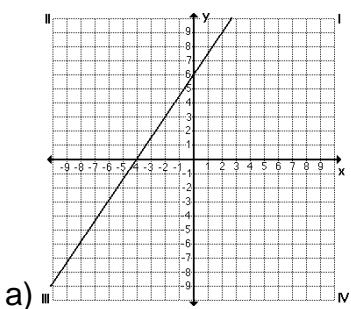
6. Write an equation of the line with each slope and y-intercept. (3 marks)

a) $m = 3, b = 0$

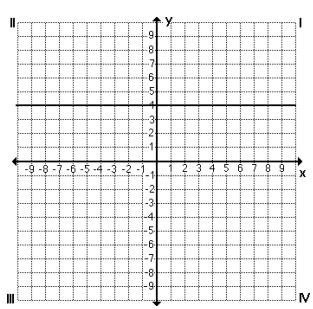
b) $m = -2, b = -3$

c) $m = \frac{3}{4}, b = +7$

7. Find the slope and y-intercept of each line, and then write its equation. (6 marks)



a)



b)



Key Question #12 (continued)

8. Graph each line. (6 marks)

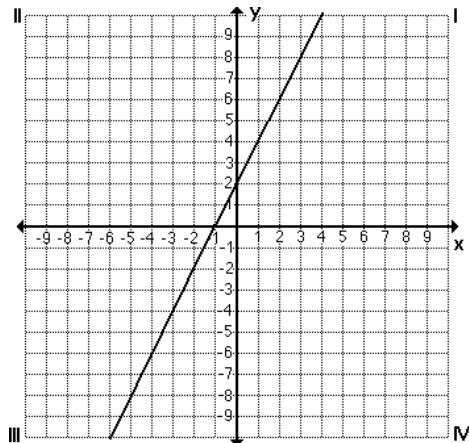
a) $y = 4x - 2$

b) $y = -\frac{1}{4}x + 1$

c) $y = -x + 3$

9. The length of time to set up is 200 min. The time to paint each sign is 25 min. Write an equation for this statement, then create a table of values and graph. (3 marks)

10. Stephen graphed the equation $y = 2x - 1$ on a Cartesian plane. Then he checked with a classmate, he realized the graph was different. Is Stephen's graph correct? If so, explain how you know. If not, explain what Stephen did wrong. (4 marks)



Scatter Plots



Lesson 13

Lesson Thirteen Concepts

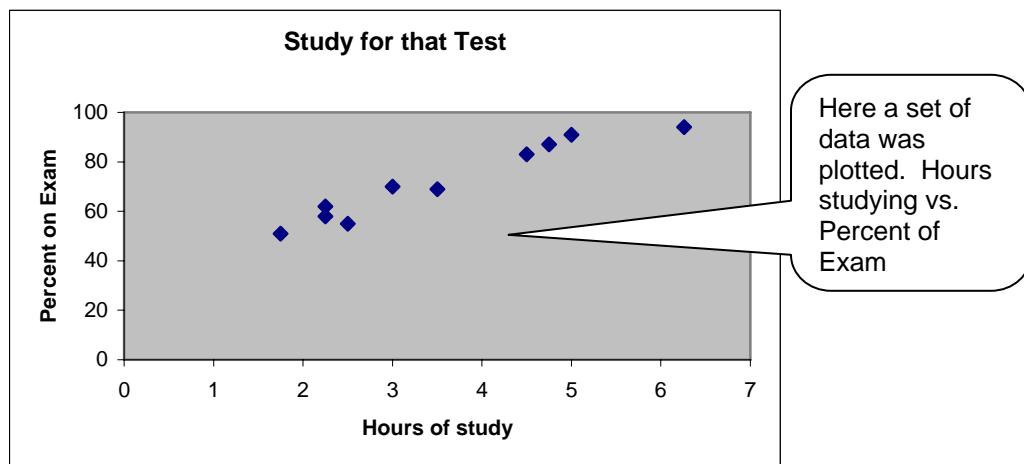
- Introduction to scatter plots
- Creating scatter plots
- Positive, negative and no correlation
- Determining the equation of best fit
- Extrapolation
- Interpolation

Scatter Plots and Line of Best Fit

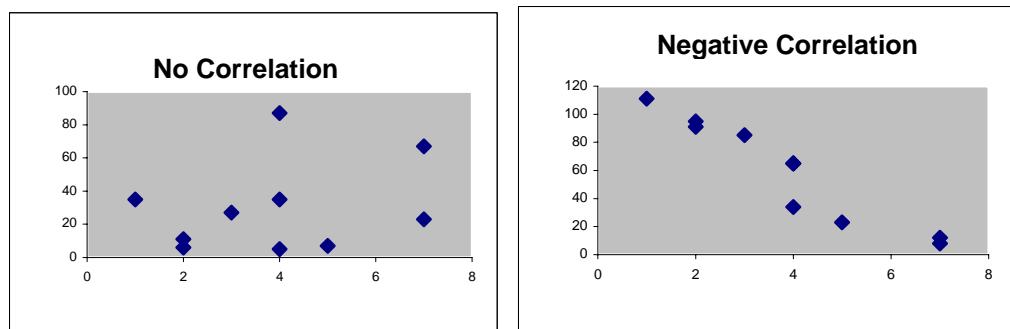
Scatter plots

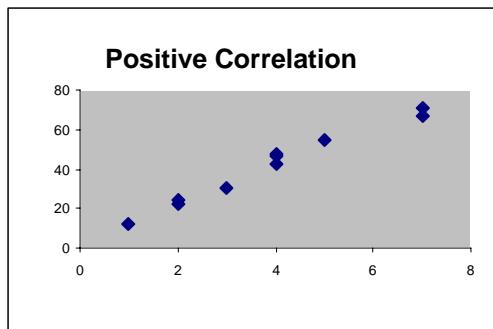
A **scatter plot** is a graph of data that is a series of points.

Example



Data in a scatter plot can have a positive/negative or no correlation.



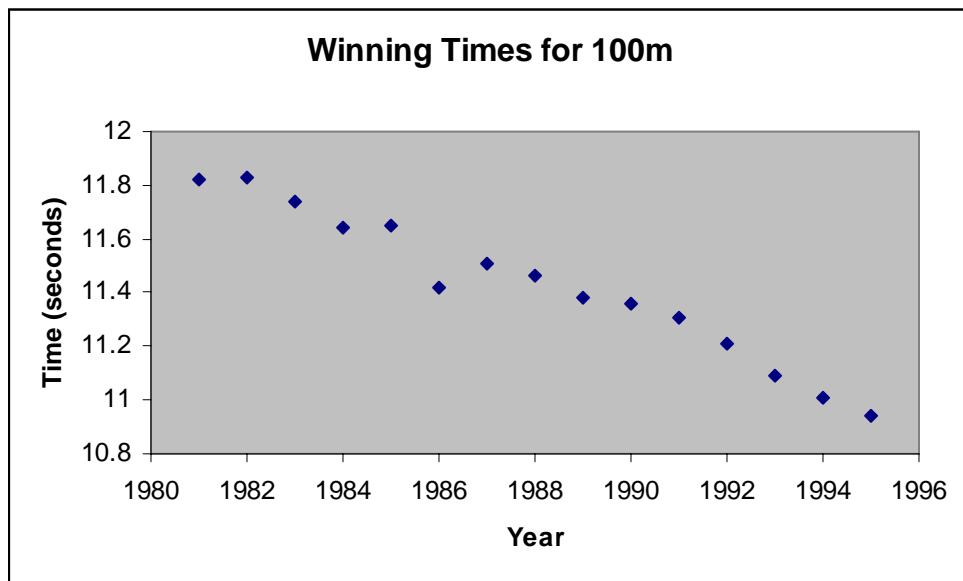


Example

Make a scatter plot for the men's times. Plot the year on the x-axes and the times on the y-axes.

