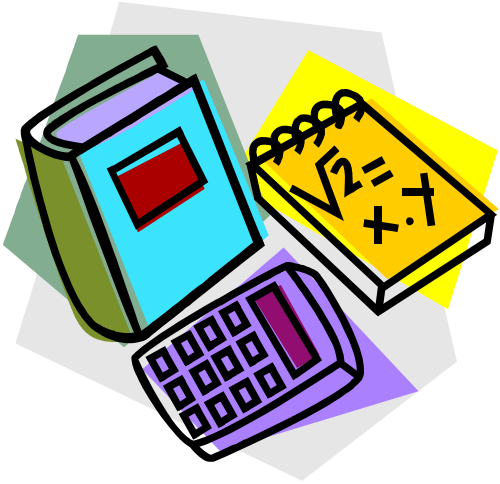
Principles of Mathematics – MPM1D

Grade 9 – Academic Mathematics



Version A

Grade 9 Mathematics (Academic)

Welcome to the Grade 9 Principals of Mathematics, MPM 1D. This full-credit course is part of the new Ontario Secondary School curriculum.

In this course, you will develop generalizations of mathematical ideas and methods through the exploration of applications, the effective use of technology, and abstract reasoning. You will investigate relationships to develop equations of straight lines in analytic geometry, explore relationships between volume and surface area of objects in measurement, and apply extended algebraic skills in problem solving. You will engage in abstract extensions of core learning that will deepen your mathematical knowledge and enrich your understanding.

Material

This course is self-contained and does not require a textbook. You will require lined paper, graph paper, a ruler, a scientific calculator and a writing utensil.

Expectations

The overall expectations you will cover in the lesson are listed on the first page of each lesson.

Lesson Description

Each lesson contains one or more concepts with each being followed by support questions. At the end of the lesson the key questions covering all concepts in the lesson are assigned and will be submitted for evaluation.

Evaluation

In each lesson, there are support questions and key questions. You will be evaluated on your answers to the key questions in each lesson, the mid-term exam and the final exam.

Support Questions

These questions will help you understand the ideas and master the skills in each lesson. They will also help you improve the way you communicate your ideas. The support questions will prepare you for the key questions.

Write your answers to the support questions in your notebook. **Do not** submit these answers for evaluation. You can check your answers against the suggested answers that are given at the end of each unit.

Key Questions

The key questions evaluate your achievement of the expectations for the lesson. Your answers will show how well you have understood the ideas and mastered the skills. They will also show how well you communicate your ideas.

You must try all the key questions and complete most of them successfully in order to pass each unit. Write your answers to the key questions on your own paper and submit them for evaluation at the end of each unit. Make sure each lesson number and question is clearly labelled on your submitted work.

Mid-term and Final Examination

The mid-term and final exams are weighted exams. Knowledge and understanding 40%; Application 30%; Communication 10%; Thinking/Inquiry 20%.

What You Must Do To Get a Credit

In order to be granted a credit in this course, you must

- ☑ Successfully complete the **Key Questions** for each unit and submit them for evaluation within the required time frame.
- ☑ Complete the **mid-term exam** after Unit 2.
- ☑ Complete and pass a **final examination**.

Your Final Mark

• Each Unit has 5 lessons each worth 2% (10% per Unit x 4 Units)	40%	} Term
• Midterm Test	30%	
• Final Examination	30%	

Unit One

Lesson One

- Introduction to ratios
- Simplifying ratios
- Equivalent ratios
- Solving proportions using ratios
- Rate
- Unit rate
- Percent
- Percent of a number

Lesson Two

- Introduction to integers
- Addition and subtraction of integers
- Multiplication and division of integers

Lesson Three

- Introduction to rational numbers
- Ordering rational numbers in numerical order
- Addition, subtraction, multiplication and division of non-fraction rational numbers
- Addition, subtraction, multiplication and division of fractions

Lesson Four

- Introduction to exponents
- Multiplying powers of the same base
- Dividing powers of the same base
- Powers of powers
- Zero exponents
- Negative exponents
- Converting into scientific notation
- Converting from scientific notation
- Multiplying numbers in scientific notation

Lesson Five

- Introduction to square root
- Pythagorean Theorem

Unit Two

Lesson Six

- Introduction to polynomials
- Like terms
- Addition and subtraction of polynomials
- Distributive law
- Multiplication and division of polynomials
- Simplifying expressions then using substitution

Lesson Seven

- Introduction to algebra
- Solving for unknowns
- Checking solutions to algebraic equations

Lesson Eight

- Introduction to problem solving
- 5 steps suggested to problem solving

Lesson Nine

- Introduction to slope
- Cartesian plane
- x and y coordinates on the Cartesian plane
- Plotting order pairs
- Quadrants of the Cartesian plane
- Recognizing positive, negative, zero and undefined slopes
- Using the rise and the run of a given line to find its slope
- Using a pair of coordinates of a line to calculate slope

Lesson Ten

- Introduction to relationships in data
- Graphing relationship in data
- Working with table of values
- Creating graphs for tables of values
- Using graphs to solve related questions

Unit Three

Lesson Eleven

- Introduction to the line
- Standard form of an equation
- Y-intercept form of an equation
- Converting to and from standard to y-intercept form
- Parallel lines
- Perpendicular lines
- Point of intersection of two lines
- x and y intercepts

Lesson Twelve

- Introduction to direct and partial variation
- The origin on a Cartesian plane
- "m" in the equation y = mx
- graphing equation of the form y = mx
- Introduction to partial variation
- "b" in the equation y = mx + b
- graphing equation of the form y = mx + b

Lesson Thirteen

- Introduction to scatter plots
- Creating scatter plots
- Positive, negative and no correlation
- Determining the equation of best fit
- Extrapolation
- Interpolation

Lesson Fourteen

- Introduction to averages (measures of central tendency)
- Mean average
- Median average
- Mode average

Lesson Fifteen

- Introduction to perimeter and circumference
- Radius and diameter
- Calculations using pi (π)
- Solving perimeter/circumference questions using formulas and substitution

Unit Four

Lesson Sixteen

- Introduction to area and surface area
- Radius and diameter
- Calculations using pi (π)
- Solving area and surface area questions using formulas and substitution

Lesson Seventeen

- Introduction to volume
- Radius and diameter
- Calculations using pi (π)
- Solving volume questions using formulas and substitution

Lesson Eighteen

- Introduction to angle geometry
- Angle types
- Angle properties
- Angle properties involving parallel lines and transversals
- Finding unknown angles with justification

Lesson Nineteen

- Introduction to Triangles and Quadrilaterals
- Triangle types
- Triangle properties
- Quadrilateral types
- Quadrilateral properties
- Finding unknown angles with justification

Lesson Twenty

- Introduction to Triangle medians and altitudes
- Median properties
- Centroid
- Altitude properties
- Orthocentre

Ratios, Rate and Percent

Lesson 1

Lesson One Concepts

- Introduction to ratios
- Simplifying ratios
- Equivalent ratios
- Solving proportions using ratios
- Rate
- Unit rate
- Percent
- Percent of a number

Ratios

A **ratio** is a comparison of two or more numbers with the same units.

What is written first in the description matches with the first number in the ratio and what is second matches with the second and so on.

Example

♦ ♦ ♦ ♥ ♥ ♥ ♥ The ratio of ♦ to ♥ is 3 to 4.

Ratios can written in the following three ways:

1. as a fraction, $\frac{3}{4}$
2. as a ratio, 3: 4
3. with words, 3 to 4

Ratios in Lowest Terms

A ratio is in lowest terms (simplest form) when the greatest common factor of the terms is one. 6:4 in lowest terms is 3:2. Both terms are divided by two to produce a ratio in lowest terms.

Example

Give the ratio of A:B in lowest terms.

A A A A B B B B B B B B B B A B A B B A B A A A B

Solution

The ratio of A:B is 10:15 or $\frac{10}{15}$.

In lowest terms both 10 and 15 are divisible by 5 so $\frac{10 \div 5}{15 \div 5} = \frac{2}{3}$ or 2:3.

Equivalent Ratios

Any two or more ratios that simplify to the same lowest terms are equivalent ratios.

Example

Show that the following ratios are equivalent:

12:16 and 60:80

Solution

12:16 has a greatest common factor of 4 reducing the ratio to 3:4 and
60:80 has a greatest common factor of 20 also reducing the ration to 3:4

$\frac{12 \div 4 = 3}{16 \div 4 = 4}$ or 3:4 and $\frac{60 \div 20 = 3}{80 \div 20 = 4}$ or 3:4

Solving Proportions

Equivalent ratios are used when trying to solve questions involving proportion.

Example

Solve each proportion.

a) $4 : 5 = x : 35$ b) $\frac{12}{20} = \frac{24}{x}$

Solution

a) To find x, cross multiplication is used then is followed by division.

$4 : 5 = x : 35$ is the same as $\frac{4}{5} = \frac{x}{35}$

(35)(4)=140 → 140 = 5x ← (5)(x)=5x

28 = x

b) $\frac{12}{20} = \frac{24}{x}$

12x = 480
x = 40

Rate

Rates compare two numbers that are expressed in different units.

Unit rate is a comparison where the second term is always one.

Example

Write the following ratios as a unit rate.

a) 24 soft drinks for 12 people.

b) $475 earned in 40 hours worked.

Solution

a) $24:12 = \frac{24}{12}$ so $\frac{24}{12} = \frac{x}{1}$

24 = 12x

2 = x

Therefore there are two soft drinks per person

b) $475:40 = \frac{475}{40}$ so $\frac{475}{40} = \frac{x}{1}$

475 = 40x
11.88 ≈ x

Therefore the unit rate is $11.88/hr.

