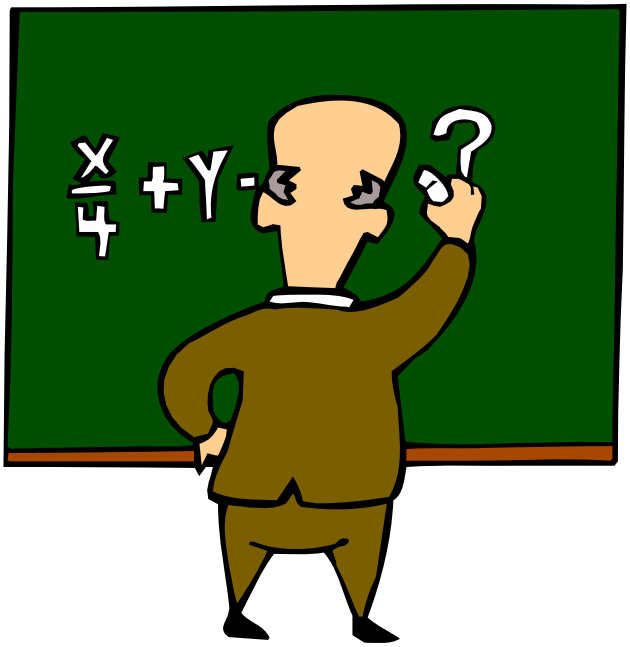
Polynomials



Lesson 6

Lesson Six Concepts

- Introduction to polynomials
- Like terms
- Addition and subtraction of polynomials
- Distributive law
- Multiplication and division of polynomials
- Simplifying expressions then using substitution

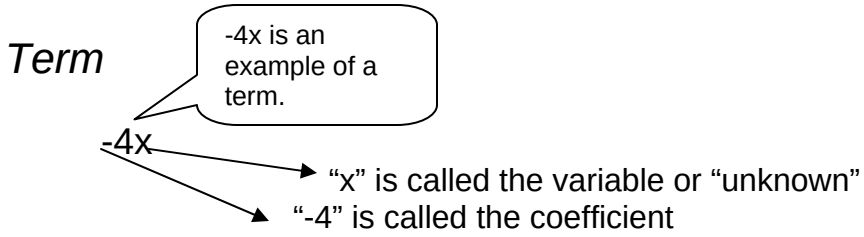
Polynomials

Polynomials are a mathematical expression with one or more terms, in which the exponents are whole numbers and the coefficients are real numbers.

Monomial is a polynomial with one term; e.g. -8 and $3x^2$.

Binomial is a polynomial with two terms; e.g. –4x +7 and $y^3 - 12$.

Trinomial is a polynomial with three terms; e.g. $4x^2 - 3x + 9$ and $-w^3 + 7w + 2$.



Adding and Subtracting Polynomials

To add and/or subtract polynomials the terms ***must*** be "like terms".

Like terms are terms that have the same variables with the same value of exponent for each variable.

Example

Underline the terms that are like terms.

a) –3x, x, $7x^2$, 2y, 12x

b) $7xy^3$, $-5x^3y$, $-y^3x$, $5xy^2$

Solution

a) $-3x$, x, $7x^2$, $2y$, $12x$

b) $7xy^3$, $-5x^3y$, $-y^3x$, $5xy^2$

The order of the variables does not matter.

Example

Simplify.

a) $(5x+1)+(3x+7)$

b) $(-a+4)-(2a-3)$

c) $(4w^2+6w-8)-(9w^2+2w+5)$

Solutions

a) $(5x+1)+(3x+7)$

The first set of brackets can be removed if there is no value or negative sign in front of the brackets.

If this is a plus sign the second set of brackets can also be removed.

Distributive Law:

a(b+c) = ab +ac

a(b-c)=ab-ac
where a, b, and c can be any real number

$$(5x+1)+(3x+7) = 5x+1+3x+7$$
$$= 5x+3x+1+7$$
$$= 8x+8$$

Group like terms and add the like terms together.

b) $(-a+4)-(2a-3) = -a+4-2a+3$

Because this sign is negative the distributive law is used.

$$= -a-2a+4+3$$
$$= -3a+7$$

c) $(4w^2+6w-8)-(9w^2+2w+5) = 4w^2+6w-8-9w^2-2w-5$

$$= 4w^2-9w^2+6w-2w-8-5$$
$$= -5w^2+4w-13$$

Support Questions

1. State the like terms in each group.

   a) $3x, 5y, 5z, x^2, -x, 3w, 3v$
   b) $4x, 3y^2, 4z, 2y, y^2, 4w$

2. Simplify.

   a) $(-3x+3)+(4x+2)$
   b) $(3-5n)-(-6n+2)$
   c) $(8a^2+2a-3)+(-5a^2+4a+7)$
   d) $(-6x^2+5x+2)-(4x^2+5-2x)$
   e) $(3-2m-n^2)+(7-6m+n^2)$
   f) $(2+6x^2)-(7-3x^2)$
   g) $(5-6w^2)-(3-w^2)$
   h) $(5x^2-3x)+(-4x+5x^2)$

3. Simplify. Then determine the value of the polynomial when n=2 and when n= –1.

   a) $(7n^2-3n+4)-(-4n^2-2n-3)$
   b) $(3n^2-7n+2)+(-2n^2+6n+3)$

Multiplying and Dividing Polynomials

To multiply and/or divide polynomials the terms ***do not have to*** be "like terms".

Example

Simplify.

a) $(4x^2)(-3x^4)$

b) $\dfrac{16x^4y^3}{-8x^3y}$

c) $\left(-2w^2y\right)(5wx)$

d) $100x^2yz^3 \div 25xyz^2$

Solutions

a) $(4x^2)(3x^4)$ $= 4 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$

$= 12x^6$

Use the multiplication of exponents rule

or $= (4)(3)\left(x^2\right)\left(x^4\right)$

$= 12x^{2+4}$

$= 12x^6$

b) $\dfrac{16x^4y^3}{-8x^3y}$ $= \dfrac{16 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y}{-8 \cdot x \cdot x \cdot x \cdot y}$

Divide common factors

$= \dfrac{\overset{-2}{\cancel{16}} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{y} \cdot y}{\underset{1}{\cancel{-8}} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{y}}$

$= \dfrac{-2 \cdot x \cdot y \cdot y}{1}$

$= -2xy^2$

or $= \dfrac{16}{-8}x^{4-3}y^{3-1}$

$= -2xy^2$

c) $\left(-2w^2y\right)(5wx)$ $= -10w^{2+1}yx$

$= -10w^3yx$

d) $100x^2yz^3 \div 25xyz^2$ $= 4x^{2-1}y^{1-1}z^{3-2}$

$= 4xz$

Any base to the zero exponent equals 1

Support Questions

4. Determine the product or quotient.

   a) $(3m^2)(-7m^5)$
   b) $(12n^2) \div (-6n)$
   c) $\dfrac{7a^5b^{10}}{-3b^5a^2}$
   d) $(-3x^3)(4xy)(-2x^2y^2)$
   e) $\dfrac{-28a^6}{-7a^5}$
   f) $(8z^3)(7z^{11})$
   g) $(-4a^3b) \div (2ab)$
   h) $(5x^2)(4x^3)$
   i) $\left(-\dfrac{3}{5}ab^2\right)\left(-\dfrac{10}{9}a^2\right)$

5. Simplify. Then determine the value of the polynomial when a=2 and when b= –1.

   a) $(2a^2b^3)^2$
   b) $(-3ab^2)(-5ab^2)$

Multiplying Polynomials with a Monomial

This process requires the use of the distributive law.

Example

Expand.

a) $2(3x - 4)$

b) $2x(-3x - 5)$

c) $(-3p^2)(3 - 5p + p^2)$

Solutions

a) $2(3x-4) = 6x-8$

(2)(3x)= 6x and (2)(-4)= -8

b) $2x(-3x-5) = -6x^2-10x$

c) $(-3p^2)(3-5p+p^2) = -9p^2+15p^3-3p^4$

Support Questions

6. Expand.

a) $x(3x-9)$
b) $(-4n)(2n-3)$
c) $b(2b^2-3b+1)$
d) $(-x)(x-2)$
e) $(6x^2)(7-3x^2)$
f) $(-4m)(m^2-m)$

Greatest Common Factor (GCF) of Polynomials

Factoring means to express a polynomial as a product of factors.

Expanding means to multiply factors together to form a product.

Example

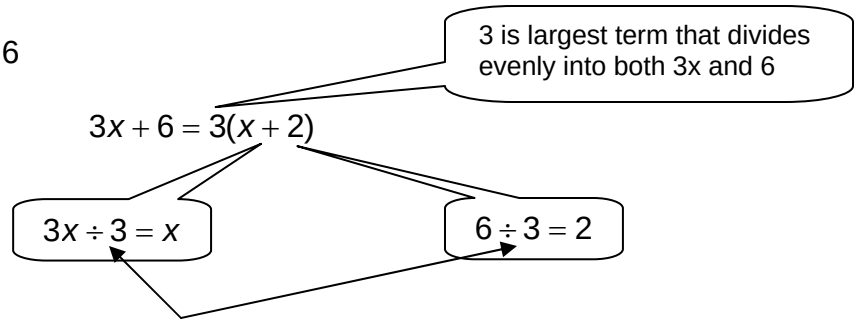
Factor fully and check by expanding.

a) $3x+6$
b) $14x^2+21x$
c) $5m^2n^2-10m^4n+30m^3n^2$

Solution

Factor fully and check by expanding.

a) $3x + 6$



3 is the Greatest Common Factor (G.C.F.)

Check by expanding

$(3)(x) = 3x$
$(3)(2) = 6$
$3x + 6$

b) $14x^2 + 21x$

$14x^2 + 21x = 7x(2x + 3)$

Divided both terms by the GCF which is 7x

Check by expanding

$(7x)(2x) = 14x^2$
$(7x)(3) = 21x$
$14x^2 + 21x$

c) $5m^2n^2 - 10m^4n + 30m^3n^2$

$5m^2n^2 - 10m^4n + 30m^3n^2 = 5m^2n(n - 2m^2 + 6mn)$

Divided both each term by the GCF which is $5m^2n$.