Year	Time	Year	Time	Year	Time
1981	11.82	1986	11.42	1991	11.31
1982	11.83	1987	11.51	1992	11.21
1983	11.74	1988	11.46	1993	11.09
1984	11.64	1989	11.38	1994	11.01
1985	11.65	1990	11.36	1995	10.94

Solution



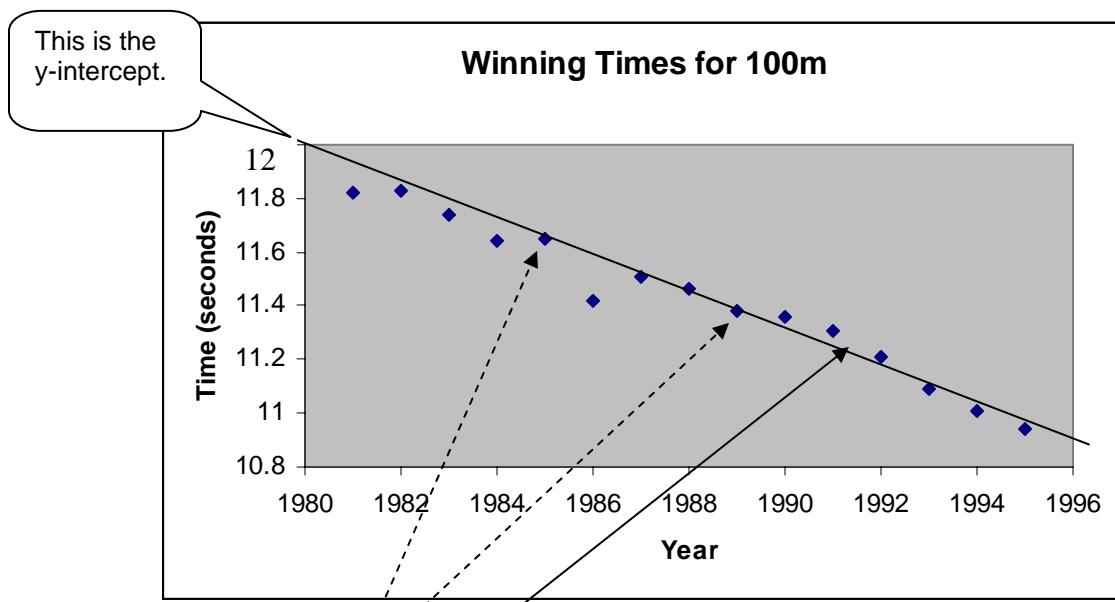
Line of Best Fit

Line of best fit is a line that passes as close as possible to a set of plotted points.

Example

Find the line of best fit for the data in the previous example.

Solution



First pick two points that represent the general center of the points plotted.

Then draw the line of best fit through those points generally dividing the plotted point even on both sides.

Example

What is the approximate equation for the line of best fit in the previous example?

Solution

Choose the coordinates of the two previously chosen points used to draw the line of best fit.

(1985, 11.7) and (1989, 11.4)

$$\text{slope } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11.4 - 11.7}{1989 - 1985}$$

$$m = \frac{-0.3}{4} = -0.075$$

$$\underline{y\text{-intercept}} = b = +12$$

Therefore the equation of the line of best fit is $y = -0.075x + 12$.

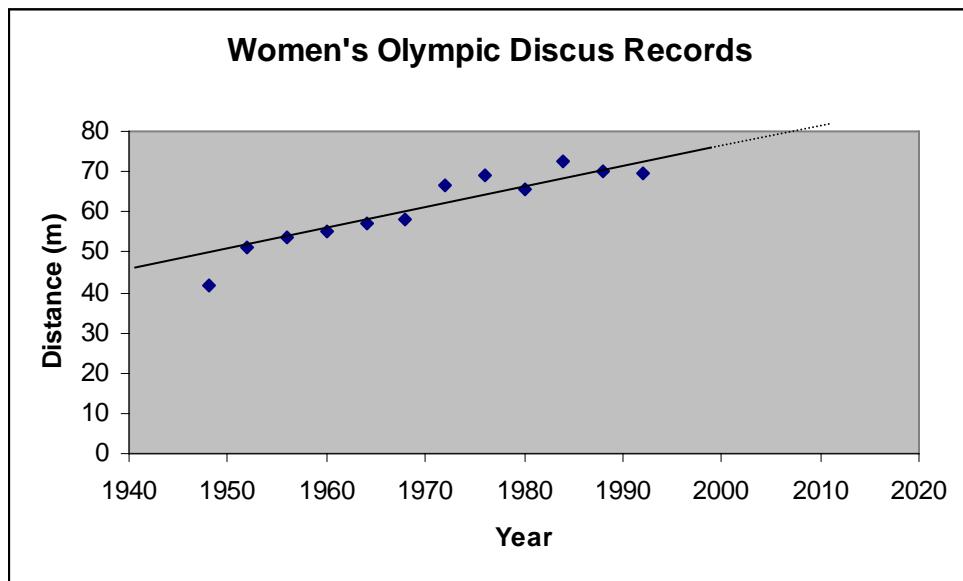
Extrapolation and Interpolation

Extrapolation is extending a graph to estimate the values that are beyond the table of values.

Interpolation is using a graph to estimate the values that are not in the table of values but are within the range of the lowest and highest values presented in the table of values.

Example

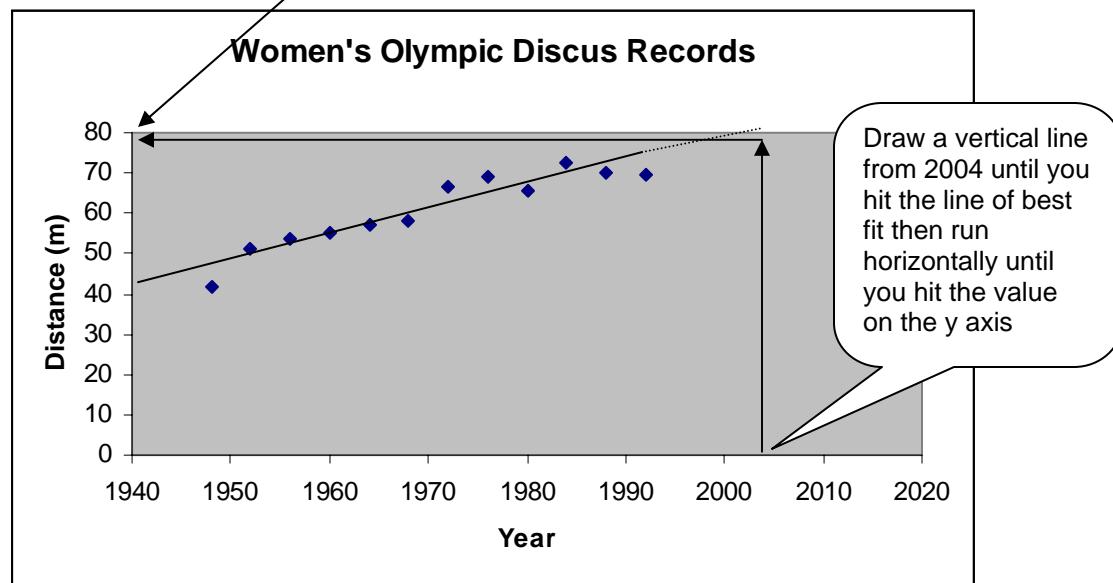
- Using extrapolation what would be the approximate distance that the discus will be thrown during the 2004 Summer Olympics?
- Using Interpolation how far was the discus thrown in 1952?



Solution

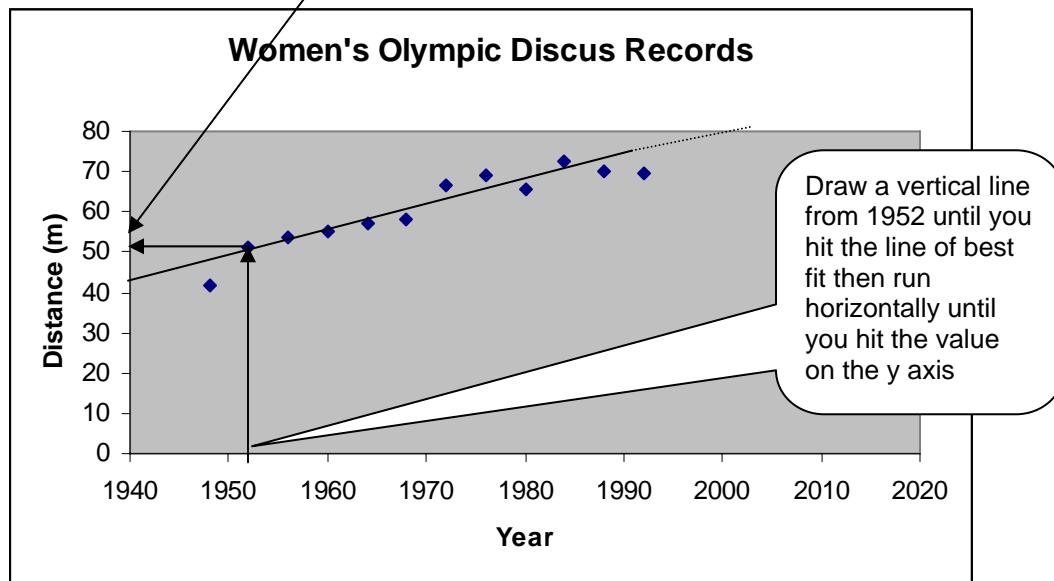
- a) Using extrapolation, what would be the approximate distance that the discus will be thrown during the 2004 Summer Olympics?

approximately 78 meters.



- b) Using Interpolation, how far was the discus thrown in 1952?