Support Questions

1. Write the following ratios in simplest form.
   a) 20:35
   b) 75:125
   c) 13:39

2. Write as a ratio in the lowest terms.
   a) 4 cups of water for every 3 cups of flour
   b) $12 saved for every $75 spent
   c) 8 people passed for every 10 writing a test

3. Write each ratio.
   a) clubs to diamonds
   b) hearts to clubs
   c) spades to diamonds
   d) spades to hearts

   ♣ ♣ ♣ ♣ ♦ ♦ ♦ ♦ ♥ ♥ ♥ ♥ ♥ ♥ ♠ ♠

4. Write each ratio in simplest terms.
   a) 2 home runs to 3 home runs
   b) $10 to $2
   c) 32 days to 8 days

5. On a hockey team 6 of 19 players are defensemen. Write each ratio in simplest terms.
   a) defensemen to team members
   b) team members to defensemen
   c) defensemen to non-defensemen

6. A gas tank in a car holds 54 litres. The cost of a fill up is $38.54.
   a) What is the unit price per litre? (round to the nearest cent)
   b) Using the answer in “a”, how much will 35 litres cost?
   c) How many litres can be purchased with $15.00?

7. Johnny had 15 hits in 40 at bats. How many at bats are needed to achieve 100 hits?

Percent

Percent means the number of parts per hundred.

Expressing a fraction as a decimal then as a percent

Example

Express the following fraction as a percent.

$\frac{4}{5}$

Solution

$\frac{4}{5} = 4 \div 5$

$4 \div 5 = 0.8$

1st convert to decimal

$0.8 \times 100 = 80\%$

Multiply the decimal by 100

Percent of a Number

Example

a) Find 35% of 200.

b) 20% of a number is 15.

c) Calculate the total cost of a shirt costing $24.99 with 7% GST.

Solution

"of" means multiplication

a) 35% of 200 = (0.35)(200)
= 70

Therefore 35% of 200 is 70

b) 20% of a number is 15

"is" means =

"a number" is an unknown (x)

$$(0.20)(x) = 15$$
$$0.20x = 15$$
$$x = \frac{15}{0.20}$$
$$x = 75$$

Therefore 20% of 75 is 15

c) Calculate the total cost of a shirt costing $24.99 with 5% GST.

Total cost = cost of item + tax

5% of 24.99 = (0.05)(24.99)
= $1.25

Rounded to the nearest penny

Total cost = 24.99 + 1.25
= $26.24

Support Questions

Write the following ratios in simplest form.

8. Express each fraction as a percent.
   a) $\frac{2}{5}$ b) $\frac{7}{8}$ c) $\frac{9}{8}$

9. Express each percent as a decimal.
   a) 34% b) 112% c) 0.9%
   d) 23.5%

10. Express each percent as a fraction in lowest terms.
    a) 55% b) 78% c) 0.5%
    d) 125%

11. Find
    a) 25% of 200 b) 70% of 350 c) 180% of 150
    d) 0.7% of 1000

Support Questions

12. Determine the number in each statement.
    a) 30% of a number is 90.
    b) 8% of a number is 24.
    c) 125% of a number is 50.

13. A pair of jeans that normally sells for $49.50 is on sale for 35% off. What is the sale price of the jeans before taxes?

14. A MP3 player costs $149.95. What is the total cost of the MP3 player including 5% GST and 8% PST.

15. The population in a town increased by 850 citizens. This represents an increase of 6.4% over last year. What was the population of the town last year.

Key Question #1

1. Write the following ratios in simplest form. (3 marks)
    a) 12:18
    b) 25:155
    c) 8:32

2. Write as a ratio. (3 marks)
    a) 4 teaspoons of sugar for every 5 cups of flour
    b) 5 t-shirts for every $200 raised
    c) 6 buckets of popcorn for every 3 tickets sold

3. Write each ratio. (4 marks)
    a) happy faces to airplanes ☺☺☺☺☺☺
    b) stars to flowers ✈✈✈✈✈
    c) flowers to airplanes ❀❀❀❀❀❀❀
    d) stars to happy faces ★★★★

4. Write each ratio in simplest terms. (3 marks)
    a) 2 goals to 4 goals
    b) $15 to $5
    c) 32 people to 2 people

Key Question #1 (continued)

5. On a soccer team 3 of 11 players are defensemen. Write each ratio in simplest terms. (3 marks)
   a) defensemen to team members
   b) team members to defensemen
   c) defensemen to non-defensemen

6. Express each fraction as a percent. (3 marks)
   a) $\frac{3}{4}$ b) $\frac{11}{8}$ c) $\frac{2}{3}$

7. Express each percent as a decimal. (4 marks)
   a) 47% b) 216%
   c) 0.75% d) 72.4%

8. Express each percent as a fraction in lowest terms. (4 marks)
   a) 34% b) 65%
   c) 0.04% d) 325%

9. Find (4 marks)
   a) 29% of 300 b) 62% of 100
   c) 180% of 2200 d) 0.2% of 1000

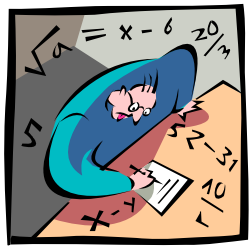
10. The regular price of a DVD player is $84.99. It is on sale for 30% off. What is the amount of the discount? What is the sale price? What is the total price after 13% sales tax is included? (3 marks)

11. The number of people in a town in 2003 was 2500. The population for 2004 increased by 2.5%. What was the population increase? What was the town's population in 2004? (3 marks)

Key Question #1 (continued)

12. An instructor used the following equation to calculate a students' final mark.
Final Mark = 75% of term + 25% of exam mark.

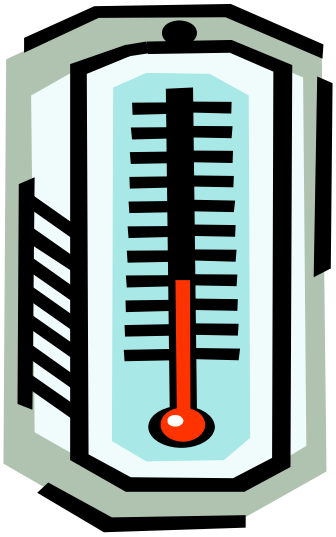
Suppose your term mark is 77%. How will your mark change as your exam mark has been taken into account? Copy and complete the table below: (3 marks)



Term mark	*Exam mark*	*Final mark*
77%	100%	
77%	90%	
77%	80%	
77%	70%	
77%	60%	
77%	50%	
77%	40%	
77%	30%	
77%	20%	
77%	10%	
77%	0%	

13. Write to explain how your final mark in question 12 changes as your examination mark goes from great to not so good. (3 marks)

Integers



Lesson 2

Lesson Two Concepts

- Introduction to integers
- Addition and subtraction of integers
- Multiplication and division of integers

Integers

Integers are the set of numbers …,-3, -2, -1, 0, 1, 2, 3,…

Adding and Subtracting Integers

Example

a) (–3) + (–5)

b) (+5) – (–7)

Solution

If these two signs are opposite then the sign becomes negative

a) (–3) + (–5) = –3 –5
= –8

b) (+5) – (–7) = +5 +7
= +12

If these two signs are the same then the sign becomes positive

Support Questions

1. Add.

a) (+4) + (–5)	b) (–7) + (–3)	c) (–8) + (+8)
d) (0) + (–6)	e) (2) + (+8)	f) (–7) + (+2)
g) (+1) + (–7)	h) (+7) + (+1)	i) (+1) + (–5)

2. Subtract

a) (+1) – (–2)	b) (–4) – (–3)	c) (–9) – (+8)
d) (0) – (–2)	e) (+5) – (+6)	f) (–8) – (–2)
g) (–7) – (–7)	h) (–7) – (+3)	i) (+1) – (–5)

Support Questions

3. For each statement, write an expression then simplify.
   a) moves forward 3 steps and moves backwards 5 steps
   b) goes up an elevator 9 floors then down 6 floors
   c) a gain of $15 followed by a loss of $8
   d) a loss of $25 followed by a gain of $32
   e) a rise in temperature of 18°C then a drop of 20°C

4. For each statement, write an expression then finish the question.
   a) A plane goes up into the air 2000m then increases another 1500m and then decreases its elevation by 1000 m. Find the height of the plane.

   b) An elevator is on the 16th floor. The elevator descends 8 floors, then ascends 14 floors to the top of the building. What floor is the elevator on?

   c) The open value of a stock is $2. Later that day the stock increase by $8 only to fall back by $4 to end the day. What is the closing value of the stock?

   d) At 8:00 am the temperature outside is 6°C. At noon the temperature had increased by 9°C. By 4:00 pm the temperature had increased another 3°C and by 9:00 pm the temperature had decreased 7°C. What was the temperature at 9:00 pm?

   e) The opening balance in a bank account is $4000. $2532 was spent on living expense. $3217 was deposited into the account from salary. What is the closing balance?

Multiplying and Dividing Integers

The product or quotient of an integer question with an even amount of negative values will always give a positive answer unless one of the values is zero.

The product or quotient of an integer question with an odd amount of negative values will always give a negative answer unless one of the values is zero.

Example

a) (+1)(–5)(–2)(–1)(+3)(–2)

b) (–3)(–2)(–1)(+1)

c) (-32) ÷ (-4)

d) $\frac{+28}{-7}$

Solution

a) (+1)(–5)(–2)(–1)(+3)(–2) = 60 or (+60)

b) (–3)(–2)(–1)(+1) = – 6

4 negative signs in question gives positive answer

c) (-32) ÷ (-4) = +8

3 negative signs in question gives negative answer

d) $\frac{+28}{-7}$ = –4

Support Questions

5. Multiply.

a) (+4)(–5)(–2)(–3) b) (–7)(–3)(+1) c) (–8)(+9)
d) (0)(–6)(–2)(–5) e) (2)(+8) f) (–1)(+2)
g) (–1)(–7) h) (+7)(–1)(+1) i) (–1)(–5)(–1)(–4)

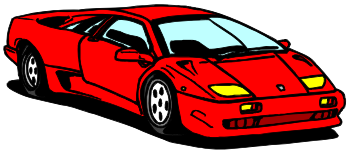
6. Divide

a) (+8)÷(–4) b) (–9)÷(–3) c) (–12)÷(+2)
d) (–15)÷(+5) e) $\frac{-36}{-9}$ f) $\frac{100}{-25}$
g) $\frac{(+8)}{(-2)}$ h) $\frac{(+42)}{(+7)}$ i) (+120)÷(–10)
j) $\frac{-72}{(+9)}$ k) (–9)÷(+3) l) $\frac{(+81)}{(-3)}$

7. For each statement, write an expression then finish the question.

a) A person paid $2 a day for 6 days on lunch. How much did this person spend in total?

b) A person made $40 a day for 8 days worked. How much did this person make in total?



c) A car traveled 400 km in 5 hours. How many km did the car travel per hour?