Check by expanding

$(5m^2n)(n) = 5m^2n^2$
$(5m^2n)(-2m^2) = -10m^4n$
$(5m^2n)(6mn) = 30m^3n^2$
$5m^2n^2 - 10m^4n + 30m^3n^2$

Support Questions

7. Factor fully.

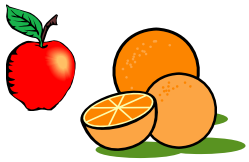
a) $3x^2 - 9$
b) $8n^2 - 4n$
c) $2b^2 - 3b$
d) $x^3 - 2x^2 + 4x$
e) $6x^2y - 3x^3y^2$
f) $3xy^2 + 9x^2y - 6xy$
g) $-4a^2b + 16ab^2 - 8ab$

8. Simplify each expression then factor.

a) $5y^3 - 3y^2 + 2y - y^3 + 11y^2 + 12y$
b) $x^2 - 2x + 3 - x^2 + 6x - 7$
c) $9 - 5t + t^2 + 5 + 6t^2 + 12t$
d) $6w^3 - 5w + 7 - 2w^2 - w + 3$

Key Question #6

1. State the like terms in each group. (2 marks)



a) $4w, 5w^2, 5z, x^2, -x, 3w, 3v$
b) $4x^2, -3x^2, 4z, 2y, y^2, 4w$

2. Simplify. (8 marks)

a) $(-12t + 2) + (7t + 5)$
b) $(6 - 4r) - (-5r + 1)$
c) $(4n^2 + 4n + 1) + (-7n^2 - 2n - 6)$
d) $(-4x^2 + 3x + 1) - (3x^2 + 7 - x)$
e) $(2 + 5m + n^2) - (5 - 1m + 4n^2)$
f) $(3 + 7x^2) + (4 - 6x^2)$
g) $(1 - 7w^2) - (-4 + 8w^2)$
h) $(6x^2 - 7x) + (-2x + 9x^2)$

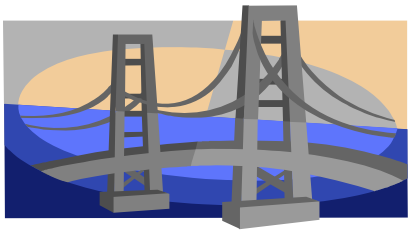
Key Question #6 (continued)

3. Simplify. Then determine the value of the polynomial when n=-3 and when n= 2. (6 marks)

   a) $(5n^2 + 3n - 4) + (-3n^2 + 4n - 1)$
   b) $(7n^2 - 5n - 2) - (-n^2 + 6n + 8)$

4. Expand. (6 marks)

   a) $2w(3w + 4)$
   b) $-4n(5n - 9)$
   c) $c(7c^2 - 5c - 6)$
   d) $(-h)(h + 6)$
   e) $(7t^2)(3 + 3t^2)$
   f) $(-6x)(-x^2 + x)$



5. Determine the product or quotient. (9 marks)

   a) $(2x^2)(-4x^5)$
   b) $(36m^2) \div (-9nx)$
   c) $\dfrac{-11a^9b^6}{5b^3a}$
   d) $(8x^3)(2x^2y)(-x^3y^5)$
   e) $\dfrac{-75w^7}{-15w^4}$
   f) $(3d^2)(9d^7)$
   g) $(-12a^2b^3) \div (3ab^3)$
   h) $(7k^2)(-k^3)$
   i) $\left(\dfrac{4}{6}a^2b^2\right)\left(-\dfrac{12}{8}ab^3\right)$

6. Simplify. Then determine the value of the polynomial when a=-1 and when b= 3. (4 marks)

   a) $(3ab^3)^3$
   b) $(-2a^2b)(-4ab)^2$

Key Question #6 (continued)

7. Factor fully. (7 marks)

   a) $7y^2 + 21$
   b) $10x^2 - 5x$
   c) $8s^2 - 7s$
   d) $w^3 - 6w^2 - 2w$
   e) $8n^2m + 4n^3m^2$
   f) $5xy^2 + 15x^2y - 10xy$
   g) $-2a^2b - 16a^2b^2 + 8ab$

8. Simplify each expression then factor. (6 marks)

   a) $7h^3 - 3h^2 + 1 - h^3 - 9h^2 + 11$
   b) $5y^2 + 3y - 8 - 5y^2 + 7y - 7$
   c) $9 - 5n + n^2 + 5 + 6n^2 + 12n$

9. When the terms of a polynomial in x are arranged from the highest to the lowest powers of x, the polynomial is in descending order. Simplify the following polynomial in descending order then evaluate for n = -0.25. (3 marks)

   $6 - \left(2n^2 + n\right) - \left(5n + n^2 - 6\right) - \left(4n + 2n^2 - 11\right)$

10. When are the sum, difference, product and quotient of two monomials also a monomial? (3 marks)

Algebra

Lesson 7

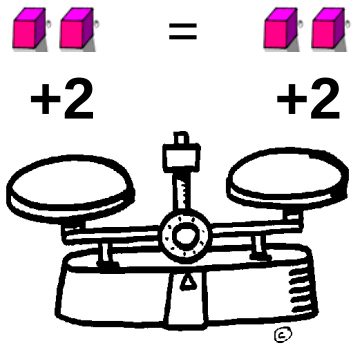
Lesson Seven Concepts

- Introduction to algebra
- Solving for unknowns
- Checking solutions to algebraic equations

Algebra

Solving Equations

When solving algebraic equations we must try to think of a scale always in equilibrium (balanced). It is important to keep the scale balanced at all times. What you do to one side of the equation must also be done to the other side of the equation.



You need to get all the terms with the variable to one side and the constants (the ones with out any letters) to the other.

It does not matter which side you choose for the isolating of each.

Example

Solve each equation algebraically. Check your solution

a) $-4-7k=12+k$

b) $3x-17=28$

c) $\frac{x}{4}+2=-3$

d) $-6n+\frac{1}{2}=n+\frac{2}{3}$

Solution

a) $-4 - 7k = 12 + k$

This side chosen for the "k's"

$-4 - 7k = 12 + k$

- 1k from both sides to keep scale balanced

$-4 - 7k \mathbf{- k} = 12 + k \mathbf{- k}$

$-4 - 8k = 12$

+4 from both sides to keep scale balanced

$-4 \mathbf{+ 4} - 8k = 12 \mathbf{+ 4}$

$-8k = 16$

Divide 8 from both sides to keep scale balanced

$\frac{-8k}{\mathbf{-8}} = \frac{16}{\mathbf{-8}}$

$k = -2$

Should now check the answer

Checking the solution

$-4 - 7k$	$12 + k$
$-4 - 7(\mathbf{-2})$	$12 + (\mathbf{-2})$
$-4 + 14$	$12 - 2$
10	10

Substitute your answer into the original equation.

same

If both side equal the same amount then your answer is correct. K = -2

b) $3x - 17 = 28$

$3x - 17 \mathbf{+17} = 28 \mathbf{+ 17}$

$3x = 45$

$\frac{3x}{\mathbf{3}} = \frac{45}{\mathbf{3}}$

$x = 15$

Checking the solution

3x – 17	28
3(**15**) - 17	28
45 - 17	28
28	28

both side equal the same amount therefore the answer is correct.

c) $\frac{x}{4}+2=-3$

$$\frac{x}{4}+2=-3$$

Write each term as a fraction

$$\frac{x}{4}+\frac{2}{1}=\frac{-3}{1}$$

$$\left(\frac{x}{4}\right)+\left(\frac{2}{1}\right)=\left(\frac{-3}{1}\right)$$

Find the (lowest common multiple) LCM of 1, and 4 (the denominators) and multiply each term by the LCM.

$$4\left(\frac{x}{4}\right)+4\left(\frac{2}{1}\right)=4\left(\frac{-3}{1}\right)$$

Divide the LCM by the denominator in each term then multiply this quotient by the numerator.

$$^{1}\cancel{4}\left(\frac{x}{\cancel{4}}\right)+4\left(\frac{2}{1}\right)=4\left(\frac{-3}{1}\right)$$

$$x+8=-12$$

$$x+8-8=-12-8$$

Continue to do the same to both sides keeping the scale balanced

$$x=-20$$

Checking the solution

$\frac{-20}{4}+2$	-3
-5 +2	-3
-3	-3

d) $-6n+\frac{1}{2}=n+\frac{2}{3}$

$$-6n+\frac{1}{2}=n+\frac{2}{3}$$

$$\frac{-6n}{1}+\frac{1}{2}=\frac{n}{1}+\frac{2}{3}$$

$$6\left(\frac{-6n}{1}\right)+6\left(\frac{1}{2}\right)=6\left(\frac{n}{1}\right)+6\left(\frac{2}{3}\right)$$

Find the (lowest common multiple) LCM of 1, 2 and 3(the denominators) and multiply each term by the LCM.

$$^{6}\cancel{6}\left(\frac{-6n}{\cancel{1}}\right)+^{3}\cancel{6}\left(\frac{1}{\cancel{2}}\right)=^{6}\cancel{6}\left(\frac{n}{\cancel{1}}\right)+^{2}\cancel{6}\left(\frac{2}{\cancel{3}}\right)$$

Divide the LCM by the denominator in each term then multiply this quotient by the numerator.

$$-36n+3=6n+4$$

$$=36m+3-3=6n+4-3$$

$$-36n=6n+1$$

$$-36n-6n+6n=6n+1$$

$$-42n=1$$

$$\frac{-42n}{-42}=\frac{1}{-42}$$

$$n=\frac{1}{-42}$$

Continue to do the same to both sides keeping the scale balanced

Checking the solution

$-6\left(-\frac{1}{42}\right)+\frac{1}{2}$	$-\frac{1}{42}+\frac{2}{3}$
$\frac{6}{42}+\frac{1}{2}$	$-\frac{1}{42}+\frac{2}{3}$
$\frac{6}{42}+\frac{21}{42}$	$\frac{-1}{42}+\frac{28}{42}$
$\frac{27}{42}$	$\frac{27}{42}$

Remember that a common denominator is needed to add fractions

Support Questions

1. Solve each equation. Check your solution

   a) $5w = 30$
   b) $3x - 5 = 16$
   c) $10 - t = 17$
   d) $\frac{n}{5} = 18$
   e) $4w - 12 = 2w + 3$
   f) $12r - 25 = 4r + 7$
   g) $5 - 6x = 2x + 5$
   h) $3 + 7c = 2c - 3$
   i) $2(w + 13) = 3(5 - w)$
   j) $4(1 - 2j) = 7(2j + 10)$
   k) $\frac{4}{3}h = 12$
   l) $\frac{n}{5} = 2 + \frac{n}{3}$
   m) $\frac{d}{4} - \frac{2}{3} = 2$
   n) $7 = -2(-3 - x)$

2. The formula for the area of a triangle is $A = \frac{bh}{2}$. The area of a triangle is 26 cm^2 and its base is 12 cm. Using algebra, what is the height of the triangle?

3. The cost of a hall rental for a wedding is \$500. Each meal at the wedding will cost an additional \$45.50.

   a) Write and algebraic equation for the total cost of the wedding.
   b) Calculate how many people attended the wedding if the wedding cost totalled \$3912.50.

Key Question #7

1. Solve each equation. Check your solution. (14 marks)

   a) $-2w = 26$
   b) $3n - 3 = 17$
   c) $15 - w = 45$
   d) $5c - 7c = 32$
   e) $6 + 6h = 2h - 5$
   f) $2.5x - 4 + 1.2x = 3.5$
   g) $1.2x + 3.3(2.4 - x) = 41$
   h) $\frac{3x}{2} + \frac{9}{5} = 12$
   i) $5 = \frac{9}{4} - \frac{r}{3}$
   j) $2(x - 2) = 12$
   k) $5(x - 3) = -15$
   l) $3(2t + 6) = 0$
   m) $2(p + 1) = 3(p - 1)$
   n) $2 + \frac{n}{3} = 12$

2. The formula for the perimeter of a rectangle with length l and width w is $P=2l+2w$. A rectangular field is 130 m long and requires 425 m of fencing to enclose it. Determine the width of the field. (3 marks)

3. Volcanoes prove that the Earth's center is hot. The formula $T = 10d + 20$ is used to estimate the temperature, T degrees Celsius, at a depth of d kilometres (km). (6 marks)

   a) What does each term on the right side of the equation represent?
   b) Estimate the depth where the temperature is 60°C.
   c) What is the approximate temperature at a depth of 4 km?

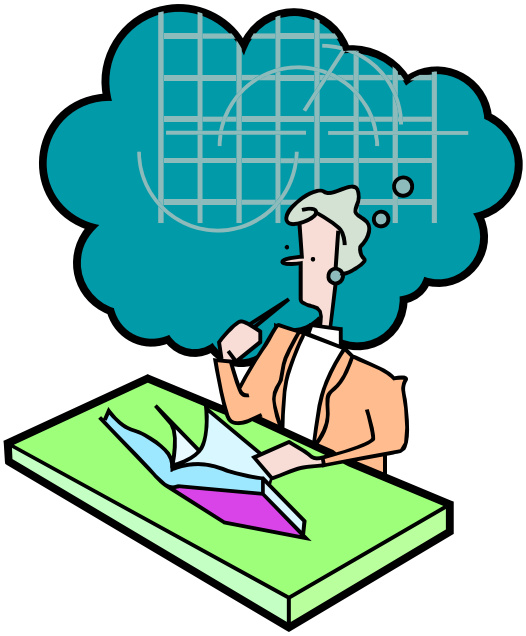
Key Question #7 (continued)

4. The cost, C dollars, to produce a school yearbook is given by the equation $C = 7500 + 8n$, where n is the number of yearbooks printed. (8 marks)

   a) What does each term on the right side of the equation represent?
   b) Suppose there is $11 500 to spend on yearbooks. How many yearbooks can be purchased?
   c) How many yearbooks can be produced for $20 000?
   d) How much would 700 yearbooks cost?