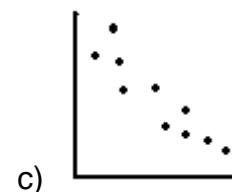
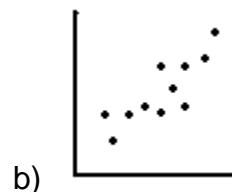
Approximately 51 metres





Support Questions

1. State whether the scatter plot has a positive, negative or no correlation.



2. The table below shows the winning times for the 800-m race at the Olympic Summer Games.

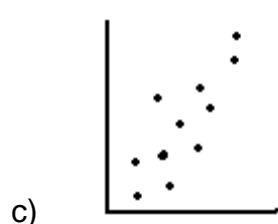
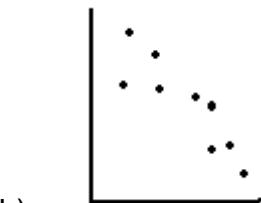
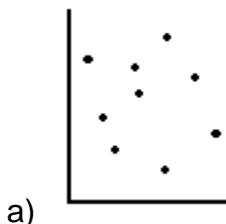
Year	Men's Time
1960	106.3
1964	105.1
1968	104.3
1972	105.9
1976	103.5
1980	105.4
1984	103
1988	103.45

- a) Construct a scatter plot and line of best fit for the data.
 b) What is the equation of the line of best fit?
 3. What is the approximate winning time in the year 2020?
 4. What approximate year does the time drop below 104 seconds?



Key Question #13

1. State whether the scatter plot has a positive, negative or no correlation.
 (3 marks)





Key Question #13 (continued)

2. The table below shows the winning heights in a men's high jump competition. (8 marks)



Year	Winning Country	Jump in Height (m)
1912	Canada	1.93
1932	United States	1.96
1952	England	1.99
1972	United States	2.16
1982	United States	2.23
2002	Australia	2.41

- a) Create a properly labelled Scatter Plot using the data given in the table.
b) Draw a line of best fit.
c) What is the approximate slope of the line of best fit?
d) Using extrapolation, what would the approximate winning height be in 2022?



3. The table below shows the percentage of high school girls who smoked more than one cigarette during the previous year. (8 marks)

	1981	1983	1985	1987	1989	1991	1993	1995	1997
Percent	34.3	29.5	25.9	25.1	24.4	22	21.3	21.6	20.4

- a) Graph the data in a scatter plot.
b) Determine the equation of the line of best fit.
c) Use your equation to predict the percentage of high school girls who smoke more than one cigarette in 2000.
d) Approximately, what percentage smoked more than one cigarette in 1982 and 1994?



Averages



Lesson 14

Lesson Fourteen Concepts

- Introduction to averages (measures of central tendency)
- Mean average
- Median average
- Mode average

Measures of Central Tendency

There are three different types of averages: **Mean**, **Median** and **Mode**.

Mean Average

Mean is a single number that is used to represent a set of numbers.

To find the mean average, all the numbers in the data set are added together and the sum is divided by the number of entries in the data set.

Example

Find the mean of the following sets of numbers.

a) $\{1,3,4,7,7,8,9,11\}$

b) $\{4,7,3,5,1,8,3,9,4,7,5,3,1\}$

Solution

a) $\{1,3,4,7,7,8,9,11\}$

$$\text{mean} = \frac{1+3+4+7+7+8+9+11}{8}$$

Divide by 8 because there are 8 numbers in the set.

$$\text{mean} = \frac{50}{8}$$

$$\text{mean} = 6.25$$

50 is the sum of all the values in the numerator.

b) $\{4,7,3,5,1,8,3,9,4,7,5,3,1\}$

$$\text{mean} = \frac{4+7+3+5+1+8+3+9+4+7+5+3+1}{13}$$

$$\text{mean} = \frac{60}{13}$$

$$\text{mean} \approx 4.62$$



Support Questions

1. Calculate the mean average for each set of numbers.
 - a) $\{3, 6, 2, 7, 3, 5, 8\}$
 - b) $\{23, 28, 32, 15, 28, 32, 35, 29, 12\}$
 - c) $\{75, 43, 57, 69, 84, 88, 94, 97, 51, 43, 45, 62, 61, 57\}$
2. Given the mean, and most of the set of data, find the missing value x in the set.
 - a) $\{4, 7, 2, 7, 9, 3, 1, 0, x\}$; mean = 3
 - b) $\{22, 26, 12, 25, 29, 31, 25, 39, 11, x\}$; mean = 24.5
 - c) $\{65, 42, 47, 60, 74, 98, 84, 96, 41, 53, 35, 72, 62, 58, x\}$; mean = 64
3. The salaries of the 2003-2004 Toronto Maple Leafs is given below. What is the mean average for a player's salary?

\$2,750,000	\$950,000	\$522,500
\$2,200,000	\$925,000	\$500,000
\$2,000,000	\$900,000	\$450,000
\$2,000,000	\$850,000	\$400,000
\$2,000,000	\$700,000	
\$1,600,000	\$650,000	
\$1,500,000	\$585,640	
\$1,400,000	\$575,000	

Median Average

Median average is the middle number of a set of numbers arranged in numerical order.

If there are an even number of values in the set of data then there will be two middle numbers and the mean of those two numbers is the median average.

Example

Find the median of the following sets of numbers.

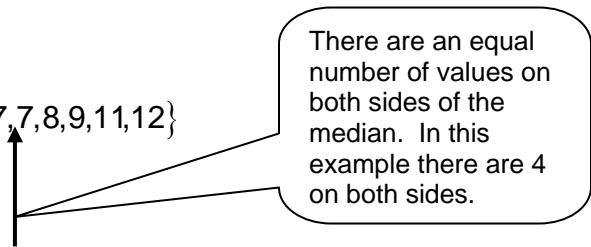
a) $\{7, 9, 12, 1, 7, 8, 3, 11, 4\}$

b) $\{4, 7, 3, 5, 1, 8, 3, 9, 4, 7, 5, 3\}$

Solution

a) $\{7, 9, 12, 1, 7, 8, 3, 11, 4\}$

Arrange in numerical order. $\{1, 3, 4, 7, 7, 8, 9, 11, 12\}$



The middle value of this set of data is 7, therefore the median average is 7. Median = 7

b) $\{4, 7, 3, 5, 1, 8, 3, 9, 4, 7, 5, 3\}$

Arrange in numerical order. $\{1, 3, 3, 3, 4, 4, 5, 5, 7, 7, 8, 9\}$

$$\text{mean} = \frac{4 + 5}{2}$$

$$\text{mean} = \frac{9}{2}$$

$$\text{mean} = 4.5$$

This time there are two middle numbers so the median is the mean of 4 and 5.

Therefore the median of the set of data is 4.5.

Median average = 4.5



Support Questions

4. Calculate the median average for each set of numbers.
- $\{3, 6, 2, 7, 3, 5, 8\}$
 - $\{23, 28, 32, 15, 28, 32, 35, 29, 12\}$
 - $\{75, 43, 57, 69, 84, 88, 94, 97, 51, 43, 45, 62, 61, 57\}$
5. The salaries of the 2003-2004 Toronto Maple Leafs is given below. What is the median average for a player's salary?

\$4,750,000	\$950,000	\$522,500
\$4,200,000	\$925,000	\$500,000
\$3,000,000	\$900,000	\$450,000
\$2,500,000	\$850,000	\$400,000
\$2,000,000	\$700,000	
\$1,600,000	\$850,000	
\$1,500,000	\$785,640	
\$1,400,000	\$475,000	

Mode Average

Mode average is the most common value in a set of numbers.

Example

Find the mode of the following sets of numbers.

- $\{7, 9, 12, 1, 7, 8, 3, 11, 4\}$
- $\{4, 7, 3, 5, 1, 8, 3, 9, 4, 7, 5, 3\}$

Solution

- a) $\{7, 9, 12, 1, 7, 8, 3, 11, 4\}$ Arrange in numerical order because it is easier to see the repeated numbers.

$$\{1, 3, 4, \underline{7}, 7, 8, 9, 11, 12\}$$

7 is the most common number in the set so therefore, the mode average = 7.

- b) $\{4, 7, 3, 8, 5, 1, 8, 3, 8, 9, 4, 7, 8, 5, 3\}$ Arrange in numerical order because it is easier to see the repeated numbers.

$\{1, 3, 3, 3, 4, 4, 5, 5, 7, 7, \underline{8}, \underline{8}, \underline{8}, 9, \}$

8 is the most common number in the set so therefore, the mode average = 8.