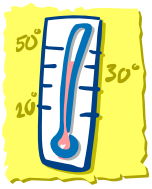
d) An individual 3 times collected 8 stamps for his collection.

Key Question #2

1. Add, subtract, multiply or divide as required. (13 marks)

a) (+4) + (–5)	b) (–7) + (–3)	c) (–8) + (+8)
d) (0) + (–6)	e) (2) + (+8)	f) (–7) + (+2)
g) (0) – (–2)	h) (+5) – (+6)	i) (–8) – (–2)
j) (–7) – (–7)	k) (–7) – (+3)	l) (+1) – (–5)
m) (+4)(–5)(–2)(–3)	n) (–7)(–3)(+1)	o) (–8)(+9)
p) (0)(–6)(–2)(–5)	q) (2)(+8)	r) (–1)(+2)
s) (+8)÷(–4)	t) (–9)÷(–3)	u) $\frac{-36}{-9}$
v) $\frac{100}{-25}$	w) (+120)÷(–10)	x) $\frac{-72}{(+9)}$
y) (–9)÷(+8)	z) $\frac{(+81)}{(-3)}$	

2. For each statement, write an expression then simplify. (4 marks)
   a) moves backwards 3 steps and moves forward 7 steps
   b) goes up 14 stairs then down 8 stairs
   c) a gain of $11 followed by a loss of $18
   d) a rise in temperature of 1°C then a drop of 9°C



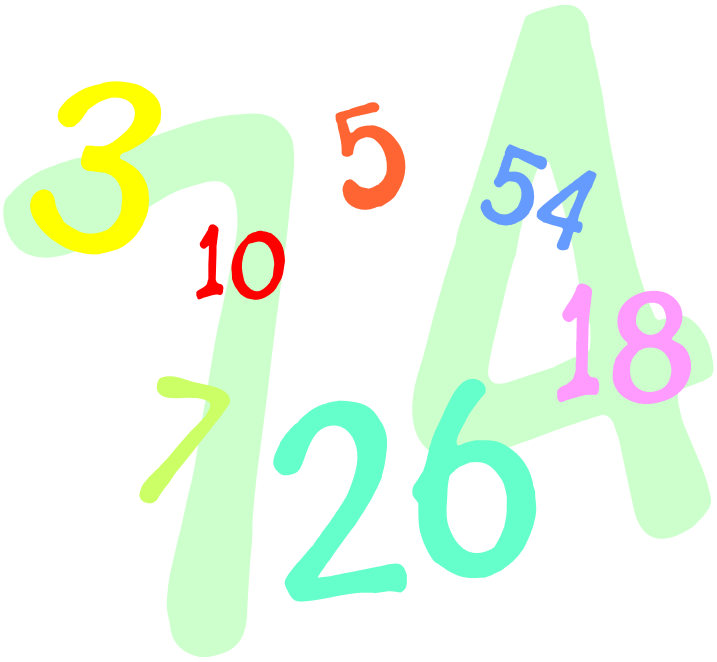
3. A hockey player was given $5 for each of 6 goals she scored. Find her total earnings. (2 marks)

4. An overall loss of $16 000 occurred over an 8 day period. Find the mean loss per day. (2 marks)

5. For each statement, write an expression then write its quotient or a product. (4 marks)
   a) A company twice suffers a loss of 5 trucks.
   b) On 5 occasions, a baseball player lost 6 baseballs.
   c) The mean daily gain when $8 000 is gained over 6 days.
   d) A chicken place sells 5 buckets of 20 pieces of chicken.

6. What did it mean when it is said that “two negatives make a positive?” Is this always true? Prove with examples. (4 marks)

Rational Numbers



Lesson 3

Lesson Three Concepts

- Introduction to rational numbers
- Ordering rational numbers in numerical order
- Addition, subtraction, multiplication and division of non-fraction rational numbers
- Addition, subtraction, multiplication and division of fractions

Rational Numbers

Rational numbers are the set of numbers that can be written in the form $\frac{m}{n}$, where *m* and *n* are integers and $n \neq 0$.

Rational numbers can always be represented on a number line.

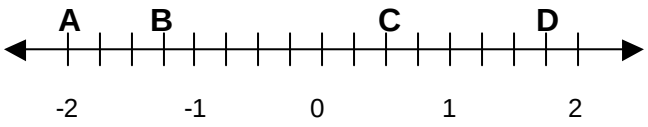
Rational numbers can be written as a decimal by dividing the numerator by the denominator.

Examples of rational numbers.

$$\text{-4, 5.7, 1.23415, 125.67, } \frac{3}{4}, -\frac{32}{5}, -4\frac{1}{8}, \frac{2}{9}$$

Example

a) State the rational number represented by the letter on the number line.



b) Which rational number is greater?

.7, .69

c) Which rational number is greater?

$$\frac{3}{5}, \frac{16}{25}$$

Solution

a) A = –2; B = –1.25; C = 0.5; D = 1.75

b) Since .7 means 70 parts per one hundred and .69 mean 69 parts per one hundred then .7 > .69

"<" means greater than and "<" means less than

c) $\frac{3}{5} = .60$ and $\frac{16}{25} = .64$,therefore, .64 > .60 or $\frac{16}{25} > \frac{3}{5}$

Support Questions

1. Which rational number is greater?

a) $\frac{2}{4}, -\frac{3}{4}$ b) $\frac{1}{2}, \frac{3}{7}$ c) $-\frac{2}{5}, 0.41$

d) $\frac{1}{3}, \frac{5}{18}$ e) $0.3, -0.33$ f) $5.9, 5.99$

g) $-\frac{7}{12}, -\frac{8}{18}$ h) $0.1, -0.11$ i) $\frac{2}{3}, \frac{66}{90}$

j) $-\frac{3}{8}, -\frac{4}{9}$

2. List in order from least to greatest.

a) $0.6, \frac{1}{9}, -\frac{6}{4}, 1.1, -\frac{1}{2}, 0$

b) $-\frac{5}{12}, -0.75, \frac{1}{5}, -1.7, -\frac{1}{6}, 0.9$

c) $\frac{2}{3}, 0.67, \frac{3}{4}, -0.71, \frac{1}{8}, 0.13$

3. Represent all of the following rational numbers on the same number line.

a) .43 b) .85 c) .62

d) .08 e) .21

Adding and Subtracting Rational Numbers (Fractions)

To add and subtract fractions a common denominator must be present. A lowest common denominator is best.

Example

Add or subtract as indicated.

a) $\frac{1}{6}+\frac{4}{6}$ b) $\frac{3}{8}+\frac{5}{6}$ c) $1\frac{2}{3}-2\frac{2}{5}$

Solution

a) $\frac{1}{6}+\frac{4}{6}=\frac{5}{6}$

Only the numerators are added. 4 + 1 = 5

The denominator is the same, as required

b) $\frac{3}{8}+\frac{5}{6}=\frac{9}{24}+\frac{20}{24}$

3 x 3 = 9

8 x 3 = 24

$=\frac{29}{24}$ or $1\frac{5}{24}$

24 is the lowest common denominator. In other words, 24 is the lowest number that both 8 and 6 divide evenly into.

c) $1\frac{2}{3}-2\frac{2}{5}=\frac{5}{3}-\frac{12}{5}$

5 x 2 + 2 = 12

3 x 1 + 2 = 5

$=\frac{25}{15}-\frac{36}{15}$

$=\frac{-11}{15}$ or $-\frac{11}{15}$

Support Questions

4. Add or subtract as indicated.

a) $-\frac{4}{7}-\frac{3}{7}$ b) $\frac{11}{14}+\frac{2}{14}$ c) $-\frac{3}{2}+\frac{1}{6}$

d) $\frac{5}{6}-\frac{3}{8}$ e) (+3.2) –(-3.9) f) (-1.8)–(-6.7)

g) 3.9 + (-4.5) h) (-1.1) – (-1.1) i) $\left(+\frac{3}{5}\right)-\left(\frac{-4}{6}\right)$

j) $\left(-\frac{7}{8}\right)+\left(\frac{5}{-6}\right)$ k) $\left(-\frac{2}{3}\right)+\left(\frac{4}{5}\right)$ l) $\left(-\frac{5}{14}\right)-\left(\frac{-7}{-8}\right)$

Multiplying and Dividing Rational Numbers (Fractions)

You should not use common denominators when multiplying or dividing fractions.