5. Suppose you were asked to explain how to solve the equation $2x - 5 = 4x + 12$ over the phone to a friend. Explain in detail the steps that you would tell your friend to solve the equation. (4 marks)

Problem Solving Algebraically



Lesson 8

Lesson Eight Concepts

- Introduction to problem solving
- 5 steps suggested to problem solving

Solving Problems using Algebraic Modeling

Problem Solving Steps

1. Use a variable to represent the unknown quantity.
2. Express any other unknown quantities in terms of this variable.
3. Write an equation and solve.
4. Check your answer to the question.
5. State the answer to the question with a therefore statement.

Example

Members of a Girl Guide troop sold boxes of cookies to raise money for their year end camp. Brianna sold 8 more boxes than her friend Nicola. They sold a total of 46 boxes. How many boxes did each sell?

Solution

1. Use a variable to represent the unknown quantity.

Let b represent the number of boxes sold by Brianna.

2. Express any other unknown quantities in terms of this variable.

If boxes sold by **Brianna** = b; then
boxes sold by **Nicola** = b – 8.

3. Write an equation and solve.

$$\begin{aligned} b + b - 8 &= 46 \\ 2b - 8 &= 46 \\ 2b - 8 + 8 &= 46 + 8 \\ 2b &= 54 \\ \frac{2b}{2} &= \frac{54}{2} \\ b &= 27 \end{aligned}$$

4. Check your answer to the question.

LHS	RHS
b+ b – 8	46
27 + 27 - 8	46
54 – 8	46
46	46

5. State the answer to the question with a therefore statement.

$\therefore$ Brianna sold 27 boxes of cookies and Nicola sold 19.

Example

An airplane travels 7 times faster than a train. The difference in their speeds is 420 km/hr. How fast is each vehicle traveling?

Solution

1. Use a variable to represent the unknown quantity.

Let v represent the speed the train travels.

2. Express any other unknown quantities in terms of this variable.

If the train travels at v; then
the airplane travels 7v.

3. Write an equation and solve.

$$7v - v = 420$$
$$6v = 420$$
$$\frac{6v}{6} = \frac{420}{6}$$
$$v = 70$$

4. Check your answer to the question.

LHS	RHS
7v - v	420
7(70) - 70	420
490 – 70	420
420	420

5. State the answer to the question with a therefore statement.

$\therefore$ the train travels at 70 km/hr and the airplane travels at 490 km/hr.

Support Questions

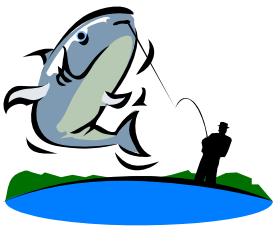
1. Find two consecutive numbers with a sum of 143.

2. A set of golf clubs and bag cost $225. The clubs cost $60 more than the bag. How much do the clubs cost?

3. Brianna and Noah ran as far as they could in 60 min. Brianna ran 2.5 km less than Noah. Together they ran 17.5 km in total. How far did each run.

4. A bottle and a cork cost $1.10. The bottle cost $0.90 more than the cork. How much does the cork cost?

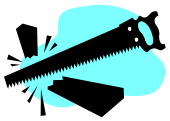
5. Don and his brother Dan went fishing. Don caught 6 times more mass of fish than his brother. Together they caught 4.2 kg of fish. What mass of fish did each brother catch?



Key Question #8

1. The combined mass of two children is 75 lbs. The first child is four times the mass of the second child. What are the masses of the two children? (3 marks)

2. 3 pieces of wood have a total length of 32 m. The second piece is twice the size of the first and the third piece is 2 more metres than triple the first. What are the lengths of the pieces of wood? (4 marks)



3. Abdul drove four trips that totalled 426 km. The second trip was 4 less than double the first. The third trip was double the second and the fourth trip was 18 more than triple the first. How far was each trip? (3 marks)

4. A rectangle has a width that is 3cm less than its length. The perimeter of the rectangle is 22 cm. What is the length and width of the rectangle? (3 marks)

5. The same numbers of each coin have a total of $6.00. There are nickels, dimes and quarters. How many of each coin are there? (4 marks)

Slope

Lesson 9

Lesson Nine Concepts

- Introduction to slope
- Cartesian plane
- x and y coordinates on the Cartesian plane
- Plotting order pairs
- Quadrants of the Cartesian plane
- Recognizing positive, negative, zero and undefined slopes
- Using the rise and the run of a given line to find its slope
- Using a pair of coordinates of a line to calculate slope

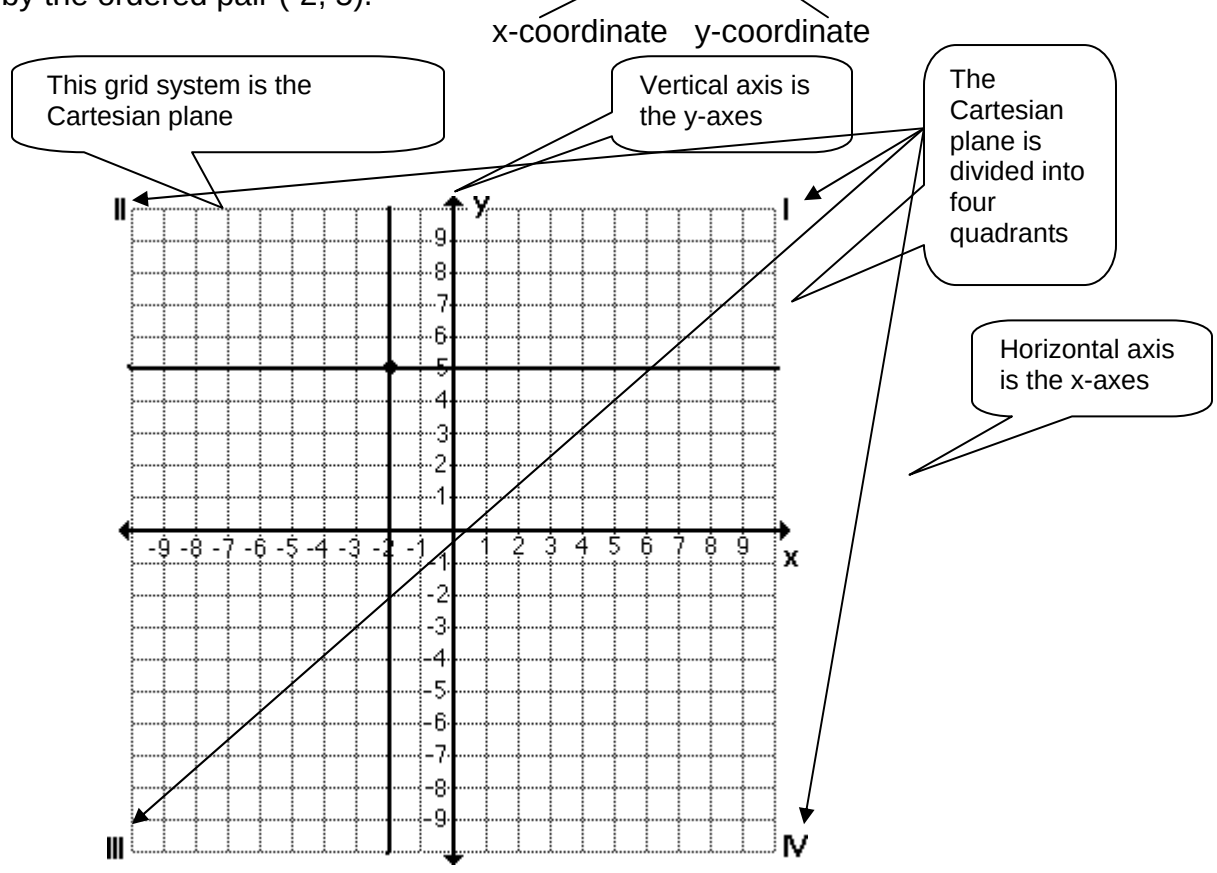
Cartesian Plane

Cartesian Plane uses the x and y axes to plot a point identified by a pair of numbers.

If it is known that a point has an x-coordinate of –2, then this point could be located anywhere along the vertical line passing through –2 on the x-axis.

If it is known that a point has a y-coordinate of 5, then this point could be located anywhere along the horizontal line passing through 5 on the y-axis.

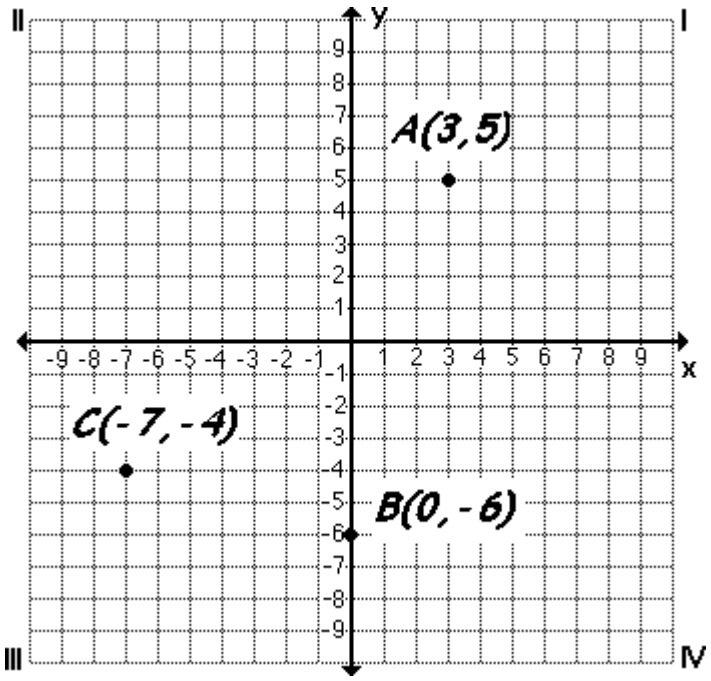
The point of intersection of the lines formed by x= -2 and y= 5 is the location of a point given by the ordered pair (-2, 5).



Example

Plot and label each point: A(3,5), B(0,-6), C(-7,-4)

Solution



Support Questions

1. Properly draw and label a Cartesian plane then plot and label the following ordered pairs.

   A(1, 1), B(1, 5), C(2, -4), D(7, 0), E(0, 8), F(-4, -8), G(-5, 9), H(-3, 0)

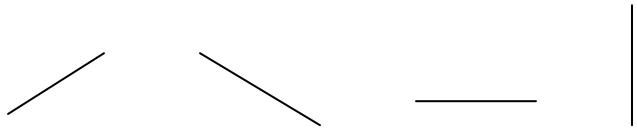
2. State the quadrant for each of the ordered pairs plotted in question one.

3. The points (-3, 5) and (3, 5) are two vertices of a square. State all other order pairs that could be the other two vertices of the square.

Slope

Slope describes the steepness of a line or line segment; the ratio of the rise of a line or line segment to its run.