Support Questions

6. Calculate the mode average for each set of numbers.
- $\{3, 6, 2, 7, 3, 5, 8\}$
 - $\{23, 28, 32, 15, 28, 32, 35, 29, 12, 28, 32\}$
 - $\{75, 43, 57, 69, 43, 88, 94, 97, 51, 43, 45, 62, 61, 57, 82, 75, 51\}$
7. The salaries of the 2003-2004 Toronto Maple Leafs is given below. What is the mode average for a player's salary?

\$4,750,000	\$950,000	\$522,500	\$4,200,000	\$925,000
\$525,000	\$3,000,000	\$900,000	\$400,000	\$2,500,000
\$850,000	\$400,000	\$2,000,000	\$850,000	\$1,600,000
\$850,000	\$1,500,000	\$785,640	\$1,400,000	\$475,000

8. Find the mean, median and mode for the given set of numbers.
- $\{63, 74, 77, 68, 71, 74, 70, 65\}$
9. Determine each measure of central tendency based on the table given below.

Annual Salary (\$)	Frequency
35 600	3
42 750	5
51 000	6
99 000	1
150 000	1



Key Question #14

1. Find the mean, median and mode for each set of numbers. (9 marks)

- a) $\{34, 47, 12, 36, 26, 34, 28, 26, 48\}$
- b) $\{21, 23, 26, 34, 21, 29, 36, 45, 32, 26, 28, 26\}$
- c) $\{8, 6, 2, 4, 7, 6, 2, 5, 3, 7, 9, 7\}$

2. Two friends went golfing. Their score card is given below. (6 marks)

Hole	1	2	3	4	5	6	7	8	9	Total
Steve	6	5	4	4	3	4	6	8	3	43
Mary	6	6	3	3	4	5	6	8	4	45

- a) Calculate the mean score for each player.
 - b) Calculate the mean score for the two players together.
 - c) Calculate the median score for each player.
 - d) Calculate the median score for the two players together.
 - e) Determine the mode for each player.
 - f) Determine the mode for the two players together.
3. The average price of gas in cents per litre is given for each province. (2 marks)

Province	Cost of Gas per Litre (cents)
British Columbia	75.2
Alberta	68.5
Saskatchewan	69.3
Manitoba	71.3
Ontario	75.3
Quebec	74.5
New Brunswick	74.6
Nova Scotia	76.3
Prince Edward Is.	75.4
Newfoundland	77.9



- a) What is the mean price of gas for all the Canadian Provinces?
- b) What is the median price of gas for all the Canadian Provinces?



Key Question #14 (continued)

4. Is each statement always true, sometimes true or sometimes false? Explain and provide an example to illustrate your understanding. (8 marks)
- If a list of numbers has a mode, it is one of the numbers in the list?
 - The median of a list of whole numbers is a whole number.
 - The mean of a list of numbers is one of the numbers in the list?
 - The mean, median, and mode of a list of numbers are not equal.
5. Given the mean, and most of the set of data, find the missing value x in the set. (4 marks)
- $\{3, 5, 6, 8, 9, 2, 1, 0, 5, x\}$; mean = 4
 - $\{6, 7, 2, 9, 4, x\}$; mean = 5
6. The April batting statistics of the 2004 Toronto Bluejays is given below. (6 marks)



Player	TEAM	POS	G	AB	R	H	2B	3B	HR	RBI	TB	BB	SO	SB	CS	OBP	SLG	AVG
1. W Wells	TOR	OF	22	95	15	20	7	0	1	6	30	7	15	1	0	.265	.316	.211
2. J Phelps	TOR	DH	22	85	11	22	3	1	1	9	30	6	24	0	0	.337	.353	.259
3. C Delgado	TOR	1B	22	83	13	22	7	0	4	13	41	12	16	0	0	.367	.494	.265
4. E Hinske	TOR	3B	21	77	12	18	3	0	2	12	27	12	15	0	1	.330	.351	.234
5. F Catalanotto	TOR	OF	21	75	5	21	6	1	0	9	29	6	14	0	0	.333	.387	.280
6. R Johnson	TOR	OF	22	73	8	18	1	1	2	10	27	6	13	0	1	.337	.370	.247
7. O Hudson	TOR	2B	21	72	11	18	6	1	1	6	29	8	15	1	1	.341	.403	.250
8. K Cash	TOR	C	19	64	7	17	6	0	2	13	29	4	19	0	0	.324	.453	.266
9. C Woodward	TOR	SS	18	58	7	18	8	2	0	6	30	6	10	0	1	.369	.517	.310
10. H Clark	TOR	OF	8	27	4	9	3	0	0	3	12	1	4	0	0	.357	.444	.333
11. C Gomez	TOR	SS	13	23	6	8	1	0	1	6	12	5	6	0	0	.464	.522	.348
12. G Myers	TOR	C	8	18	0	4	2	0	0	1	6	2	4	0	0	.300	.333	.222
13. D Berg	TOR	OF	8	15	0	1	0	0	0	1	1	0	3	0	0	.067	.067	.067

- What is the team's mean batting average (AVG)?
- What is the team's mode for doubles(2B)?
- What is the team's mean at bats (AB)?
- What is the team's median for strikeouts (SO)?
- What is the team's mean slugging percents (SLG)?
- What is the team's mean, median and mode for home runs (HR)?

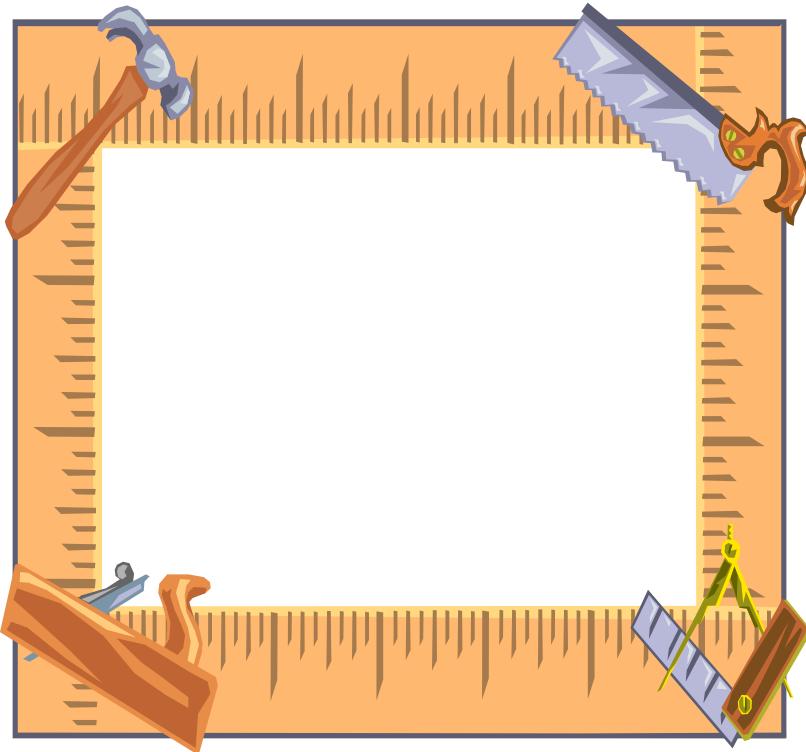


Key Question #14 (continued)

7. Determine each measure of central tendency based on the table given below. (3 marks)

Annual Salary (\$)	Frequency
35 600	2
42 750	4
51 000	7
99 000	2
150 000	3

Perimeter



Lesson 15

Lesson Fifteen Concepts

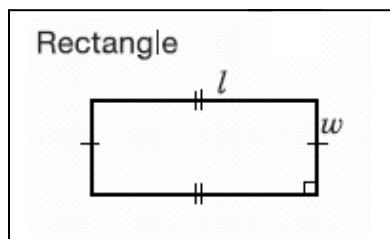
- Introduction to perimeter and circumference
- Radius and diameter
- Calculations using pi (π)
- Solving perimeter/circumference questions using formulas and substitution

Perimeter

Perimeter

Perimeter is the distance around an object. If that object is a circle then the perimeter is called **circumference**.

Formulas to be used to calculate perimeter.