Example

Multiply or divide as indicated.

a) $\frac{2}{7}\times\frac{8}{9}$

b) $2\frac{1}{6}\times 3\frac{7}{8}$

c) $\frac{4}{5}\div\frac{8}{11}$

Solution

a) $\frac{2}{7}\times\frac{8}{9} = \frac{16}{63}$

Both 14 and 6 can be divided by 2 so it is usually best to simplify before multiplying.

b) $2\frac{2}{6}\times 3\frac{7}{8} = \frac{14}{6}\times\frac{31}{8} = \frac{\cancel{14}^{7}}{\cancel{6}_{3}}\times\frac{31}{8} = \frac{217}{24}$

c) $\frac{4}{5}\div\frac{8}{11} = \frac{4}{5}\times\frac{11}{8} = \frac{\cancel{4}^{1}}{5}\times\frac{11}{\cancel{8}_{2}} = \frac{11}{10}$ or $1\frac{1}{10}$

Support Questions

5. Multiply or divide as indicated. Simplify all fraction answers.

a) $\left(-\frac{4}{7}\right)\left(-\frac{3}{7}\right)$ b) $\frac{\left(\frac{11}{7}\right)}{+\left(\frac{2}{5}\right)}$ c) $-\frac{3}{2}\times\frac{1}{6}$

d) $\frac{5}{6}\div\frac{3}{8}$ e) (+3.2) ÷(-3.9) f) (-1.8)×(-6.7)

g) 3.9 ÷ (-4.5) h) (-1.1) × (-1.1) i) $\left(+\frac{3}{5}\right)\left(\frac{-4}{6}\right)$

j) $\left(-\frac{7}{8}\right)\div\left(\frac{5}{-6}\right)$ k) $\left(-\frac{2}{3}\right)\times\left(\frac{6}{5}\right)$ l) $\left(-\frac{5}{14}\right)\div\left(\frac{-7}{-10}\right)$

Key Question #3

1. Which rational number is greater? (5 marks)

a) $\frac{1}{3},\frac{4}{9}$ b) $\frac{22}{25},\frac{21}{24}$ c) $-\frac{2}{5},-.41$

d) $\frac{4}{3},\frac{23}{18}$ e) $0.01,\frac{3}{200}$ f) $4.7,4.71$

g) $-\frac{6}{11},-\frac{5}{9}$ h) $0.2,-0.22$ i) $\frac{2}{7},\frac{22}{70}$

j) $-\frac{5}{8},-\frac{6}{9}$

2. List in order from least to greatest. (3 marks)

a) $0.51,\frac{1}{2},-\frac{5}{4},1.1,\frac{1}{3},0$

b) $\frac{4}{13},0.9,\frac{1}{5},-1.27,-\frac{7}{5},0.93$

c) $\frac{1}{4},0.76,\frac{2}{5},-0.17,\frac{2}{7},0.31$

Key Question #3 (continued)

3. Represent all of the following rational numbers on the same number line. (5 marks)

a) .72 b) .81 c) $\frac{23}{100}$

d) .06 e) $\frac{3}{5}$

4. Add, subtract, multiply or divide as indicated and simplify if needed. (24 marks)

a) $-\frac{2}{5}-\frac{4}{5}$ b) $\frac{2}{8}+\frac{4}{8}$ c) $-\frac{4}{3}+\frac{2}{5}$

d) $\frac{4}{5}-\frac{4}{9}$ e) (+5.1) +(-3.1) f) (1.6)–(-2.4)

g) 8.1 + (-5.7) h) (-2.2) – (-2.2) i) $\left(+\frac{3}{7}\right)-\left(\frac{-4}{7}\right)$

j) $\left(-\frac{5}{6}\right)+\left(\frac{7}{-8}\right)$ k) $\left(-\frac{1}{3}\right)+\left(\frac{3}{7}\right)$ l) $\left(-\frac{5}{6}\right)-\left(\frac{-7}{-4}\right)$

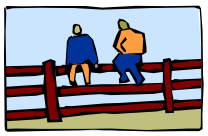
m) $\left(-\frac{4}{3}\right)\left(-\frac{3}{8}\right)$ n) $\frac{\left(\frac{10}{6}\right)}{-\left(\frac{3}{12}\right)}$ o) $-\frac{4}{5}\times\frac{1}{8}$

p) $\frac{4}{7}\div\frac{3}{8}$ q) (+7.1) ÷(-4.9) r) (-0.8)×(-6.3)

s) 8.9 ÷ (-9.5) t) (-2.2) × (-2.2) u) $\left(+\frac{3}{4}\right)\left(\frac{-1}{6}\right)$

v) $\left(\frac{3}{8}\right)\div\left(\frac{1}{-6}\right)$ w) $\left(-\frac{1}{2}\right)\times\left(\frac{4}{9}\right)$ x) $\left(-\frac{5}{21}\right)\div\left(\frac{-7}{-10}\right)$

5. Cliff fenced his yard using lengths of fencing that were 2.4 m, 5.7 m and 4.3 m respectively. The fencing cost $20.23/m. How much did the fence cost? (3 marks)



6. Add 1 to the numerator of the fraction $\frac{1}{5}$. How does the size of the fraction change? (2 marks)

Key Question #3 (continued)

7. Add 1 to the denominator of the fraction $\frac{1}{5}$. How does the size of the fraction change? (2 marks)

8. How does the size of the fraction $\frac{1}{5}$ change when both the numerator and the denominator are increased by 1? (2 marks)

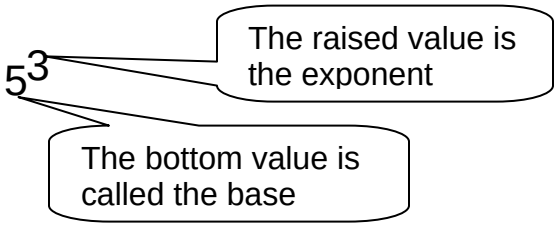
Exponents

Lesson 4

Lesson Four Concepts

- Introduction to exponents
- Multiplying powers of the same base
- Dividing powers of the same base
- Powers of powers
- Zero exponents
- Negative exponents
- Converting into scientific notation
- Converting from scientific notation
- Multiplying numbers in scientific notation

Exponents



5^3 ⟶ Together the base and exponent are called a "power".

5^3 in expanded form is $5 \times 5 \times 5$.

Example

Evaluate each power.

a) 5^3

b) 10^5

Solution

a) $5^3 = 5 \times 5 \times 5$
$= 125$

b) $10^5 = 10 \times 10 \times 10 \times 10 \times 10$
$= 100\,000$

Support Questions

1. Evaluate.

a) 4^3 b) 8^2 c) $(0.5)^3$

d) $\left(\frac{3}{4}\right)^2$ e) $\left(-\frac{4}{5}\right)^3$ f) -6^2

g) 0.7^5 h) $(-6)^2$ i) $\left(\frac{1}{2}\right)^3$

Multiplying Powers

Exponent Law for Multiplying Powers: To multiply powers with the same base, keep the base and add the exponents. $x^n \times x^m = x^{n+m}$, where $x \neq 0$, n and m are natural numbers.

Example

Write each product as a single power.

a) $4^3 \times 4^5$

b) $10^5 \times 10^2$

c) 6×6^3

Solution

a) $4^3 \times 4^5 = 4^{3+5} = 4^8$

b) $10^5 \times 10^2 = 10^{5+2} = 10^7$

c) $6 \times 6^3 = 6^{1+3} = 6^4$

Any number without a visible exponent has an exponent of one. i.e. $6 = 6^1$

Dividing Powers

Exponent Law for Dividing Powers: To divide powers with the same base, keep the base and subtract the exponents. $x^n \div x^m = x^{n-m}$, where x ≠ 0, n and m are natural numbers and n > m.

Example

Write each quotient as a single power.

a) $6^8 \div 6^5$

b) $10^5 \div 10$

Solution

a) $6^8 \div 6^5 = 6^{8-5} = 6^3$

b) $10^5 \div 10 = 10^{5-1} = 10^4$

Support Questions

2. Write each quotient or power as a single power.

a) $4^3 \div 4$ b) $9^2 \times 9^5$ c) $(0.5)^3 \times (0.5)^2$

d) $\left(\frac{2}{3}\right)^7 \div \left(\frac{2}{3}\right)$ e) $\left(\frac{5}{6}\right)^3 \times \left(\frac{5}{6}\right)$ f) $\frac{(-6)^7}{(-6)^3}$

g) $0.4^5 \times 0.4^7$ h) $(-2)^2 \times (-2)^3 \times (-2)^4$

Zero and Negative Exponents

Zero exponent: x^0 is defined to be equal to 1; that is, $x^0 = 1$ (x ≠ 0).

Negative Integer Exponent: x^{-n} is defined to be the reciprocal of x^n; that is $x^{-n} = \frac{1}{x^n}$ (x ≠ 0 and n is an integer).

Example - Evaluate each power.

a) $(-3)^0$

b) 0.25^0

c) 5^{-2}

d) $\frac{1}{2^{-3}}$

Solution

Any positive value or any negative value in brackets with an exponent of zero is always equal to zero

a) $(-3)^0 = 1$

b) $0.25^0 = 1$

When a negative exponent occurs we rewrite the question with one as the numerator and the restated question without the negative exponent sign in the denominator

c) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

d) $\frac{1}{2^{-3}} = 2^3 = 8$

This time the negative exponent is in the denominator so the denominator is rewritten as the numerator without the negative exponent

Support Questions

3. Write each power as a positive then evaluate.

a) 4^{-2} b) $(-5)^{-3}$ c) $\frac{1}{3^{-2}}$

d) 10^{-1}

4. Evaluate.

a) -2^{-2} b) $(-2)^{-2}$ c) 2^2

d) 2^{-2}

5. Write each expression as a single power then evaluate.

a) $2^{-5} \times 2^2$ b) $5^2 \times 5^{-2}$

c) $\left(\frac{1}{2}\right)^2 \div \left(\frac{1}{2}\right)^3$ d) $\frac{3^{-2}}{3^{-2}} \times \frac{3^2}{3^4}$

Support Questions (continued)

6. Use the law of exponents to evaluate each expression.

a) $5^{-2} \times 5^{3} \times 5^{-1}$ b) $(-4)^{3} \times (-4)^{-6} \times (-4)$ c) $\dfrac{8^{-2}}{8^{-3}}$

Power of Powers

Exponent Law for a Power of Power is $\left(x^{m}\right)^{n} = x^{mn}$, where m and n are Integers.