Slope is either one of the following four types:

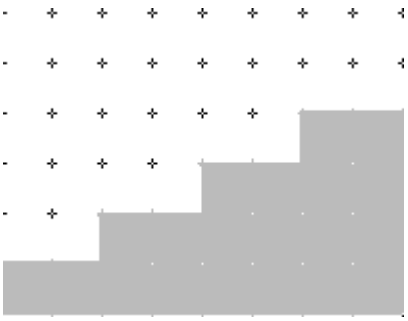


a) positive b) negative c) no slope d) undefined

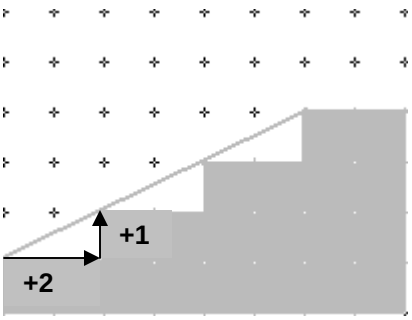
The formula for determining slope is: $\text{slope}=\frac{\text{rise}}{\text{run}}$.

Example

a) Draw a board that would lie on the staircase
b) State the slope of the staircase



Solution

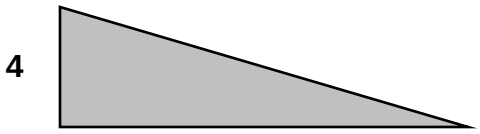


$$\text{slope}=\frac{\text{rise}}{\text{run}}$$

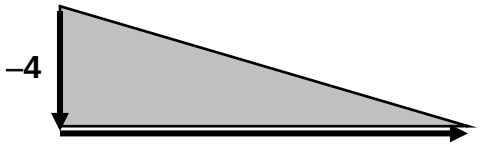
$$\text{slope}=\frac{+1}{+2}=\frac{1}{2}$$

Example

c) State the slope of the hypotenuse of the triangle given below.



Solution



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{slope} = \frac{-4}{12}$$

$$\text{slope} = -\frac{1}{3}$$

Reduced to simplest form.

Support Questions

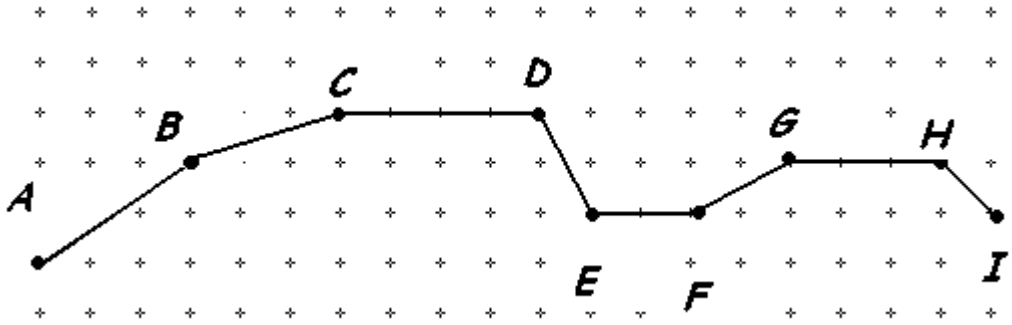
4. Find the slope of each staircase.

a) 

b) 

Support Questions

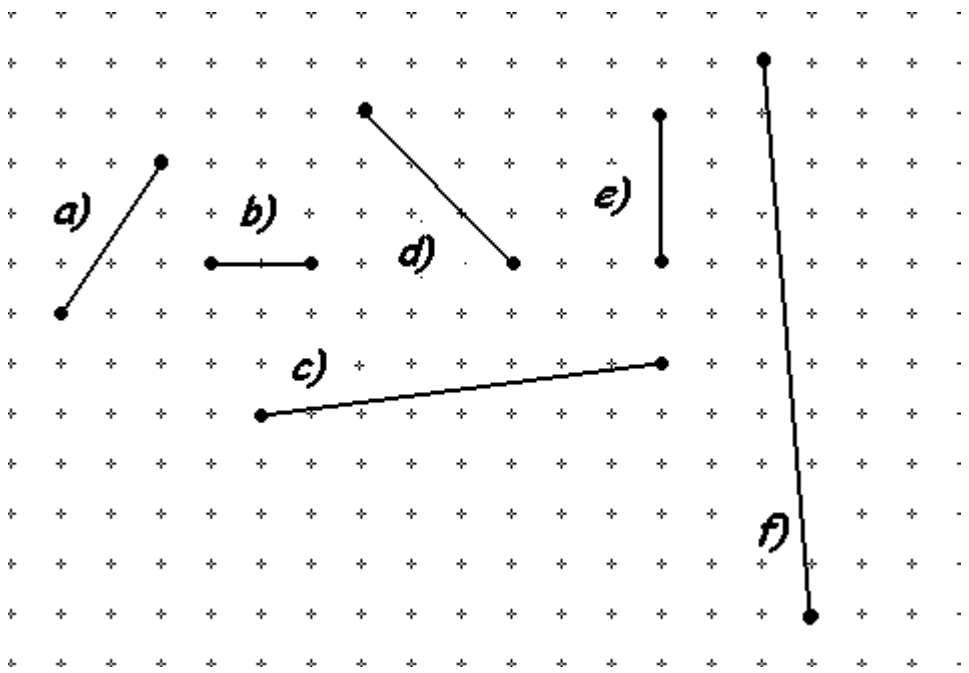
5. This drawing represents the side view of road passing through a mountain range. The road moves from left to right. Calculate the slope of each line segment.



6. The slope of a line segment is –6. What is a possible rise and run?

7. The slope of a line segment is 4. What is the rise if the run is 3?

8. State the slope of each line segment.



Slope between two points on a Cartesian Plane

The slope of line segment can be determined if two ordered pairs of the line segment are known.

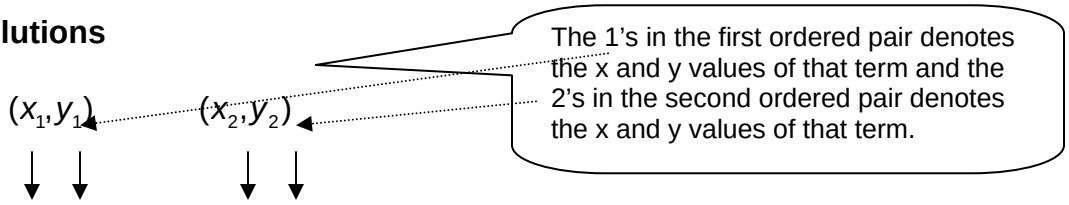
The following formula is used: $\text{slope} = \frac{y_2 - y_1}{x_2 - x_z}$

Example

a) Find the slope of the line containing the coordinate pairs A(7,3) and B(-9,7).

b) Find the slope of the line containing the coordinate pairs A(-3,1) and B(4,6).

Solutions



a) (7 , 3) and (-9, 7)

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(7) - (3)}{(-9) - (7)}$$

Substitute the values into the equation for slope.

$$= \frac{4}{-16}$$

Fraction needs to be simplified.

$$= -\frac{1}{4}$$

Therefore the slope is $-\frac{1}{4}$.

b) (-3,1) and (4,6)

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(6) - (1)}{(4) - (-3)}$$

$$= \frac{+5}{+7}$$

$$= \frac{5}{7}$$

Therefore the slope is $\frac{5}{7}$.

Support Questions

9. Find the slope of the line containing the ordered pairs.

   a) A(4,-5), B(-4,6)

   b) C(3,8), D(7,2)

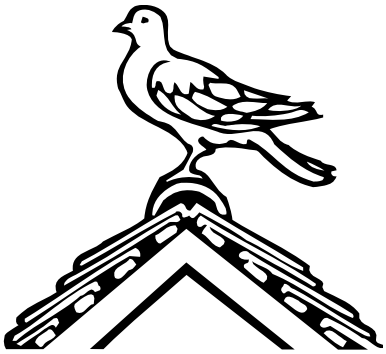
   c) E(0,3), F(2,1)

   d) G(5,4), H(1,-3)

   e) I(2,3), J(-6,-2)

   f) L(3,7), M(3,-2)

   g) N(4,-6),P(2,-6)



10. Find the missing value in the ordered pair.

    a) (3,-2), (x,6); slope = -4

    b) (1,1), (7,y); slope = 2

    c) (x,4), (3,-1); slope = -5

    d) (8,y), (-2,-3)= $\frac{1}{5}$

Key Question #9

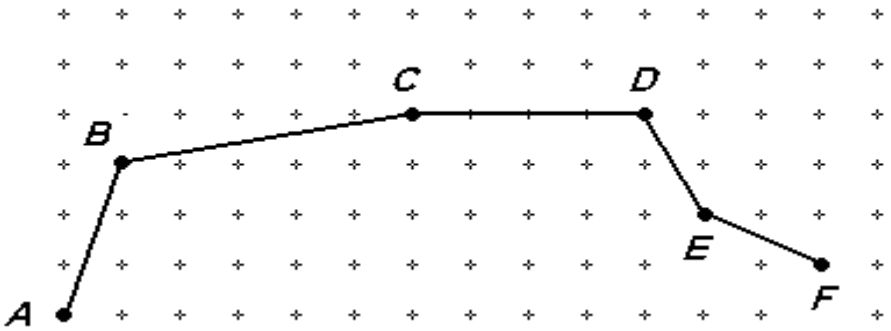
1. Properly draw and label a Cartesian plane then plot and label the following ordered pairs. (6 marks)

   A(2, 1), B(1, -5), C(3, 4), D(-7, 0), E(0, -8), F(1, -8), G(-4, -9), H(3, 0)

2. State the quadrant for each of the ordered pairs plotted in question one. (2 marks)

3. The points (-4, 2) and (4, 2) are two vertices of a square. State all other order pairs that could be the other two vertices of the square. (4 marks)

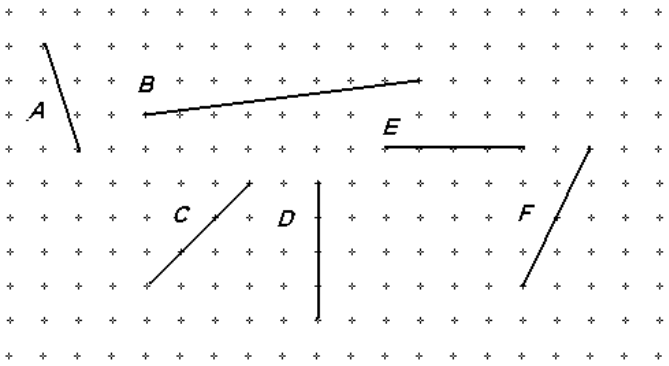
4. This drawing represents the side view of road passing through a mountain range. The road moves from left to right. Calculate the slope of each line segment. (6 marks)



5. The slope of a line segment is 3. What is a possible rise and run? (2 marks)

6. The slope of a line segment is -5. What is the rise if the run is 4? (2 marks)

7. State the slope of each line segment. (6 marks)



8. Find the slope of the in line containing the ordered pairs. (7 marks)

   a. A(2,-5), B(-2,6)

   b. C(3,6), D(5,2)

   c. E(1,2), F(2,1)

   d. G(-3,4), H(1,-2)

   e. I(2,3), J(-6,3)

   f. L(4,5), M(4,-7)

   g. N(2,-5),P(3,-8)

Key Question #9 (continued)

9. Find the missing value in the ordered pair. (4 marks)

a) (-3,-4), (0,y); slope = 2

b) (-2,2), (x,5); slope = $\frac{3}{7}$

c) (x,5), (1,-5); slope = $\frac{-5}{2}$

e) (-6,y), (8,2)= $-\frac{1}{7}$

10. Decide if the following statements are true, sometimes true or not true. Explain. (8 marks)

a) An ordered pair with 1 positive co-ordinate and 1 negative coordinate lies in the 3rd quadrant.
b) An ordered pair with both co-ordinates positive coordinates lies in the 1st quadrant.
c) An ordered pair where the x and y coordinate are the same lies in the 1st or 3rd quadrant.
d) The order pairs on a vertical line have the same y coordinate.