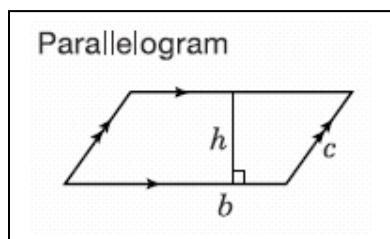
$$P = 2l + 2w$$

or

$$P = 2(l + w)$$

or

$$P = l + w + l + w$$



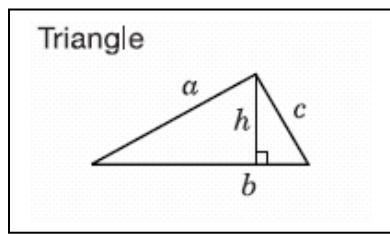
$$P = 2b + 2c$$

or

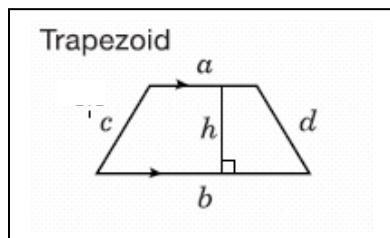
$$P = 2(b + c)$$

or

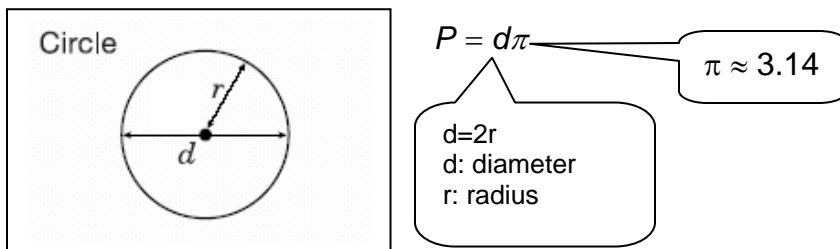
$$P = b + c + b + c$$



$$P = a + b + c$$

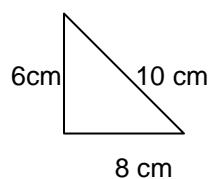


$$P = a + b + c + d$$

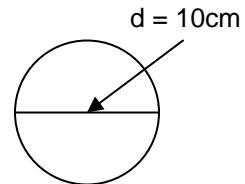
**Example**

Find the perimeter of each of the following shapes.

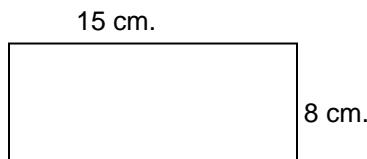
a)



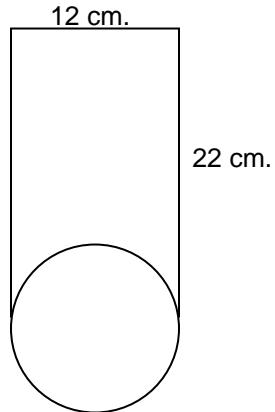
b)



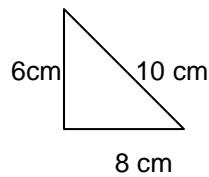
c)



d)

**Solution**

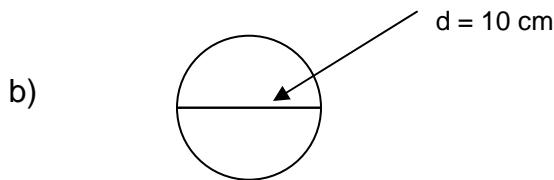
a)



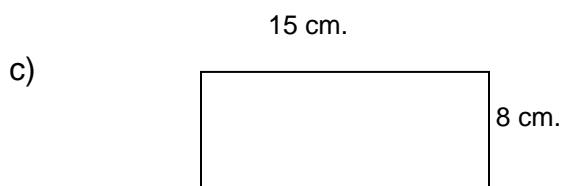
$$P = a + b + c$$

$$P = 6 + 8 + 10$$

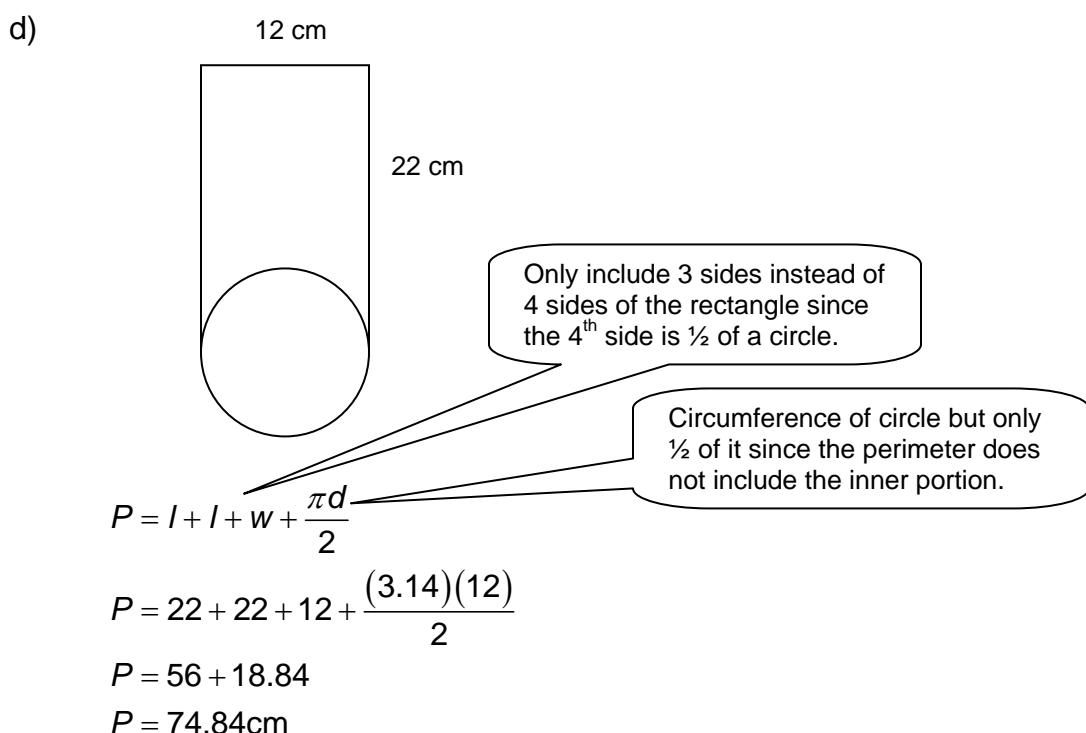
$$P = 24\text{cm}$$



$$\begin{aligned}
 P &= \pi d \\
 P &= (3.14)(10) \\
 P &= 31.4 \text{ cm}
 \end{aligned}$$

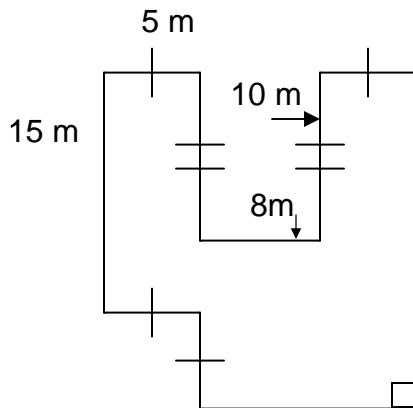


$$\begin{aligned}
 P &= 2(l + w) \\
 P &= 2(8 + 15) \\
 P &= 2(23) \\
 P &= 46 \text{ cm}
 \end{aligned}$$

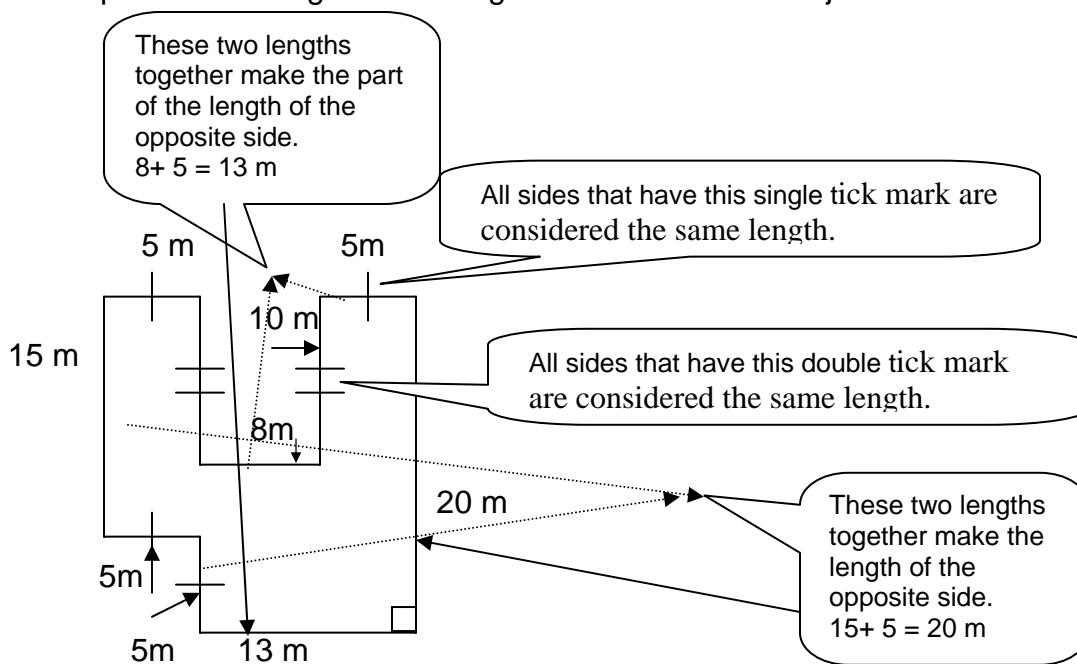


Example

Find the perimeter.