Example

Write as a power with a single exponent.

a) $\left(6^{3}\right)^{2}$

b) $\left((-2)^{4}\right)^{3}$

c) $\left(5^{2}\right)^{3} \times \left(5^{4}\right)^{6}$

Solution

Multiply the exponents together

a) $\left(6^{3}\right)^{2} = 6^{3\times 2} = 6^{6}$

Remember to add the exponents here

b) $\left((-2)^{4}\right)^{3} = (-2)^{4\times 3} = (-2)^{12}$

c) $\left(5^{2}\right)^{3} \times \left(5^{4}\right)^{6} = 5^{6} \times 5^{24} = 5^{30}$

Support Questions

7. Write as a power with a single exponent.

a) $\left(7^{-2}\right)^{-1}$ b) $\left(5^{3}\right)^{5}$ c) $\left(4^{0}\right)^{6}$

d) $\left[(-2)^{3}\right]^{-2}$ e) $\left(3^{2}\right)^{3} \times \left(3^{4}\right)^{5}$ f) $\left[(-16)^{2}\right]^{4} \times \left[(-16)^{3}\right]^{4}$

g) $\left(6^{5}\right)^{2} \div \left(6^{3}\right)^{3}$ h) $\left(10^{3}\right)^{6} \div \left(10^{2}\right)^{9}$ i) $\left(3^{6}\right)^{0}$

Scientific Notation

Scientific Notation: a number expressed as the product of a number greater than –10 and less than –1 or greater than 1 and less than 10, and a power of 10.

Scientific Notation is used to express either very large number or very small numbers.

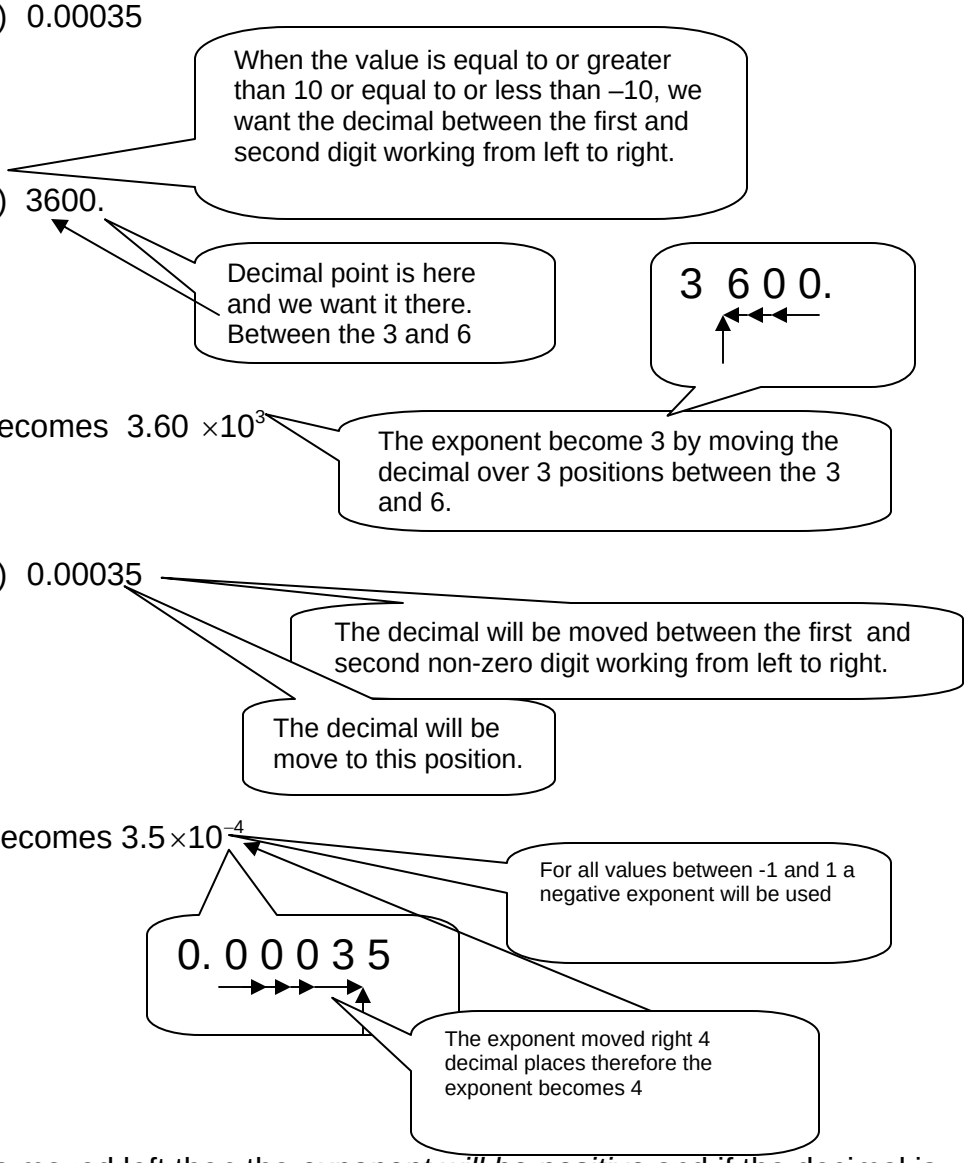
Example

Write the following numbers in scientific notation.

a) 3650

b) 0.00035

Solution



If the decimal is moved left then the ***exponent will be positive*** and if the decimal is moved right the ***exponent will be negative***.

Support Questions

8. Write in scientific notation.

a) 1400 b) 0.0124 c) 1 000 000
d) 0.000 000 17 e) 12 500 f) 135 000
g) –0.001 27 h) –345 i) 12.5×10^{4}
j) -49×10^{-2} k) 0.000 82 l) 2 47 000 000 000

9. Write as a numeral.

a) 2.3×10^{-5} b) -4.6×10^{-2} c) -7.8×10^{3}
d) 4.82×10^{7}

10. Simplify to scientific notation.

a) $\left(3.45\times10^{3}\right)\left(7.25\times10^{-7}\right)$ b) $\dfrac{7.12\times10^{8}}{5.35\times10^{4}}$

c) $3.1\times10^{-2}\times4.89\times10^{-3}$

Key Question #4

1. Evaluate. (9 marks)

a) 3^{4} b) 7^{1} c) $(0.3)^{2}$
d) $\left(\dfrac{4}{5}\right)^{2}$ e) $\left(-\dfrac{4}{5}\right)^{3}$ f) -2^{4}
g) 0.3^{5} h) $(-2)^{4}$ i) $\left(\dfrac{2}{3}\right)^{3}$

2. Write each quotient or power as a single power. (8 marks)

a) $5^{4}\div5$ b) $7^{2}\times7^{5}$ c) $(.5)^{4}\times(.5)^{4}$
d) $\left(\dfrac{3}{5}\right)^{6}\div\left(\dfrac{3}{5}\right)$ e) $\left(\dfrac{3}{8}\right)^{3}\times\left(\dfrac{3}{8}\right)^{2}$ f) $\dfrac{(-3)^{9}}{(-3)^{5}}$
g) $0.9^{5}\times0.9^{2}$ h) $(-4)^{2}\times(-4)^{5}\times(-4)$

3. Write each power as a positive then evaluate. (4 marks)

a) 2^{-3} b) $(-4)^{-2}$
c) $\dfrac{1}{5^{-3}}$ d) 10^{-2}

Key Question #4 (continued)

4. Evaluate. (4 marks)

a) -3^{-2} b) $(-2)^{-3}$

c) 3^{3} d) 3^{-3}

5. Write each expression as a single power then evaluate. (4 marks)

a) $3^{-4} \times 3^{2}$ b) $2^{1} \times 2^{-1}$

c) $\left(\frac{1}{3}\right)^{2} \div \left(\frac{1}{3}\right)^{-1}$ d) $\frac{4^{-2}}{4^{-2}} \times \frac{4^{2}}{4^{4}}$

6. Use the law of exponents to evaluate each expression. (3 marks)

a) $10^{-2} \times 10^{3} \times 10^{-2}$ b) $(-5)^{2} \times (-5)^{-4} \times (-5)$ c) $\frac{7^{-3}}{7^{-4}}$

7. Write as a power with a single exponent. (9 marks)

a) $\left(3^{-3}\right)^{-1}$ b) $\left(5^{4}\right)^{3}$ c) $\left(10^{0}\right)^{5}$

d) $\left[\left(-6\right)^{4}\right]^{-2}$ e) $\left(2^{3}\right)^{2} \times \left(2^{3}\right)^{5}$ f) $\left[(-10)^{3}\right]^{4} \times \left[(-10)^{3}\right]^{2}$

g) $\left(7^{3}\right)^{2} \div \left(7^{4}\right)^{-1}$ h) $\left(9^{1}\right)^{6} \div \left(9^{2}\right)^{6}$ i) $\left(8^{9}\right)^{0}$

8. Write in scientific notation. (12 marks)

a) 2300 b) 0.024 c) 1 000

d) 0.000 045 200 e) 38 500 f) –5 000

g) –0.001 78 h) –591 i) 85×10^{6}

j) $-.49 \times 10^{-2}$ k) 0.012 8 l) 2 47 000 000

9. Write as a numeral. (4 marks)

a) 3.78×10^{-2} b) -4.6×10^{2}

c) -7.84×10^{4} d) 6.92×10^{6}

10. Simplify to scientific notation. (3 marks)

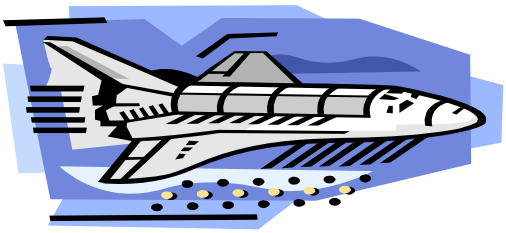
a) $\left(4.72 \times 10^{5}\right)\left(3.99 \times 10^{6}\right)$ b) $\frac{2.94 \times 10^{11}}{6.87 \times 10^{3}}$ c) $-4.53 \times 10^{2} \times 8.37 \times 10^{-3}$

11. The mass of the Earth is approximately 6.0×10^{24}. The mass of the Sun is about 3.3×10^{7} times greater than the mass of the Earth. What is the mass of the Sun? (2 marks)

Key Question #4 (continued)

12. The mass of the Sun is about 2.7×10^7 times as great as the mass of the Moon. What is the mass of the Moon? (2 marks)