Relationships In Data

Lesson 10

Lesson Ten Concepts

- Introduction to relationships in data
- Graphing relationship in data
- Working with table of values
- Creating graphs for tables of values
- Using graphs to solve related questions

Relationships in Data

Tables and graphs of data help to show the relationships between quantities. In mathematics the relationship between a pair of quantities is called a **relation**.

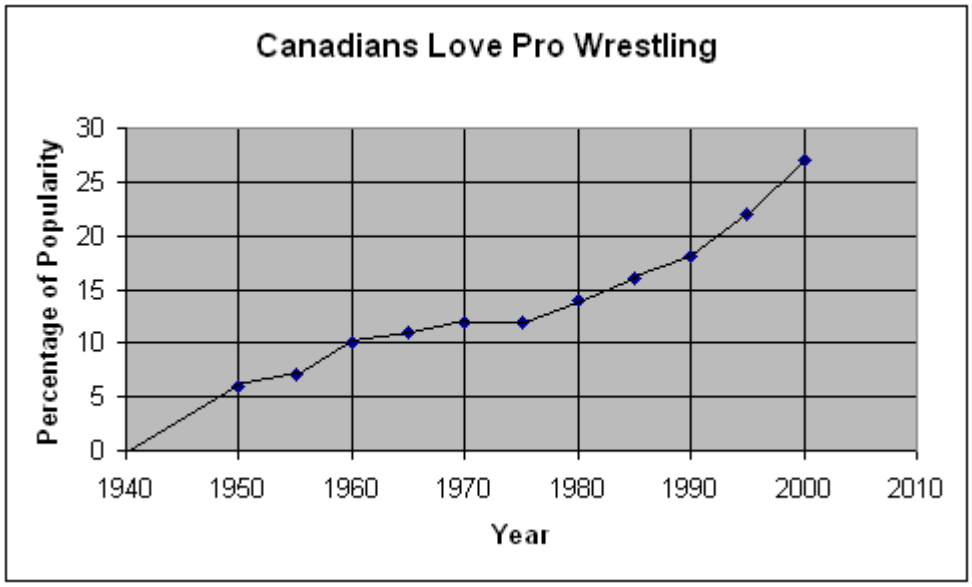
Example

Use the graph below to answer each question.

a) State the percentage of Canadians who enjoy professional wrestling for each of the following years. Show the popularity in a table.

1960 1970 1980 1990 2000

b) When did the popularity of professional wrestling reach 16% of all Canadians?



Solution

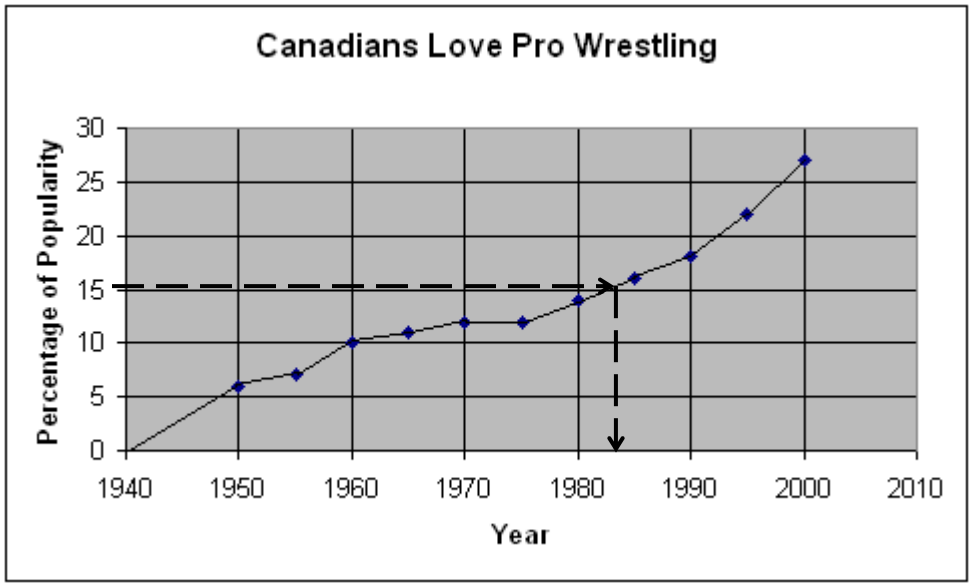
a) State the percentage of Canadians who enjoy professional wrestling for each of the following years? Show the popularity in a table.

1960 1970 1980 1990 2000

Year	1960	1970	1980	1990	2000
Popularity (%)	10	12	14	18	27

b) When did the popularity of professional wrestling reach 16% of Canadians?

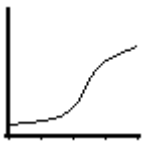
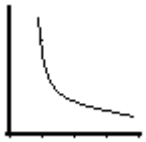
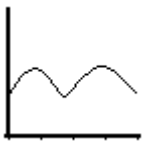
Approximately 1984



Example

Which graph below best represents each situation?

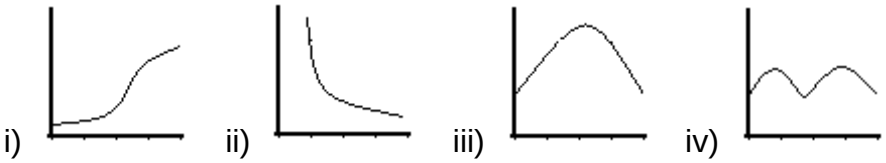
a) the height of a person over time
b) the height of roller coaster over time
c) the amount of hours of sunlight over a year
d) the number of D.V.D. players sold compared to selling price

i) 
ii) 
iii) 
iv) 

Solution

Which graph below best represents each situation?

a) the height of a person over time
b) the height of roller coaster over time
c) the amount of hours of sunlight over a year
d) the number of D.V.D. players sold compared to selling price



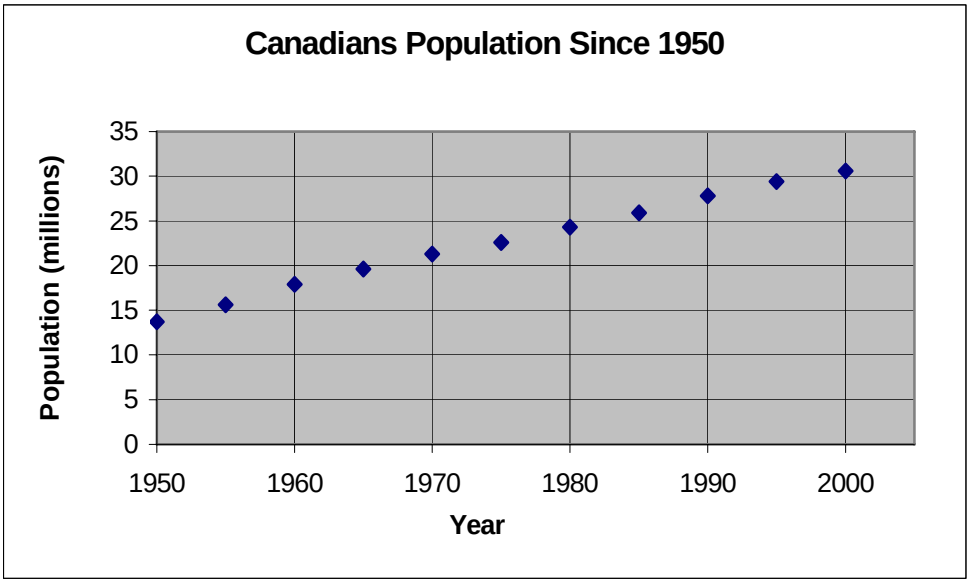
a **d** **c** **b**

Support Questions

1. Use the graph below to answer each question.

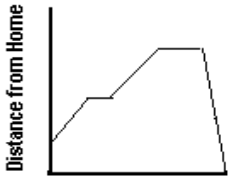
a) What was Canada's population in the following years?
1955 1965 1975 1985 1995.

b) When did Canada's population reach 25 million?

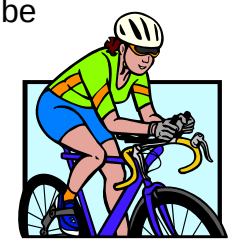


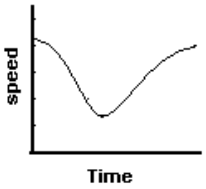
Support Questions

2. Brianna walks to her grandparents. This graph shows her distance from home during one of her walks. Describe her walk.



3. Noah is riding his bike from his grandparents to his home. Describe Noah's possible ride home





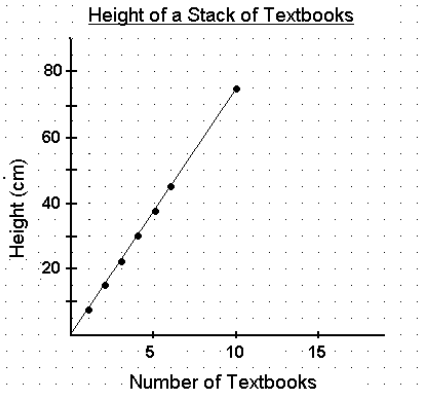
4. Refer to the graph given below:

a) How many textbooks make the following heights?

45 cm, 60cm, 75cm

b) Approximately how high are the following stacks of textbooks?

7 books, 9 books, 4 books, 2 books



Support Questions

5. Construct a graph of these data.

Number of stairs climbed	4	6	14	22	28
Height (cm)	75	112.5	262.5	412.5	525

a) Did you join the points? Explain.
b) What is the height of 15 stairs?
c) How many stairs will reach a height of 444 cm?
d) If the number of stairs is doubled will the height double? Explain.

6. Construct a graph of these data.

Radius of a circle, (*cm*)	1	2	3	4	5
Area of a circle. (cm^2)	3.14	12.56	28.26	50.24	78.5

a) Did you join the points? Explain.
b) What is the approximate radius of circle with an area of $100cm^2$?
c) What is the approximate area of circle with a radius of 4.5 cm?
d) If the radius is doubled will the area double? Explain.

Graphing Relations

Relations can be either linear or non-linear. **Linear** means the relation forms a single straight line and **non-linear** produces anything that is not a single straight line.

The following formula is used: slope $= \frac{y_2 - y_1}{x_2 - x_z}$

Example

Draw a graph of the relation described by the equation.

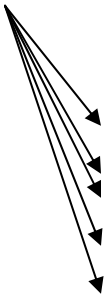
a) $y = -3x + 1$

b) $y = n^2 - 2$

Solutions

a) $y = -3x + 1$

First step is to make a table of values and choose values to place in **x** column of table.

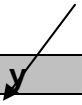


x	y
-2	
-1	
0	
1	
2	

Second step is to substitute each value into the equation to determine the **y** value.