**Solution**

It is important to recognize the lengths of all side of the object.



So the Perimeter is

$$\begin{aligned} P &= 5 + 10 + 8 + 10 + 5 + 20 + 13 + 5 + 5 + 15 \\ &= 96 \text{ m} \end{aligned}$$

or

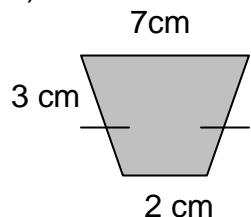
$$\begin{aligned} P &= 4(5) + 2(10) + 20 + 15 + 13 + 8 \\ &= 96 \text{ m} \end{aligned}$$



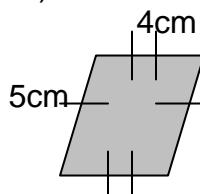
Support Questions

1. Calculate the perimeter for each of the following objects.

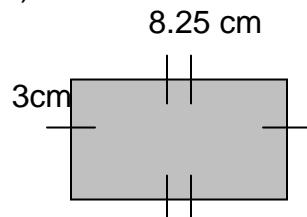
a)



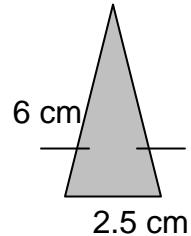
b)



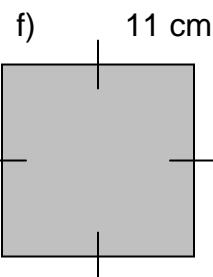
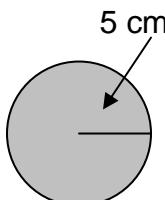
c)



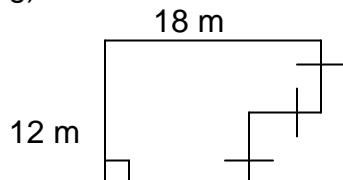
d)



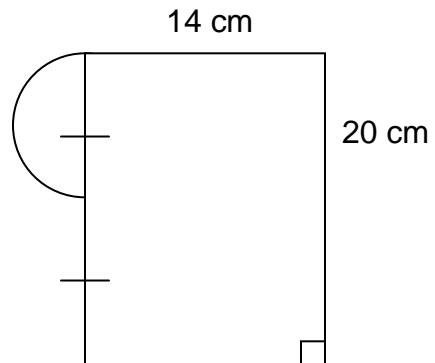
e)



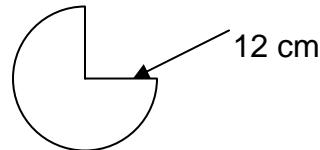
g)



h)



i)

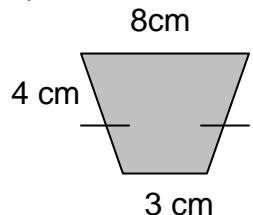




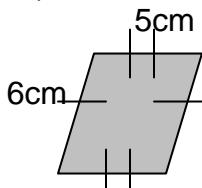
Key Question #15

1. Calculate the perimeter for each of the following objects. (12 marks)

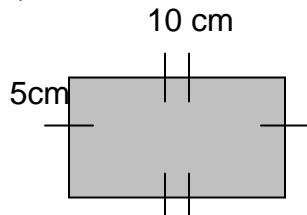
a)



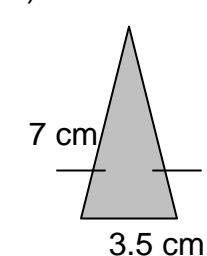
b)



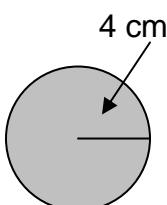
c)



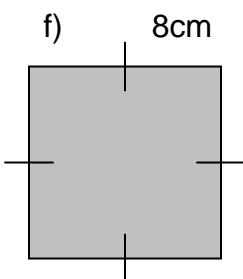
d)



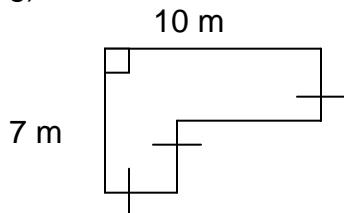
e)



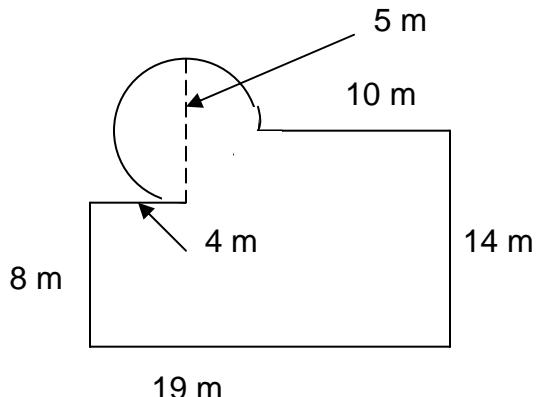
f)



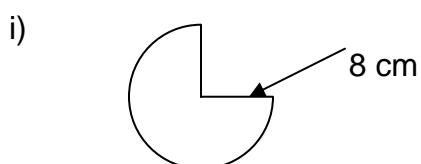
g)



h)



i)





Key Question #15 (continued)

2. Explain how you could find the circumference of a circle given its area. (3 marks)
3. A rectangular yard has an area of 24m^2 . What are four possible perimeters for this yard? (4 marks)
4. When a circle's radius doubles so does its circumference. True or False? Prove your answer with an example. (4 marks)
5. What is the total amount of edging that would be required to cover all edges of a rectangular prism measuring 10 cm by 15 cm by 30 cm? (4 marks)



Lesson 11

1. a) slope = -4 , y-int = 3 b) slope = 1 , y-int = -4

c) slope = $-\frac{2}{3}$, y-int = 7 d) slope = $\frac{3}{5}$, y-int = -1

2. a) $(4, 4)$ b) $(12, -8)$ (8, -5)

$$y = -2x + 4$$

$$(-4) = -2(4) + 4$$

$$-4 = -8 + 4$$

$$-4 = -4$$

$$y = -\frac{3}{4}x + 1$$

$$(-8) = -\frac{3}{4}(12) + 1$$

$$-8 = -9 + 1$$

$$-8 = -8$$

$$y = -\frac{3}{4}x + 1$$

$$-5 = -\frac{3}{4}(8) + 1$$

$$-5 = -6 + 1$$

$$-5 = -5$$

c) $(1, -1)$ (4, -6)

$$y = -x - 2$$

$$-1 = -(-1) - 2$$

$$-1 = 1 - 2$$

$$-1 = -1$$

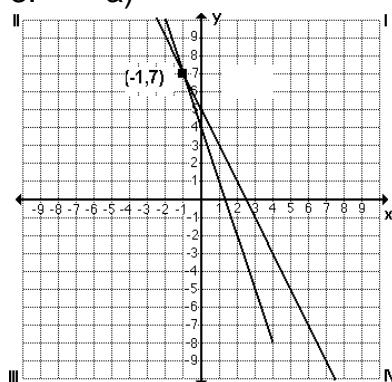
$$y = -x - 2$$

$$-6 = -(4) - 2$$

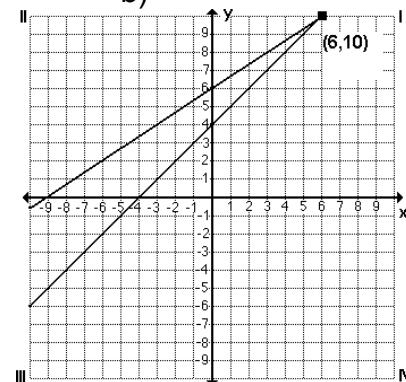
$$-6 = -6$$

3.