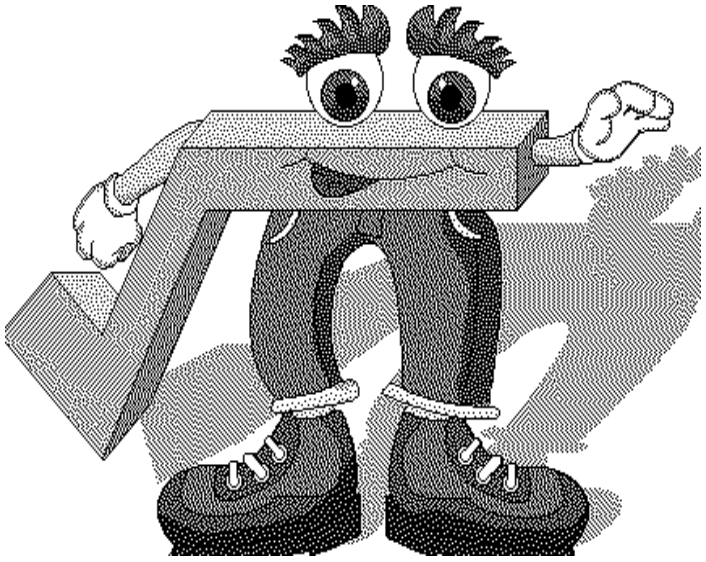
13. Two spacecraft named Veger I and Veger II were launched to visit the outer planets of our solar system. These spacecraft have now traveled so far they have left our solar system. Veger I is traveling at 57 000 km/h and in 275 000 years it will reach the star of Sirius. How far away is Sirius? (2 marks)



14. Brenda wrote a number in scientific notation. She made a mistake. Instead of writing 4.5×10^5 she wrote 4.5×10^{-5}. How many times as large as the correct number was Brenda's number? (2 marks)

15. If two numbers in scientific notation are divided and their quotient is also in scientific notation, how is the power of 10 in the quotient related to the power of 10 in the two original numbers? (2 marks)

Square Root



Lesson 5

Lesson Five Concepts

- Introduction to square root
- Pythagorean Theorem

Square Root

Square root is a number, when multiplied by itself, results in a given number.

Example

Find the square root.

a) $\sqrt{25}$

b) $\sqrt{235}$

Solution

a) $\sqrt{25} = \pm 5$

-5 and +5, when multiplied to itself, result in 25

b) $\sqrt{235} \approx \pm 15.33$

square roots questions always give 2 answers

Support Questions

1. What are the square roots of each number?

a) 9	b) 64	c) 1
d) 121	e) 23	f) 58
g) 2314	h) -16	i) 0.25
j) 1.25	k) $\frac{1}{16}$	l) $\frac{4}{9}$

2. Determine the values that satisfy the equations.

a) $n^2 = 25$	b) $x^2 = 121$
c) $w^2 = \frac{16}{81}$	d) $t^2 = \frac{1}{4}$

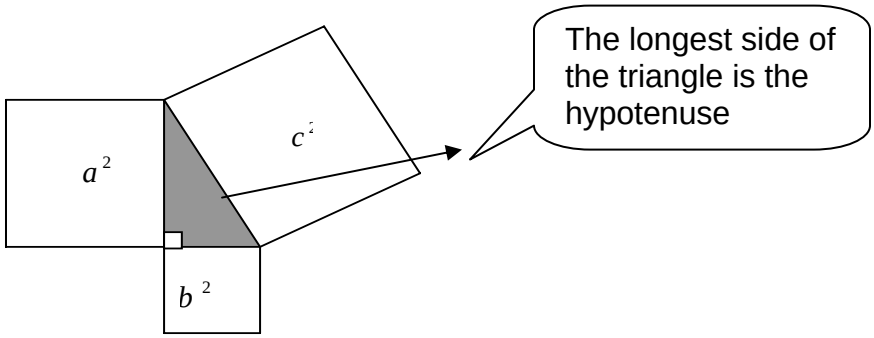
3. Determine the square roots of each number.

a) 0.09	b) 9	c) 900
d) 90 000	e) 9 000 000	f) 900 000 000

Pythagorean Theorem

The Pythagorean Theorem is one method used to calculate an unknown side of a right angle triangle if the other two side are known.

Pythagorean Theorem states $a^2 + b^2 = c^2$. Where the area of square "a" plus the area of square "b" is equal to the area of square "c".



Since the formula for the area of a square is $A = s^2$ then to find the length of the *a* side of a square we square root the value of the area.

Example

Find the length of a side of a square that has an area of 100 cm^2.

Solution

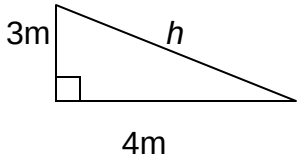
The area of square was 100 then the side would be 10.

$\sqrt{100} = \pm 10$

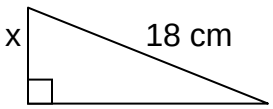
We only use the positive answer since you cannot have negative length.

Example

a) Find the length of the hypotenuse in the given triangle using Pythagorean Theorem.



b) Find the length of the missing side in the given triangle using Pythagorean Theorem.



Solution

a) Using Pythagorean Theorem $a^2 + b^2 = c^2$

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$
$$25 = c^2$$
$$5 = c$$

The area of square created by the hypotenuse is 25 so the length of its side is the square root of 25.

Therefore, the length of the hypotenuse is 5 m.

b) Using Pythagorean Theorem $a^2 + b^2 = c^2$

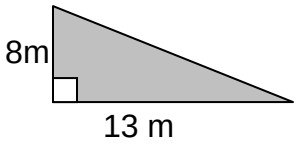
$$a^2 + b^2 = c^2$$
$$x^2 + 12^2 = 18^2$$
$$x^2 = 18^2 - 12^2$$
$$x^2 = 180$$
$$x \approx 13.4 \text{ cm}$$

Therefore, the length of the missing side is 13.4 cm.

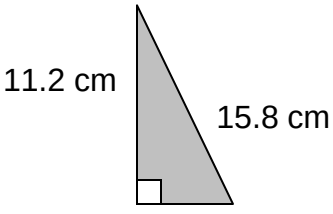
Support Questions

4. Calculate the length of the third side of each triangle. Round to one decimal place.

a)

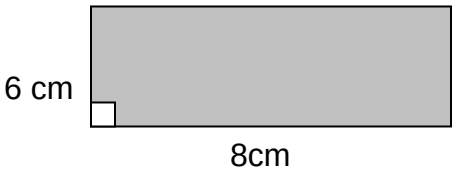


b)

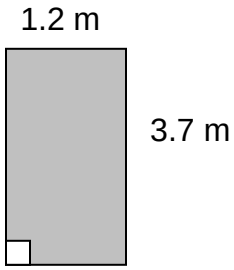


5. Calculate the diagonal of each rectangle. Round to one decimal place.

a)



b)



Key Question #5

1. What are the square roots of each number? (4 marks)

a) 14 b) 81 c) 100

d) 1 e) 0.62 f) 1.73

g) $\frac{1}{25}$ h) $\frac{36}{64}$

2. Simplify each expression. (3 marks)

a) $\sqrt{25}+\sqrt{36}$ b) $\sqrt{25+36}$ c) $\sqrt{\sqrt{25}+36}$

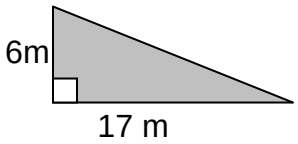
3. Determine the values that satisfy the equations. (4 marks)

a) $n^2 = 81$ b) $x^2 = 70$

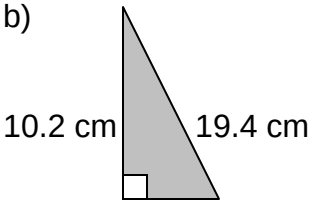
c) $w^2 = \frac{81}{121}$ d) $t^2 = \frac{1}{49}$

4. Calculate the length of the third side of each triangle. Round to one decimal place. (4 marks)

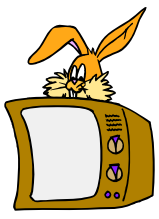
a)



b)



5. Televisions are sold by the size of their screen's diagonal. What is the diagonal size of a television that has a monitor screen height of 22 inches and a monitor screen width of 38 inches? (2 marks)



6. Plot each pair of ordered pairs and calculate the distance between them. (6 marks)

a) A(1,3), B(5,7) b) C(5,0), D(7,2) c) E(-2,3), F(1,-2)

Key Question #5 (continued)

7. For a sloping ladder to be safe, the distance from the wall to the base of the ladder must be $\frac{1}{4}$ of the vertical distance from the ground to the top of the ladder. A 16 m ladder is placed against a wall. How far up the wall will the ladder reach? (4 marks)

8. Does Pythagorean Theorem work for non-right angle triangles? Prove your answer with examples. (4 marks)

Support Question Answers

Lesson 1

1. a) 4:7 b) 3:5 c) 1:3

2. a) 4:3 b) 4:25 c) 4:5

3. a) 4:4 b) 6:4 c) 2:4
   d) 2:6

4. a) 2:3 b) 5:1 c) 4.1

5. a) 6:19 b) 19:6 c) 6:13

6. a)
$$\frac{54L}{38.54} = \frac{1L}{x}$$
$$54x = 38.54$$
$$\frac{54x}{54} = \frac{38.54}{54}$$
$$x = \$0.71/L$$
   b) $35L \times .71 = \$24.85$
   c) $\$15.00 \div .71 \approx 21L$

7. a)
$$\frac{15}{40} = \frac{100}{x} \quad 15x = 4000 \quad x = 267 \text{ at bats}$$

8. a) $2 \div 5 = 0.4$
   $0.40 \times 100 = 40\%$
   b) $7 \div 8 = 0.875$
   $0.875 \times 100 = 87.5\%$
   c) $9 \div 8 = 1.125$
   $1.125 \times 100 = 112.5\%$