For x = -2

y = –3x+1
= –3(-2)+1
= 6 + 1
= 7



x	y
-2	7
-1	
0	
1	
2	

For x = -1

$y = -3x+1$
$= -3(-1)+1$
$= 3 + 1$
$= 4$

x	y
-2	7
-1	4
0	
1	
2	

For x = 0

$y = -3x+1$
$= -3(0)+1$
$= 0 + 1$
$= 1$

x	y
-2	7
-1	4
0	1
1	
2	

For x = 1

$y = -3x+1$
$= -3(1)+1$
$= -3 + 1$
$= -2$

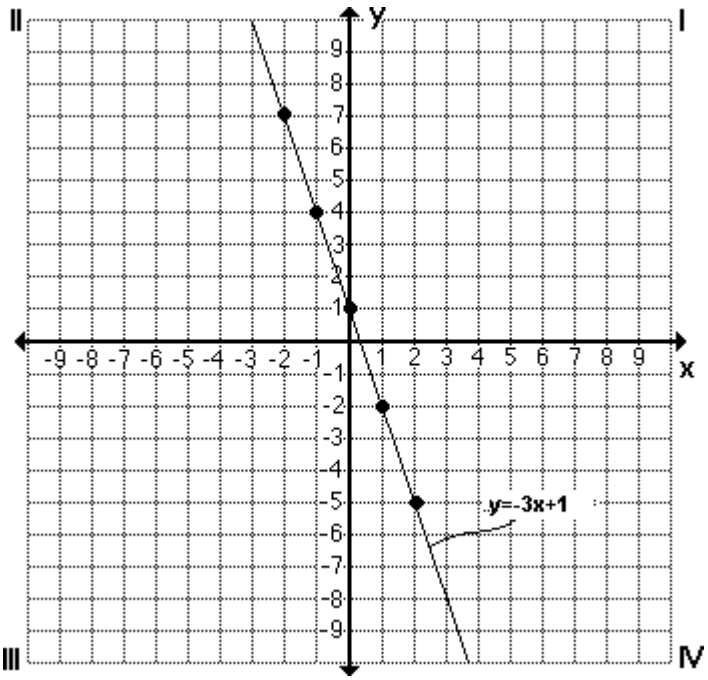
x	y
-2	7
-1	4
0	1
1	-2
2	

For x = 2

$y = -3x+1$
$= -3(2)+1$
$= -6 + 1$
$= -5$

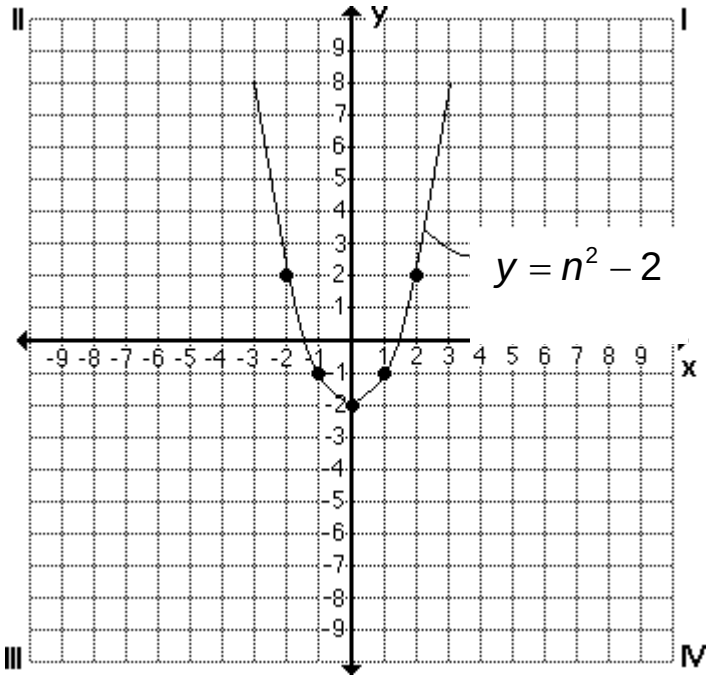
x	y
-2	7
-1	4
0	1
1	-2
2	-5

Next, plot the coordinates (x, y) on a grid and join the points with a straight edge and label the equation.



b) $y = n^2 - 2$

x	y
-2	2
-1	-1
0	-2
1	-1
2	-2



Support Questions

7. Complete the table of values.

a) $y = 2x - 5$

x	y
-4	
-1	
2	
5	

b) $y = -x + 6$

x	y
-2	
-1	
0	
1	

c) $y = w^2 - 1$

w	y
-1	
0	
1	
2	

Support Questions

8. The cost of D dollars, to print and bind y copies of a yearbook is given by the equation D = 60 +10n.

a) Make a table of values to show the costs for up to 400 Yearbooks.
b) Use the table to draw a graph.
c) Use the graph to estimate the cost of 325 copies.
d) Use the graph to estimate how many copies can be made for $1250.

9. Complete a table of values and graph each relation.

a) $t = s^2 + 25$ b) $y = \frac{1}{3}n - 15$ c) $p = 5.25w - .25$

10. Which of the following ordered pairs satisfy the relation modeled by $y = -\frac{1}{2}x + 7$.

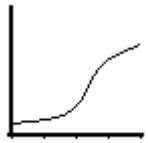
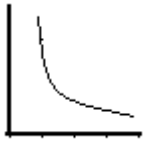
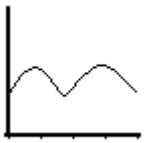
Show by substituting the values into the relation.

a) (-3, 6) b) (4,5) c) (7,3.5) d) (-6, 10) e) (-4, 6.5)

Key Question #10

1. Which graph below best represents each situation? (2 marks)

a) the height of a person over time
b) the height of roller coaster over time
c) the amount of hours of sunlight over a year
d) the number of D.V.D. players sold compared to selling price

i)  ii)  iii)  iv) 

2. Complete a table of values and graph each relation. (12 marks)

a) $C = 3n + 10$ b) $y = \frac{2}{5}x - 4$ c) $W = 4.75n - 0.50$

d) $P = 2l + 12$ e) $m = n^2 + 3$ f) $b = 2a^2 + 4$

Key Question #10 (continued)

3. The time that passes between the time you see lightning and you hear the thunder depends on your distance from the lightning. With each km from the lightning 3 seconds pass. (5 marks)

   a) Make a table of values for distances from 0 to 5 km.
   b) Graph this relation. State whether it is a linear or non-linear relation.
   c) Using your graphed relation, how much time passes before you hear lightning that occurs 4.5 km away?
   d) Using your graphed relation, how far away are you if you hear the thunder in 1.5 seconds?

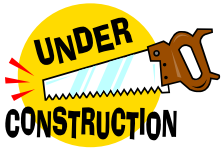
4. The amount a taxi driver charges a costumer is given by the equation $A = 1.25k + 5.25$, where A is the total amount charged and k is the kilometres driven. (6 marks)

   a) What do the numbers in the equation represent?
   b) Make a table of values for distances from 0 to 10 km.
   c) Graph this relation. State whether it is a linear or non-linear relation.
   d) Using your graphed relation, how much is charged if a person goes 7.5 km?
   e) Using your graphed relation, how far can a person go in a taxi for $15?

5. A sales person earns a monthly salary of $500 and 8% commission on all sales for that month. (6 marks)

   a) Write an equation to model this relation.
   b) Make a table of values.
   c) Graph this relation. State whether it is a linear or non-linear relation.
   d) Using your graphed relation, what were the sales if the sales persons' total earnings for the month were $800?
   e) Using your graphed relation, how much did the sales person earn if there was $2250 in sales?

6. Ashlee repairs DVD players. She charges $25 to inspect the problem and $20/h to repair the device. (5 marks)



   a) Write an equation to model this relation.
   b) Make a table of values.
   c) Graph this relation.
   d) Using your graphed relation, how long did it take Ashlee to repair the DVD player if she charged $55?
   e) Using your graphed relation, how much should Ashlee charge if it takes her 4.5 hours to repair the DVD player?

Support Question Answers

Lesson 6

1. a) $3x, -x$ b) $3y^2, y^2$

2. a)
$$\begin{aligned}&(-3x+3)+(4x+2)\\&=-3x+3+4x+2\\&=-3x+4x+3+2\\&=x+5\end{aligned}$$

b)
$$\begin{aligned}&(3-5n)-(-6n+2)\\&=3-5n+6n-2\\&=-5n+6n+3-2\\&=n+1\end{aligned}$$

c)
$$\begin{aligned}&(8a^2+2a-3)+(-5a^2+4a+7)\\&=8a^2+2a-3-5a^2+4a+7\\&=8a^2-5a^2+2a+4a-3+7\\&=3a^2+6a+4\end{aligned}$$

d)
$$\begin{aligned}&(-6x^2+5x+2)-(4x^2+5-2x)\\&=-6x^2+5x+2-4x^2-5+2x\\&=-6x^2-4x^2+5x+2x+2-5\\&=-10x^2+7x-3\end{aligned}$$

e)
$$\begin{aligned}&(3-2m-n^2)+(7-6m+n^2)\\&=3-2m-n^2+7-6m+n^2\\&=10-8m\end{aligned}$$

f)
$$\begin{aligned}&(2+6x^2)-(7-3x^2)\\&=2+6x^2-7+3x^2\\&=2-7+6x^2+3x^2\\&=-5+9x^2\end{aligned}$$

g)
$$\begin{aligned}&(5-6w^2)-(3-w^2)\\&=5-6w^2-3+w^2\\&=5-3-6w^2+w^2\\&=2-5w^2\end{aligned}$$

h)
$$\begin{aligned}&(5x^2-3x)+(-4x+5x^2)\\&=5x^2-3x-4x+5x^2\\&=5x^2+5x^2-3x-4x\\&=10x^2-7x\end{aligned}$$

3. a)
$$\begin{aligned}&(7n^2-3n+4)-(-4n^2-2n-3)\\&=7n^2-3n+4+4n^2+2n+3\\&=7n^2+4n^2-3n+2n+4+3\\&=11n^2-n+7\end{aligned}$$
for $\ldots n=2$
$$\begin{aligned}11n^2-n+7&=11(2)^2-(2)+7\\&=44-2+7\\&=51\end{aligned}$$
for $\ldots n=-1$
$$\begin{aligned}11n^2-n+7&=11(-1)^2-(-1)+7\\&=11+1+7\\&=19\end{aligned}$$

b)
$$\begin{aligned}&(3n^2-7n+2)+(-2n^2+6n+3)\\&=3n^2-7n+2-2n^2+6n+3\\&=3n^2-2n^2-7n+6n+2+3\\&=n^2-n+5\end{aligned}$$
for $\ldots n=2$
$$\begin{aligned}n^2-n+5&=(2)^2-(2)+5\\&=4-2+5\\&=7\end{aligned}$$
for $\ldots n=-1$
$$\begin{aligned}n^2-n+5&=(-1)^2-(-1)+5\\&=1+1+5\\&=7\end{aligned}$$

4. a)
$$\begin{aligned}&(3m^2)(-7m^5)\\&=-21m^7\end{aligned}$$

b)
$$\begin{aligned}&(12n^2)\div(-6n)\\&=-2n\end{aligned}$$

c)
$$\begin{aligned}&\frac{7a^5b^{10}}{-3b^5a^2}\\&=-\frac{7}{3}a^3b^5\end{aligned}$$

d)
$$\begin{aligned}&(-3x)^3(4xy)(-2x^2y^2)\\&=24x^6y^3\end{aligned}$$

e)
$$\begin{aligned}&\frac{-28a^6}{-7a^5}\\&=4a\end{aligned}$$

f)
$$\begin{aligned}&(8z^3)(7z^{11})\\&=56z^{14}\end{aligned}$$

g)
$$\begin{aligned}&(-4a^3b)\div(2ab)\\&=-2a^2\end{aligned}$$

h)
$$\begin{aligned}&(5x^2)(4x^3)\\&=20x^5\end{aligned}$$

i)
$$\begin{aligned}&\left(-\frac{3}{5}ab^2\right)\left(-\frac{10}{9}a^2\right)\\&=\frac{30}{45}a^3b^2\\&=\frac{2}{3}a^3b^2\end{aligned}$$

5. a)

$$\begin{aligned}&(2a^2b^3)^2\\&=(2a^2b^3)(2a^2b^3)\\&=4a^4b^6\end{aligned}$$

$$\begin{aligned}&for\ \ldots a=2,\ldots b=-1\\&4a^4b^6=4(2)^4(-1)^6\\&=4(16)(1)\\&=64\end{aligned}$$

b)

$$\begin{aligned}&(-3ab^2)(-5ab^2)\\&=15a^2b^4\end{aligned}$$

$$\begin{aligned}&for\ \ldots a=2,\ldots b=-1\\&15a^2b^4=15(2)^2(-1)^4\\&=(60)(1)\\&=60\end{aligned}$$