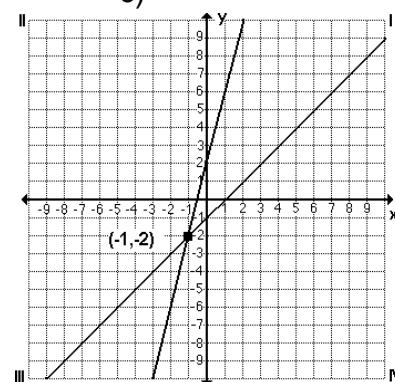
a)



b)



c)



4. a)

$$\begin{aligned}
 y &= x + 1 \\
 -x + y &= x - x + 1 \\
 -x + y &= +1 \\
 -x + y - 1 &= 1 - 1 \\
 -x + y - 1 &= 0 \\
 x - y + 1 &= 0
 \end{aligned}$$

b)

$$\begin{aligned}
 y &= -\frac{3}{7}x - 2 \\
 7(y) &= 7\left(-\frac{3}{7}x - 2\right) \\
 7y &= -3x - 14 \\
 3x + 7y &= -14 \\
 3x + 7y + 14 &= 0
 \end{aligned}$$

c)

$$\begin{aligned}
 y &= 5x - \frac{1}{2} \\
 2(y) &= 2(5x) - 2\left(\frac{1}{2}\right) \\
 2y &= 10x - 1 \\
 -10x + 2y &= -1 \\
 -10x + 2y + 1 &= 0 \\
 10x - 2y - 1 &= 0
 \end{aligned}$$

d)

$$\begin{aligned}
 y &= -3x + 7 \\
 3x + y &= 7 \\
 3x + y - 7 &= 0
 \end{aligned}$$

5.

a)

$$\begin{aligned}
 3x - 2y + 6 &= 0 \\
 -2y + 6 &= -3x \\
 -2y &= -3x - 6 \\
 \frac{-2y}{-2} &= \frac{-3x - 6}{-2} \\
 y &= \frac{3}{2}x + 3
 \end{aligned}$$

b)

$$\begin{aligned}
 2x + 5y - 10 &= 0 \\
 5y - 10 &= -2x \\
 5y &= -2x + 10 \\
 \frac{5y}{5} &= \frac{-2x + 10}{5} \\
 y &= \frac{-2}{5}x + 2
 \end{aligned}$$

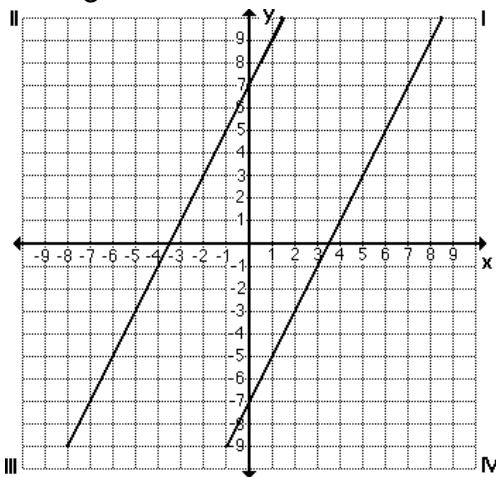
c)

$$\begin{aligned}
 5x - 2y + 7 &= 0 \\
 -2y + 7 &= -5x \\
 -2y &= -5x - 7 \\
 \frac{-2y}{-2} &= \frac{-5x - 7}{-2} \\
 y &= \frac{5}{2}x + \frac{7}{2}
 \end{aligned}$$

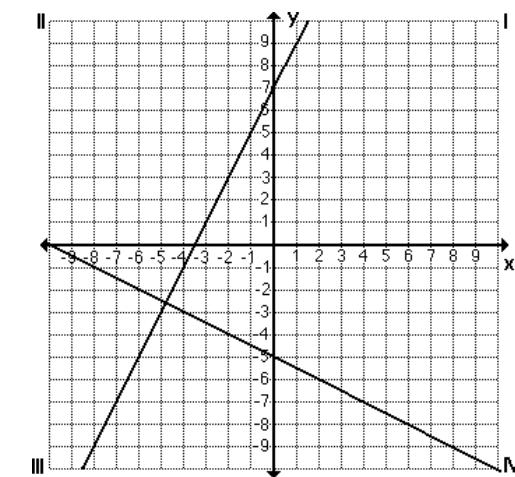
6.

$$y = \frac{2}{3}x + 2$$

$$y = \frac{2}{3}x - 5$$



$$\begin{aligned}
 y &= 2x + 7 \\
 y &= -\frac{1}{2}x - 6
 \end{aligned}$$



8. a)

b)

c)

$$\begin{array}{l}
 \text{y - intercept} \\
 \text{set } x = 0 \\
 -2y + 6 = 0 \\
 -2y = -6 \\
 y = 3
 \end{array}$$

$$\begin{array}{l}
 \text{y - intercept} \\
 \text{set } x = 0 \\
 5y - 10 = 0 \\
 5y = 10 \\
 y = 2
 \end{array}$$

$$\begin{array}{l}
 \text{y - intercept} \\
 \text{set } x = 0 \\
 -2y + 7 = 0 \\
 -2y = -7 \\
 y = \frac{7}{2}
 \end{array}$$

$$\begin{array}{l}
 \text{x - intercept} \\
 \text{set } y = 0 \\
 3x + 6 = 0 \\
 3x = -6 \\
 x = -2
 \end{array}$$

$$\begin{array}{l}
 \text{x - intercept} \\
 \text{set } y = 0 \\
 2x - 10 = 0 \\
 2x = 10 \\
 x = 5
 \end{array}$$

$$\begin{array}{l}
 \text{x - intercept} \\
 \text{set } y = 0 \\
 5x + 7 = 0 \\
 5x = -7 \\
 x = -\frac{7}{5}
 \end{array}$$

9. a)

b)

$$\begin{array}{l}
 \text{y - intercept} \\
 \text{set } x = 0 \\
 y = 1
 \end{array}$$

$$\begin{array}{l}
 \text{y - intercept} \\
 \text{set } x = 0 \\
 y = -2
 \end{array}$$

$$\begin{array}{l}
 \text{x - intercept} \\
 \text{set } y = 0 \\
 0 = x + 1 \\
 -1 = x
 \end{array}$$

$$\begin{array}{l}
 \text{x - intercept} \\
 \text{set } y = 0 \\
 0 = \frac{-3}{7}x - 2 \\
 \frac{3}{7}x = -2 \\
 7(\frac{3}{7})x = 7(-2) \\
 3x = -14 \\
 x = \frac{-14}{3}
 \end{array}$$

Lesson 12

1. a) -4

b) 1

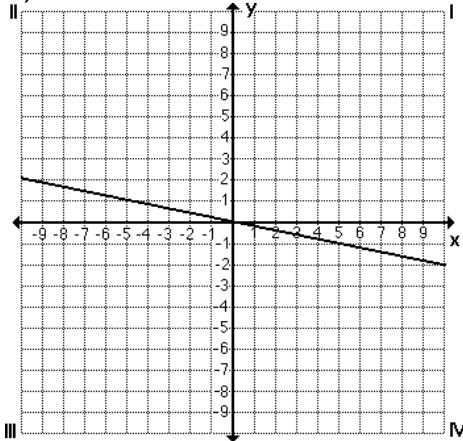
c) $-\frac{1}{3}$ d) $\frac{2}{5}$ 2. a) $y = -5x$ b) $y = -\frac{3}{7}x$ c) $y = \frac{1}{4}x$ d) $y = x$

3. a) $y = \frac{1}{3}x$

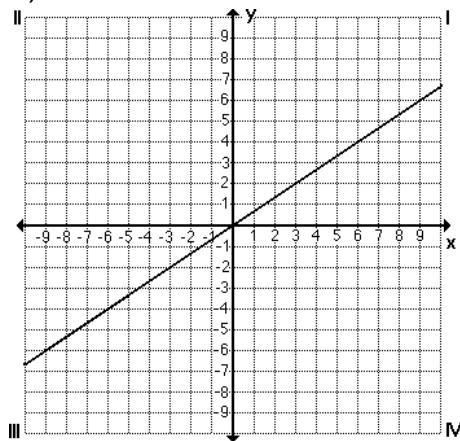
b) $y = -\frac{5}{4}x$

4.

a)



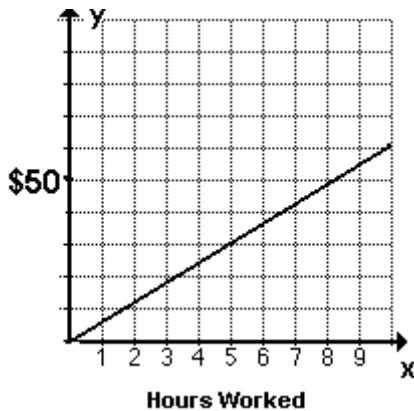
b)



5.