9. a) $34 \div 100 = 0.34$
   b) $112 \div 100 = 1.12$
   c) $0.9 \div 100 = 0.009$
   d) $23.5 \div 100 = 0.235$

10. a) $55 \div 100 = 0.55$
$$0.55 = \frac{55}{100} = \frac{11}{20}$$
    b) $78 \div 100 = 0.78$
$$0.78 = \frac{78}{100} = \frac{39}{50}$$
    c) $0.5 \div 100 = 0.005$
$$0.005 = \frac{5}{1000} = \frac{1}{200}$$
    d) $125 \div 100 = 1.25$
$$1.25 = \frac{125}{100} = \frac{5}{4}$$

11. a) $25 \div 100 = 0.25$
$0.25 \times 200 = 50$

b) $70 \div 100 = 0.7$
$0.7 \times 350 = 245$

c) $1.8 \times 150 = 270$
$.007 \times 1000 = 7$

12. a) $0.30 \times n = 90$
$0.30n = 90$
$\frac{0.30}{0.30}n = \frac{90}{0.30}$
$n = 300$

b) $0.08 \times n = 24$
$0.08n = 24$
$\frac{0.08}{0.08}n = \frac{24}{0.08}$
$n = 300$

c) $1.25 \times n = 50$
$1.25n = 50$
$\frac{1.25}{1.25}n = \frac{50}{1.25}$
$n = 40$

13. $0.35 \times 49.50 = 17.33$
$49.50 - 17.33 = \$32.17$

14. $\text{GST} = 149.95 \times 0.05$
$\text{GST} = 7.50$

$\text{PST} = 149.95 \times 0.08$
$\text{PST} = 12.00$

$\text{Sales Tax} = 7.50 + 12.00$
$\text{Sales Tax} = \$19.50$
$\text{Total Price} = \$169.00$

15. $0.064 \times n = 850$
$0.064n = 850$
$\frac{0.064}{0.064}n = \frac{850}{0.064}$
$n = 13281$

Lesson 2

1. a) –1 b) –10 c) 0
d) –6 e) 10 f) –5
g) –6 h) 8 i) –4

2. a) 3 b) –1 c) –17
   d) 2 e) –1 f) –6
   g) 0 h) –10 i) 6

3. a) +3 – 5 = -2 b) +9 – 6 = 3 c) +15 – 8 = 7
   d) –25 + 32 = 7 e) +18 – 20 = -2

4 a) +2000 +1500 – 1000 = 2500
   b) 16 – 8 + 14 = 22
   c) 2 + 8 – 4 = 6
   d) 6 + 9 + 3 – 7 = 11
   e) 4000 – 2532 + 3217 = 4685

5. a) –120 b) 21 c) –72
   d) 0 e) 16 f) –2
   g) 7 h) –7 i) 20

6. a) –2 b) 3 c) –6
   d) –3 e) 4 f) –4
   g) –4 h) 6 i) –12
   j) –8 k) –3 l) –27

7. a) $2 \times 6 = 12$ b) $40 \times 8 = 320$
   c) $400 \div 5 = 80$ d) $3 \times 8 = 24$

Lesson 3

1. a)

$$\frac{2}{4} = 0.5 \ , \ -\frac{3}{4} = -0.75$$

$0.5 > -0.75$ therefore $\frac{2}{4} > -\frac{3}{4}$

b)

$$\frac{1}{2} = 0.5 \ , \ \frac{3}{7} \approx 0.43$$

$0.5 > 0.43$ therefore $\frac{1}{2} > \frac{3}{7}$

c)

$$-\frac{2}{5} = -0.40$$

$0.41 > -0.40$ therefore $0.41 > -0.40$

d)

$$\frac{1}{3} = 0.3\overline{3} \quad , \quad \frac{5}{18} \approx 0.2\overline{7}$$

$0.33 > 0.27$ therefore $\frac{1}{3} > \frac{5}{18}$

e)

$0.3 = 0.30$
$0.3 > -0.33$

f)

$5.9 = 5.90$
$5.99 > 5.90$ therefore $5.99 > 5.9$

g)

$$-\frac{7}{12} = -0.58\overline{3} \quad , \quad \frac{8}{18} \approx -0.4\overline{4}$$

$-0.4\overline{4} > -0.58\overline{3}$ therefore $-\frac{8}{18} > -\frac{7}{12}$

h)

$0.1 = 0.10$
$0.10 > -0.11$

i)

$$\frac{2}{3} = 0.6\overline{6} \quad , \quad \frac{66}{90} \approx 0.7\overline{3}$$

$0.7\overline{3} > 0.6\overline{6}$ therefore $\frac{66}{90} > \frac{2}{3}$

j)

$$-\frac{3}{8} = -0.375 \quad , \quad -\frac{4}{9} \approx -0.44\overline{4}$$

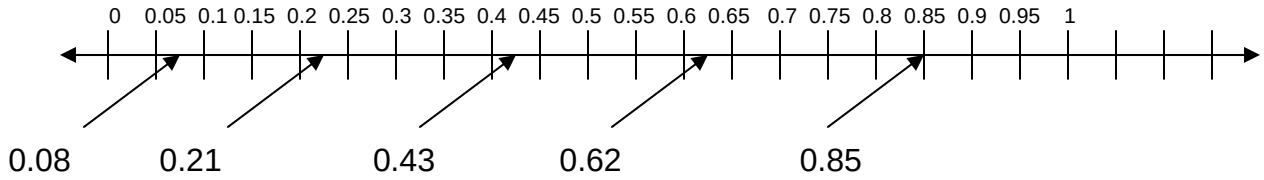
$-0.375 > -0.444$ therefore $-\frac{3}{8} > -\frac{4}{9}$

2. a) $-\frac{6}{4},\ -\frac{1}{2},\ 0,\ .1,\ \frac{1}{9},\ 0.6,\ 1$

b) $-1.7,\ -0.75,\ -\frac{5}{12},\ -\frac{1}{6},\ \frac{1}{5},\ 0.9$

c) $-0.71,\ \frac{1}{8},\ 0.13,\ \frac{2}{3},\ 0.67,\ \frac{3}{4}$

3.



4. a) $-\frac{4}{7}-\frac{3}{7}=\frac{-4}{7}-\frac{3}{7}=\frac{-7}{7}=-1$

b) $\frac{11}{14}+\frac{2}{14}=\frac{13}{14}$

c) $-\frac{3}{2}+\frac{1}{6}=\frac{-3^{x3}}{2^{x3}}+\frac{1}{6}=\frac{-9}{6}+\frac{1}{6}=\frac{-8}{6}=\frac{-4}{3}$ or $-1\frac{1}{3}$

d) $\frac{5}{6}-\frac{3}{8}=\frac{5^{x4}}{6^{x4}}-\frac{3^{x3}}{8^{x3}}=\frac{20}{24}-\frac{9}{24}=\frac{11}{24}$

e) 7.1 f) 4.9 g) –0.6 h. 0

i) $\left(+\frac{3}{5}\right)-\left(\frac{-4}{6}\right)=\frac{3}{5}+\frac{4}{6}=\frac{3^{x6}}{5^{x6}}+\frac{4^{x5}}{6^{x5}}=\frac{18}{30}+\frac{20}{30}=\frac{38}{30}=\frac{19}{15}$ or $1\frac{4}{15}$

j) $\left(-\frac{7}{8}\right)+\left(\frac{5}{-6}\right)=\frac{-7}{8}+\frac{-5}{6}=\frac{-7^{x3}}{8^{x3}}+\frac{-5^{x4}}{6^{x4}}=\frac{-21}{24}+\frac{-20}{24}=\frac{-41}{24}$ or $-1\frac{17}{24}$

k) $\left(-\frac{2}{3}\right)+\left(\frac{4}{5}\right)=\frac{-2}{3}+\frac{4}{5}=\frac{-2^{x5}}{3^{x5}}+\frac{4^{x3}}{5^{x3}}=\frac{-10}{15}+\frac{12}{15}=\frac{2}{15}$

l) $\left(-\frac{5}{14}\right)-\left(\frac{-7}{-8}\right)=\frac{-5}{14}-\frac{7}{8}=\frac{-5^{x4}}{14^{x4}}-\frac{7^{x7}}{8^{x7}}=\frac{-20}{56}-\frac{49}{56}=\frac{-69}{56}$ or $-1\frac{13}{56}$

5. a) $\left(-\frac{4}{7}\right)\left(-\frac{3}{7}\right) = \frac{-4}{7} \times \frac{-3}{7} = \frac{12}{49}$

b) $\frac{\left(\frac{11}{7}\right)}{+\left(\frac{2}{5}\right)} = \frac{11}{7} \div \frac{2}{5} = \frac{11}{7} \times \frac{5}{2} = \frac{55}{14}$ or $3\frac{13}{14}$

c) $\frac{-\cancel{3}^{1}}{2} \times \frac{1}{\cancel{6}^{2}} = \frac{-1}{2} \times \frac{1}{2} = \frac{-1}{4}$ or $-\frac{1}{4}$

d) $\frac{5}{6} \div \frac{3}{8} = \frac{5}{6} \times \frac{8}{3} = \frac{5}{\cancel{6}^{3}} \times \frac{\cancel{8}^{4}}{3} = \frac{20}{9}$ or $2\frac{2}{9}$

e) ≈ -0.82 f) ≈ 12.06 g) ≈ -0.87 h) 1.21

g) $\left(+\frac{3}{5}\right)\left(\frac{-4}{6}\right) = \frac{-12}{30} = \frac{-2}{5}$ or $-\frac{2}{5}$

h) $\left(-\frac{7}{8}\right) \div \left(\frac{5}{-6}\right) = \frac{-7}{8} \times \frac{-6}{5} = \frac{-7}{\cancel{8}_{4}} \times \frac{-\cancel{6}^{3}}{5} = \frac{21}{20}$ or $1\frac{1}{20}$

i) $\left(\frac{3}{5}\right)\left(\frac{-4}{6}\right) = \frac{\cancel{3}^{1}}{5} \times \frac{-4}{\cancel{6}_{2}} = \frac{-4}{10} = \frac{-2}{5}$

j) $\left(-\frac{7}{8}\right) \div \left(\frac{5}{-6}\right) = \frac{-7}{\cancel{8}_{4}} \times \frac{-\cancel{6}^{3}}{5} = \frac{21}{20}$ or $1\frac{1}{20}$

i) $\frac{-2}{3} \times \frac{6}{5} = \frac{-2}{\cancel{3}^{1}} \times \frac{\cancel{6}^{2}}{5} = \frac{-4}{5}$ or $-\frac{4}{5}$

j) $\left(-\frac{5}{14}\right) \div \left(\frac{-7}{-10}\right) = \frac{-5}{14} \times \frac{10}{7} = \frac{-5}{\cancel{14}^{7}} \times \frac{\cancel{10}^{5}}{7} = \frac{-25}{49}$ or $-\frac{25}{49}$