6. a) $x(3x-9)=3x^2-9x$

b) $(-4n)(2n-3)=-8n^2+12n$

c) $b(2b^2-3b+1)=2b^3-3b^2+b$

d) $(-x)(x-2)=-x^2+2x$

e) $(6x^2)(7-3x^2)=42x^2-18x^4$

f) $(-4m)(m^2-m)=-4m^3+4m^2$

7. a) $3(x^2-3)$ b) $4n(2n-1)$ c) $b(2b-3)$
d) $x(x^2-2x+4)$ e) $3x^2y(2-xy)$ f) $3xy(y+3x-2)$
g) $-4ab(a-4b+2)$

8. a)

$$\begin{aligned}&5y^3-3y^2+2y-y^3+11y^2+12y\\&=4y^3+8y^2+14y\\&=2y(2y^2+4y+7)\end{aligned}$$

b)

$$\begin{aligned}&x^2-2x+3-x^2+6x-7\\&=4x-4\\&=4(x-1)\end{aligned}$$

c)

$$\begin{aligned}&9-5t+t^2+5+6t^2+12t\\&=7t^2+7t+14\\&=7(t^2+t+2)\end{aligned}$$

d)

$$6w^3 - 5w + 7 - 2w^2 - w + 3$$
$$= 6w^3 - 2w^2 - 6w + 10$$
$$= 2(3w^3 - w^2 - 3w + 5)$$

Lesson 7

1. a)

$$5w = 30$$
$$\frac{5w}{5} = \frac{30}{5}$$
$$w = 6$$

b)

$$3x - 5 = 16$$
$$3x - 5 + 5 = 16 + 5$$
$$3x = 21$$
$$\frac{3x}{3} = \frac{21}{3}$$
$$x = 7$$

c)

$$10 - t = 17$$
$$10 - 10 - t = 17 - 10$$
$$-t = 7$$
$$\frac{-1t}{-1} = \frac{7}{-1}$$
$$t = -7$$

d)

$$\frac{n}{5} = 18$$
$$5\left(\frac{n}{5}\right) = 5(18)$$
$$n = 90$$

e)

$$4w - 12 = 2w + 3$$
$$4w - 12 + 12 = 2w + 3 + 12$$
$$4w = 2w + 15$$
$$4w - 2w = 2w - 2w + 15$$
$$2w = 15$$
$$\frac{2w}{2} = \frac{15}{2}$$
$$w = \frac{15}{2}$$

f)

$$12r - 25 = 4r + 7$$
$$12r - 25 + 25 = 4r + 7 + 25$$
$$12r = 4r + 32$$
$$12r - 4r = 4r - 4r + 32$$
$$8r = 32$$
$$\frac{8r}{8} = \frac{32}{8}$$
$$r = 4$$

g)

$$5 - 6x = 2x + 5$$
$$5 - 6x - 6x = 2x - 6x + 5$$
$$5 = -4x + 5$$
$$5 - 5 = -4x + 5 - 5$$
$$0 = 4x$$
$$0 = x$$

h)

$$3+7c=2c-3$$
$$3-3+7c=2c-3-3$$
$$7c=2c-6$$
$$7c-2c=2c-2c-6$$
$$5c=-6$$
$$\frac{5c}{5}=\frac{-6}{5}$$
$$c=-\frac{6}{5}$$

i)

$$2(w+13)=3(5-w)$$
$$2w+26=15-3w$$
$$2w+26-26=15-26-3w$$
$$2w=-11-3w$$
$$2w+3w=-11-3w+3w$$
$$5w=-11$$
$$\frac{5w}{5}=\frac{-11}{5}$$
$$w=-\frac{11}{5}$$

j)

$$4(1-2j)=7(2j+10)$$
$$4-8j=14j+70$$
$$4-8j+8j=14j+8j+70$$
$$4=22j+70$$
$$4-70=22j+70-70$$
$$-66=22j$$
$$\frac{-66}{22}=\frac{22j}{22}$$
$$-3=j$$

k)

$$\frac{4}{3}h=12$$
$$\frac{4h}{3}=12$$
$$3\left(\frac{4h}{3}\right)=3(12)$$
$$4h=36$$
$$\frac{4h}{4}=\frac{36}{4}$$
$$h=9$$

l)

$$\frac{n}{5}=2+\frac{n}{3}$$
$$15\left(\frac{n}{5}\right)=15(2)+15\left(\frac{n}{3}\right)$$
$$3n=30+5n$$
$$3n-5n=30+5n-5n$$
$$-2n=30$$
$$\frac{-2n}{-2}=\frac{30}{-2}$$
$$n=-15$$

m)

$$\frac{d}{4}-\frac{2}{3}=2$$
$$12\left(\frac{d}{4}\right)-12\left(\frac{2}{3}\right)=12(2)$$
$$3d-8=24$$
$$3d-8+8=24+8$$
$$3d=32$$
$$\frac{3d}{3}=\frac{32}{3}$$
$$d=\frac{32}{3}$$

n)

$$\begin{aligned} 7 &= -2(-3-x) \\ 7 &= 6+2x \\ 7-6 &= 6-6+2x \\ 1 &= 2x \\ \frac{1}{2} &= \frac{2x}{2} \\ \frac{1}{2} &= x \end{aligned}$$

2.

$$\begin{aligned} A &= \frac{bh}{2} \\ 26 &= \frac{(12)(h)}{2} \\ 2(26) &= 2\left(\frac{12h}{2}\right) \\ 52 &= 12h \\ \frac{52}{12} &= \frac{12h}{12} \\ \frac{13}{3} &= h \end{aligned}$$

Therefore the height of the triangle is $\frac{13}{3} cm$.

3. a) C = 45.50m+500

b)

$$\begin{aligned} 3912.50 &= 45.50m+500 \\ 3912.50-500 &= 45.50m+500-500 \\ 3412.50 &= 45.50m \\ \frac{3412.50}{45.50} &= \frac{45.50m}{45.50} \\ 75 &= m \end{aligned}$$

Therefore 75 people would attend.

Lesson 8

1. Let *n* be the first consecutive number.

   If the first consecutive number is “n” then the next number is “n+1”

$$\begin{aligned} n+n+1&=143\\ 2n+1&=143\\ 2n+1-1&=143-1\\ 2n&=142\\ \frac{2n}{2}&=\frac{142}{2}\\ n&=71 \end{aligned}$$

   $\therefore$ the consecutive numbers are 71 and 72

2. Let *b* be the cost of the bag

   If “b” is the cost of the bag then the clubs are “b+60”

$$\begin{aligned} b+b+60&=225\\ 2b+60&=225\\ 2b+60-60&=225-60\\ 2b&=165\\ \frac{2b}{2}&=\frac{165}{2}\\ b&=82.5 \end{aligned}$$

   $\therefore$ the bag cost \$82.50 and the golf clubs cost \$142.50.

3. Let *b* be the distance ran by Brianna

If “b” is the distance ran by Brianna then Noah ran b+2.5

$$\begin{aligned} b + b + 2.5 &= 17.5 \\ 2b + 2.5 &= 17.5 \\ 2b + 2.5 - 2.5 &= 17.5 - 2.5 \\ 2b &= 15 \\ \frac{2b}{2} &= \frac{15}{2} \\ b &= 7.5 \end{aligned}$$

$\therefore$ Brianna ran 7.5 km and Noah ran 10 km.

4. Let *c* be the cost of the cork

If “c” is the cost of the bottle then costs c+ 0.90

$$\begin{aligned} c + c + 0.90 &= 1.10 \\ 2c + 0.90 &= 1.10 \\ 2c + 0.90 - 0.90 &= 1.10 - 0.90 \\ 2c &= .20 \\ \frac{2c}{2} &= \frac{.20}{2} \\ c &= .10 \end{aligned}$$

$\therefore$ the cork cost $0.10.

5. Let *d* be the mass of the fish Dan caught.

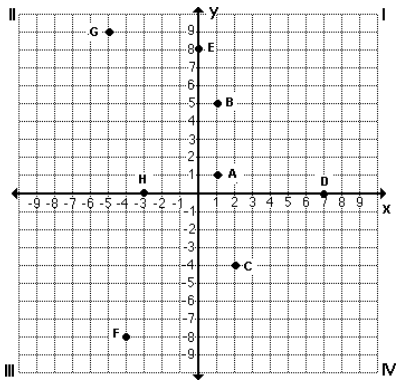
If “d” is the mass of the fish Dan caught then Don’s fish’s mass is 6d.

$$\begin{aligned} 6d + d &= 4.2 \\ 7d &= 4.2 \\ \frac{7d}{7} &= \frac{4.2}{7} \\ d &= .6 \end{aligned}$$

$\therefore$ Don caught 3.6 kg of fish and Dan caught .6 kg of fish.

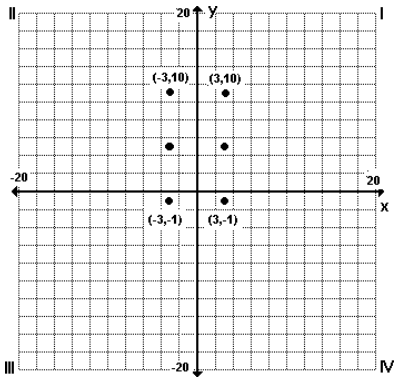
Lesson 9

1.



2. a) I b) I c) IV d) I and IV e) I and II f) III g) II h) II and III

3.



4. a) rise = +1 run = +1 $m = \frac{rise}{run} = \frac{1}{1} = 1$

b) rise = -2 run = +3 $m = \frac{rise}{run} = \frac{-2}{+3} = -\frac{2}{3}$

5.

$\overline{AB}$ *slope* $= m = \dfrac{rise}{run} = \dfrac{2}{3}$

$\overline{BC}$ *slope* $= m = \dfrac{rise}{run} = \dfrac{1}{3}$

$\overline{CD}$ *slope* $= m = \dfrac{rise}{run} = \dfrac{0}{4} = 0$

$\overline{DE}$ *slope* $= m = \dfrac{rise}{run} = \dfrac{-2}{+1} = -2$

$\overline{EF}$ *slope* $= m = \dfrac{rise}{run} = \dfrac{0}{2} = 0$

$\overline{FG}$ *slope* $= m = \dfrac{rise}{run} = \dfrac{1}{2}$

$\overline{GH}$ *slope* $= m = \dfrac{rise}{run} = \dfrac{0}{3} = 0$

$\overline{HI}$ *slope* $= m = \dfrac{rise}{run} = \dfrac{-1}{1} = -1$

6. Answers may vary. $\dfrac{-18}{3} = -6$ so the a possible rise is –18 and a possible run is +3.

7.

$$\frac{x}{3} = 4$$

$$3\left(\frac{x}{3}\right) = 3(4)$$

$$x = 12$$

Therefore the rise is +12.