$E = 6h$

h	E
0	0
2	12
4	24
10	60



6. a) $y = -5x + 1$

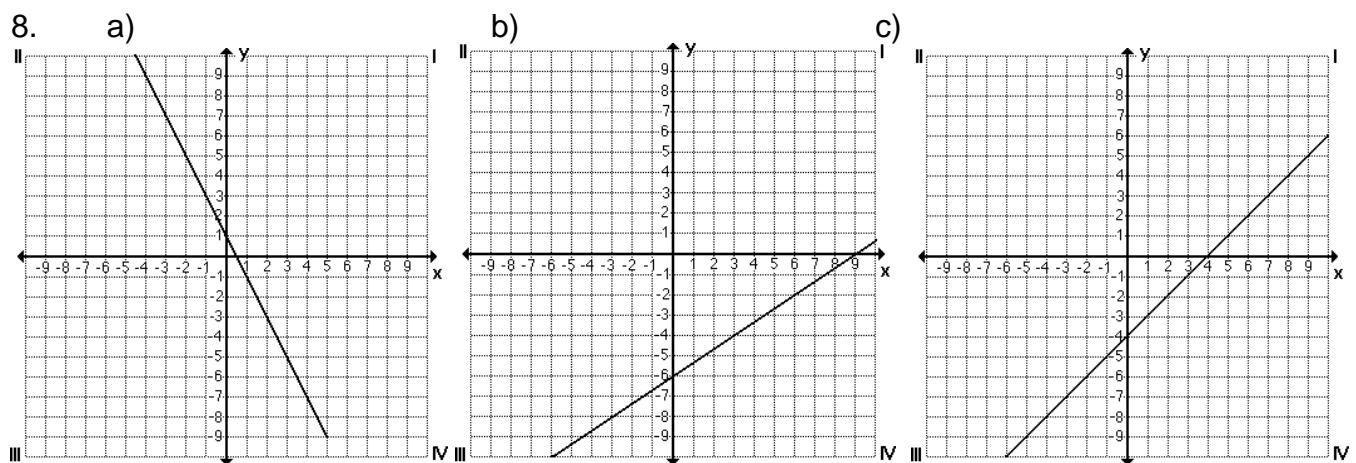
b) $y = -\frac{3}{7}x - 4$

c) $y = \frac{1}{4}x + 6$

7. a) $y = \frac{7}{5}x + 5$

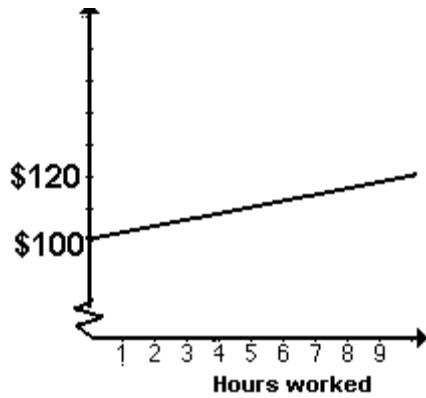
b) $y = -\frac{1}{3}x - 1$

8.



9. $E = 2h + 100$

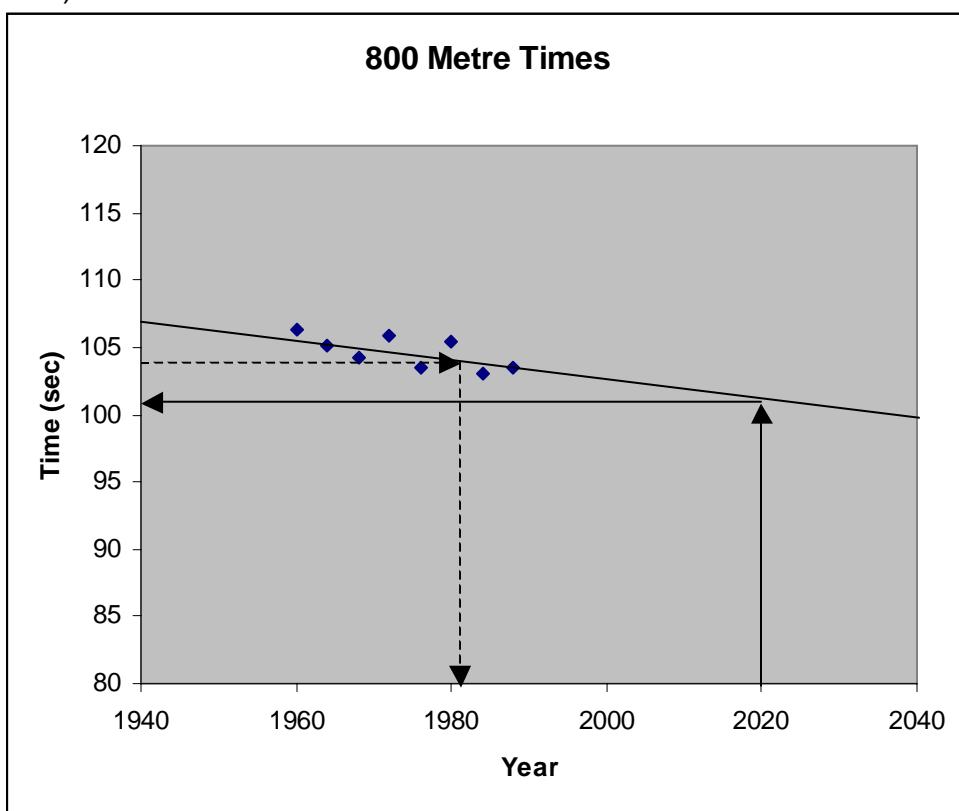
h	E
0	100
2	104
4	108
10	120



Lesson 13

1. a) no correlation b) positive correlation c) negative correlation

2. a)



b)

Equation of best fit

Use the coordinates (1964, 105.1) and (1988, 103.45)

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{103.45 - 105.1}{1988 - 1964} = \frac{-1.65}{24} = -0.06875$$

$$y - \text{int} = 107$$

$$y = -0.06875x + 107$$

b) ≈ 101 secc) ≈ 1981

Lesson 14

1. a) $\frac{3 + 6 + 2 + 7 + 3 + 5 + 8}{7} \approx 4.86$

b) $\frac{23 + 28 + 32 + 15 + 28 + 32 + 35 + 29 + 12}{9} = 26$

c) $\frac{75 + 43 + 57 + 69 + 84 + 88 + 94 + 97 + 51 + 43 + 45 + 62 + 61 + 57}{14} \approx 66.1$

2. a)

$$\frac{33+x}{9} = 3$$

$$33+x = 27$$

$$x = -6$$

b)

$$\frac{220+x}{10} = 24.5$$

$$220+x = 245$$

$$x = 25$$

c)

$$\frac{887+x}{15} = 64$$

$$887+x = 960$$

$$x = 73$$

3.
$$\left(\begin{array}{l} 2750000 + 2200000 + 2000000 + 2000000 + 2000000 + 1600000 + 1500000 + 1400000 + 950000 + 925000 \\ + 900000 + 850000 + 700000 + 650000 + 585640 + 575000 + 522500 + 500000 + 450000 + 400000 \end{array} \right) / 20$$

$$= \$1\,172\,907$$

4. a) $\{2,3,3,\underline{5},6,7,8\}$ median = 5

b) $\{12,15,23,28,\underline{28},29,32,32,35\}$ median = 28

c)

$$\{43,43,45,51,57,57,\underline{61},\underline{62},69,75,84,88,94,99\}$$

$$\frac{61+62}{2} = \frac{123}{2} = 61.5 \quad \text{median} = 61.5$$

5.
$$\frac{925000 + 900000}{2} = \frac{1825000}{2} = 912500 \quad \text{median} = \$912\,500$$

6. a) 3 (occurs 2 times) b) 28 or 32 (occur three times) c) 43 (occurs 3 times)

7. \$850 000

8.

$$\text{mean} = \left(\frac{63+74+77+68+71+74+70+65}{8} \right) = 70.25$$

$$\text{median} = \{63, 65, 68, \underline{70}, \underline{71}, 74, 74, 77\} = 71$$

$$\text{mode} = 74$$

9.

$$\text{mean} = \frac{35600 \times 3 + 42750 \times 5 + 51000 \times 6 + 99000 + 150000}{16} \approx \$54721.88$$

$$\text{median} = \frac{42750 + 51000}{2} = \$46875$$

$$\text{Mode} = 51000$$

Lesson 15

1. a) $P = 7 + 3 + 2 + 3$
 $P = 15 \text{ cm}$
- b) $P = 2(b + c)$
 $P = 2(4 + 5)$
 $P = 2(9)$
 $P = 18 \text{ cm}$
- c) $P = 2(l + w)$
 $P = 2(8.25 + 3)$
 $P = 2(11.25)$
 $P = 22.5 \text{ cm}$
- d) $P = 6 + 6 + 2.5$
 $P = 14.5 \text{ cm}$
- e) $C = 2\pi r$
 $C = 2(3.14)(5)$
 $C = 31.4 \text{ cm}$
- f) $P = 4(l)$
 $P = 4(11)$
 $P = 44 \text{ cm}$
- g) $P = 12 + 18 + 6 + 6 + 6 + 12$
 $P = 60 \text{ m}$
- h) $P = 14 + 20 + 14 + 10 + \frac{2\pi r}{2}$
 $P = 58 + \frac{2(3.14)(5)}{2}$
 $P = 73.7 \text{ cm}$
- i) $P = \frac{3}{4}(2\pi r) + 12 + 12$
 $P = \frac{3}{4}(2)(3.14)(12) + 12 + 12$
 $P = 56.52 + 24$
 $P = 80.52 \text{ cm}$