Lesson 4

1. a) 64 b) 64 c) 0.125
   d) $\frac{9}{16}$ e) $-\frac{64}{125}$ f) –36
   g) 0.16807 h) 36 i) $\frac{1}{8}$

2. a) $4^3 \div 4 = 4^{3-1} = 4^2$ b) $9^2 \times 9^5 = 9^{2+5} = 9^7$
   c) $(.5)^3 \times (.5)^2 = (.5)^{3+2} = (.5)^{3+2} = (.5)^5$ d) $\left(\frac{2}{3}\right)^7 \div \left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^6$
   e) $\left(\frac{5}{3}\right)^3 \times \left(\frac{5}{6}\right) = \left(\frac{5}{6}\right)^{3+1} = \left(\frac{5}{6}\right)^4$ f) $(-6)^7 \div (-6)^3 = (-6)^{7-3} = (-6)^4$
   g) $0.4^5 \times 0.4^7 = 0.4^{5+7} = 0.4^{12}$ h) $(-2)^2 \times (-2)^3 \times (-2)^4 = (-2)^{2+3+4} = (-2)^9$

3. a)
   $4^{-2} = \frac{1}{4^2}$
   $\frac{1}{4^2} = \frac{1}{16}$

   b)
   $(-5)^{-3} = \frac{1}{(-5)^3}$
   $\frac{1}{(-5)^3} = -\frac{1}{125}$

   c)
   $\frac{1}{3^{-2}} = 3^2$
   $3^2 = 9$

   d)
   $10^{-1} = \frac{1}{10}$

4. a) $-2^{-2} = -\frac{1}{2^2} = -\frac{1}{4}$ b) $(-2)^{-2} = \frac{1}{(-2)^2} = \frac{1}{4}$
   c) $2^2 = 4$ d) $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

5. a) $2^{-5} \times 2^2 = 2^{-5+2} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ b) $5^2 \times 5^{-2} = 5^{2-2} = 5^0 = 1$
   c) $\left(\frac{1}{2}\right)^2 \div \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{-1} = \frac{1^{-1}}{2^{-1}} = \frac{2^1}{1^1} = \frac{2}{1} = 2$
   d) $\frac{3^{-2}}{3^{-2}} \times \frac{3^2}{3^4} = \frac{3^{-2+2}}{3^{-2+4}} = \frac{3^0}{3^2} = \frac{1}{9}$

6. a) $5^{-2} \times 5^3 \times 5^{-1} = 5^{-2+3-1} = 5^0 = 1$

b) $(-4)^3 \times (-4)^{-6} \times (-4) = (-4)^{3-6+1} = (-4)^{-2} = \dfrac{1}{(-4)^2} = \dfrac{1}{16}$

c) $\dfrac{8^{-2}}{8^{-3}} = \dfrac{8^3}{8^2} = 8^{3-2} = 8^1 = 8$

7. a) $\left(7^{-2}\right)^{-1} = (7)^{(-2)(-1)} = (7)^2$

b) $(5^3)^5 = (5)^{(3)(5)} = 5^{15}$

c) $\left(4^0\right)^6 = 4^{(0)(6)} = 4^0 = 1$

d) $\left[(-2)^3\right]^{-2} = (-2)^{(3)(-2)} = (-2)^{-6}$

e) $(3^2)^3 \times (3^4)^5 = \left(3\right)^{(2)(3)} \times (3)^{(4)(5)} = (3)^6 \times (3)^{20} = (3)^{6+20} = (3)^{26}$

f) $\left[(-16)^2\right]^4 \times \left[(-16)^3\right]^4 = (-16)^{(2)(4)} \times (-16)^{(3)(4)} = (-16)^8 \times (-16)^{12} = (-16)^{8+12} = (-16)^{20}$

g) $(6^5)^2 \div (6^3)^3 = (6)^{(5)(2)} \div (6)^{(3)(3)} = (6)^{10} \div (6)^9 = (6)^{10-9} = (6)^1$

h) $(10^3)^6 \div (10^2)^9 = (10)^{(3)(6)} \div (10)^{(2)(9)} = (10)^{18} \div (10)^{18} = (10)^{18-18} = (10)^0 = 1$

i) $(3^6)^0 = (3)^{(6)(0)} = (3)^0 = 1$

8. a) $1400 = 1.4 \times 10^3$ b) $0.0124 = 1.24 \times 10^{-2}$ c) $1\,000\,000 = 1.0 \times 10^6$

d) $0.00000017 = 1.7 \times 10^{-7}$ e) $12500 = 1.25 \times 10^4$ f) $135000 = 1.35 \times 10^5$

g) $-0.00127 = -1.27 \times 10^{-3}$ h) $-345 = -3.45 \times 10^2$ i) $12.5 \times 10^4 = 1.25 \times 10^5$

j) $-49 \times 10^{-2} = -4.9 \times 10^{-1}$ k) $0.00082 = 8.2 \times 10^{-4}$

l) $247000000000 = 2.47 \times 10^{11}$

9. a) $2.3 \times 10^{-5} = 0.000023$ b) $-4.6 \times 10^{-2} = -0.046$ c) $-7.8 \times 10^3 = -7800$

d) $4.82 \times 10^7 = 48200000$

$(3.45 \times 10^3)(7.25 \times 10^{-7})$
$= 3.45 \times 7.25 \times 10^3 \times 10^{-7}$

10. a) $= 25.0125 \times 10^{3+(-7)}$
$= 25.0125 \times 10^{-4}$
$= 2.5 \times 10^{-3}$

$$\frac{7.12\times10^{8}}{5.35\times10^{4}}$$

b) $=\left(7.12\div5.35\right)\times10^{8}\div10^{4}$

$=1.3308\times10^{8-4}$

$=1.33\times10^{4}$

$3.1\times10^{-2}\times4.89\times10^{-3}$

$=3.1\times4.89\times10^{-2}\times10^{-3}$

c) $=15.159\times10^{(-2)+(-3)}$

$=15.159\times10^{-5}$

$=1.52\times10^{-4}$

Lesson 5

1. a) $\sqrt{9}=\pm3$ b) $\sqrt{64}=\pm8$ c) $\sqrt{1}=\pm1$

   d) $\sqrt{121}=\pm11$ e) $\sqrt{23}\approx\pm4.8$ f) $\sqrt{58}\approx\pm7.62$

   g) $\sqrt{2314}\approx\pm48.1$ h) $\sqrt{-16}=$ *undefined* i) $\sqrt{.25}=\pm0.5$

   j) $\sqrt{1.25}\approx\pm1.12$ k) $\sqrt{\frac{1}{16}}=\frac{\sqrt{1}}{\sqrt{16}}=\pm\frac{1}{4}$ l) $\sqrt{\frac{4}{9}}=\frac{\sqrt{4}}{\sqrt{9}}=\pm\frac{2}{3}$

2. a)

$$n^2=25$$
$$\sqrt{n^2}=\sqrt{25}$$
$$n=\pm5$$

b)

$$x^2=121$$
$$\sqrt{x^2}=\sqrt{121}$$
$$x=\pm11$$

c)

$$w^2=\frac{16}{81}$$
$$\sqrt{w^2}=\sqrt{\frac{16}{81}}$$
$$\sqrt{w^2}=\frac{\sqrt{16}}{\sqrt{81}}$$
$$w=\pm\frac{4}{9}$$

d)

$$t^2 = \frac{1}{4}$$
$$\sqrt{t^2} = \sqrt{\frac{1}{4}}$$
$$\sqrt{t^2} = \frac{\sqrt{1}}{\sqrt{4}}$$
$$t = \pm\frac{1}{2}$$

3. a) $\sqrt{0.09} = \pm 0.3$ b) $\sqrt{9} = \pm 3$ c) $\sqrt{900} = \pm 30$

d) $\sqrt{90000} = \pm 300$ e) $\sqrt{9000000} = \pm 3000$ f) $\sqrt{900000000} = \pm 30000$

4. a)

$$a^2 + b^2 = c^2$$
$$(8)^2 + (13)^2 = c^2$$
$$64 + 169 = c^2$$
$$233 = c^2$$
$$\sqrt{233} = \sqrt{c^2}$$
$$15.3 \approx c$$

b)

$$a^2 + b^2 = c^2$$
$$(11.2)^2 + b^2 = (15.8)^2$$
$$125.44 + b^2 = 249.64$$
$$125.44 - 125.44 + b^2 = 249.64 - 125.44$$
$$b^2 = 124.2$$
$$\sqrt{b^2} = \sqrt{124.2}$$
$$b \approx 11.1$$

5. a)

$$a^2 + b^2 = c^2$$
$$(6)^2 + (8)^2 = c^2$$
$$36 + 64 = c^2$$
$$100 = c^2$$
$$\sqrt{100} = \sqrt{c^2}$$
$$10 = c$$

b)

$$a^2 + b^2 = c^2$$
$$(1.2)^2 + (3.7)^2 = c^2$$
$$1.44 + 13.69 = c^2$$
$$15.13 = c^2$$
$$\sqrt{15.13} = \sqrt{c^2}$$
$$3.9 \approx c$$