8.

a) $slope = m = \frac{rise}{run} = \frac{3}{2}$

b) $slope = m = \frac{rise}{run} = \frac{0}{2} = 0$

c) $slope = m = \frac{rise}{run} = \frac{1}{8}$

d) $slope = m = \frac{rise}{run} = \frac{-3}{+3} = -1$

e) $slope = m = \frac{rise}{run} = \frac{-3}{0} = undefined$

f) $slope = m = \frac{rise}{run} = \frac{-11}{1} = -11$

9.

a) $slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(6) - (-5)}{(-4) - (4)} = \frac{11}{-8} = -\frac{11}{8}$

b) $slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(2) - (8)}{(7) - (3)} = \frac{-6}{4} = -\frac{3}{2}$

c) $slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(1) - (3)}{(2) - (0)} = \frac{-2}{2} = -1$

d) $slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-3) - (4)}{(1) - (5)} = \frac{-7}{-4} = \frac{7}{4}$

e) $slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-2) - (3)}{(-6) - (2)} = \frac{-5}{-8} = \frac{5}{8}$

f) $slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-2) - (7)}{(3) - (3)} = \frac{-9}{0} = undefined$

g) $slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-6) - (-6)}{(2) - (4)} = \frac{0}{-2} = 0$

10.

a) $slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(6)-(-2)}{(x)-(3)} = \frac{8}{x-3} = \frac{-4}{1}$

$$(8)(1) = (-4)(x-3)$$
$$8 = -4x + 12$$
$$8 - 12 = -4x + 12 - 12$$
$$-4 = -4x$$
$$\frac{-4}{-4} = \frac{-4x}{-4}$$
$$1 = x$$

b) $slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(y)-(1)}{(7)-(1)} = \frac{y-1}{6} = \frac{2}{1}$

$$(6)(2) = (1)(y-1)$$
$$12 = y - 1$$
$$12 + 1 = y - 1 + 1$$
$$13 = y$$

c) $slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1)-(4)}{(3)-(x)} = \frac{-5}{3-x} = \frac{-5}{1}$

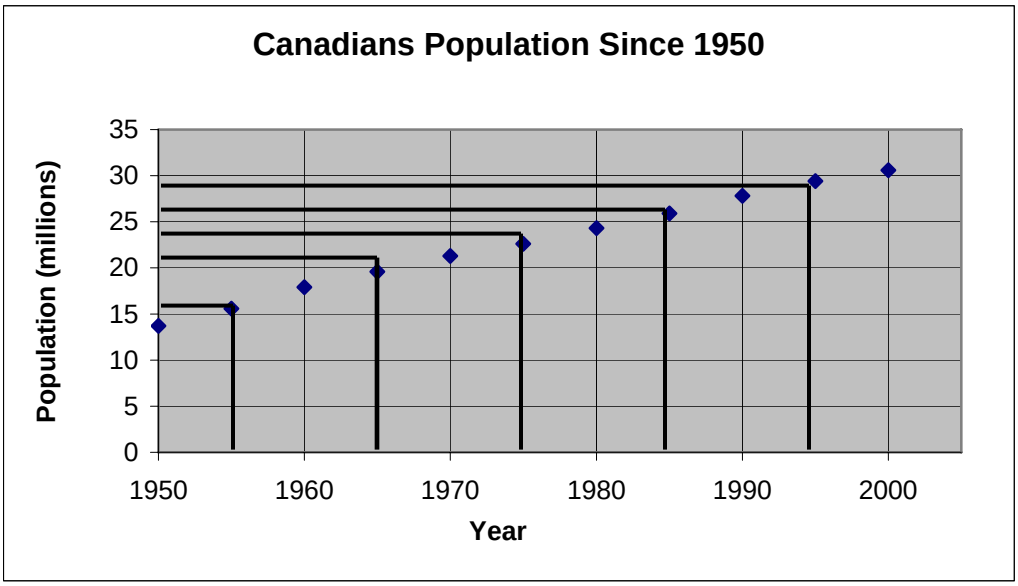
$$(-5)(1) = (-5)(3-x)$$
$$-5 = -15 + 5x$$
$$-5 + 15 = -15 + 15 = 5x$$
$$10 = 5x$$
$$\frac{10}{5} = \frac{5x}{5}$$
$$2 = x$$

d) $slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-3)-(y)}{(-2)-(8)} = \frac{-3-y}{-10} = \frac{1}{5}$

$$(-10)(1) = (5)(-3-y)$$
$$-10 = -15 - 5y$$
$$-10 + 15 = -15 + 15 - 5y$$
$$5 = -5y$$
$$\frac{5}{-5} = \frac{-5y}{-5}$$
$$-1 = y$$

Lesson 10

1. a) 16 million; 19 million; 23 million; 26 million; 29 million
   b) ≈1982



2. Walked at a steady rate, then took a brief rest then walked slightly slower but steadily then reached her grandparents and finally walked quickly back to home.

3. Noah was riding his bike up a hill then once he reached the top rode his bike quickly down the hill.

4. a) 6,8 and 10 textbooks b) approx 48, 68, 30 and 15 cm.

5.

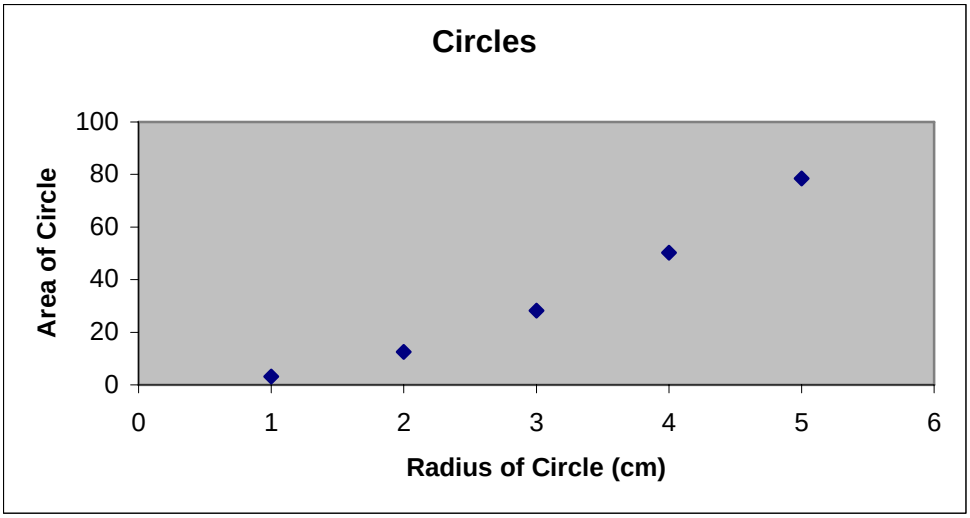
a) Yes, points are joined to see the possible values between points.

b) ≈ 290 cm

c) ≈ 23 stairs

d) yes because this is a linear relationship.

6.



a) Yes, points are joined to see the possible values between points.

b) ≈ 5.4 cm

c) $\approx 62cm^2$

d) No, this is not a linear relationship.

7. a) $y = 2x - 5$

X	2x -5	Y
-4	2(-4) -5	-13
-1	2(-1) -5	-7
2	2(2) -5	-1
5	2(5) -5	5

b) $y = -x + 6$

X	-x + 6	Y
-2	-(-2) + 6	8
-1	-(-1) + 6	7
0	-(0) + 6	6
1	-(1) + 6	5

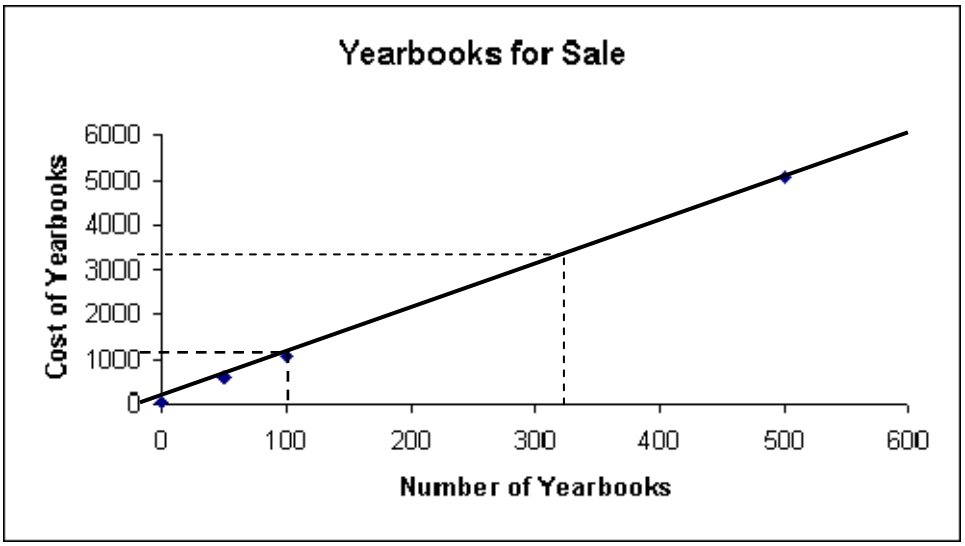
c) $y = w^2 - 1$

X	w^2 - 1	Y
-1	(-1)^2 -1	0
0	(0)^2 -1	-1
1	(1)^2 -1	0
2	(2)^2 -1	3

8.

X	Y
0	60
10	160
50	560
100	1060

b)

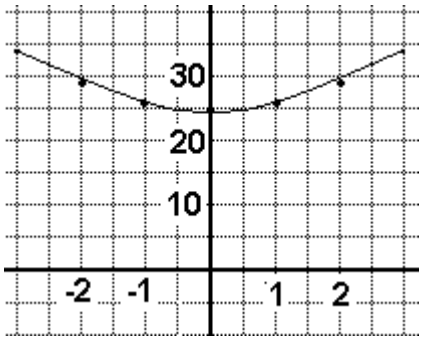


c) ≈ $3300

d) ≈ 100 yearbooks

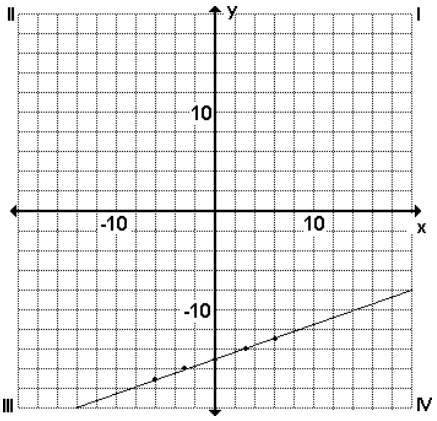
9. a)

X		Y
-2	$(-2)^2 + 25$	29
-1	$(-1)^2 + 25$	26
0	$(0)^2 + 25$	25
1	$(1)^2 + 25$	26
2	$(2)^2 + 25$	29



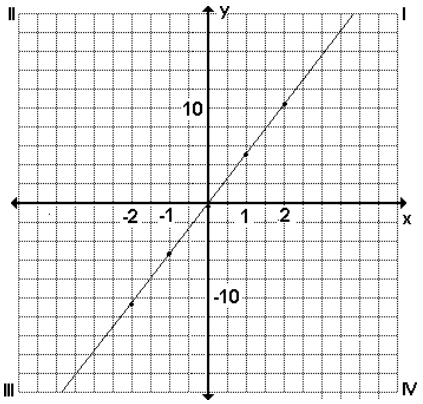
b)

X		Y
-6	$\frac{1}{3}(-6)-15$	-17
-3	$\frac{1}{3}(-3)-15$	-16
0	$\frac{1}{3}(0)-15$	-15
3	$\frac{1}{3}(3)-15$	-14
6	$\frac{1}{3}(6)-15$	-13



c)

X		Y
-2	$5.25(-2)-.25$	-10.75
-1	$5.25(-1)-.25$	-5.50
0	$5.25(0)-.25$	-.25
1	$5.25(1)-.25$	5.00
2	$5.25(2)-.25$	10.25



10. c) (7, 3.5)

$$y = -\frac{1}{2}x + 7$$
$$3.5 = -\frac{1}{2}(7) + 7$$
$$3.5 = -3.5 + 7$$
$$3.5 = 3.5$$

b) (4, 5)

$$5 = -\frac{1}{2}(4) + 7$$
$$5 = -2 + 7$$
$$5 = 5$$

e) (-6, 10)

$$10 = -\frac{1}{2}(-6) + 7$$
$$10 = 3 + 7$$
$$10 = 10$$

