Chapter 10: Angles and Triangles

Learning Goals:

- Construct circles, triangles and quadrilaterals, and describe their properties.
- Investigate, describe and use the relationships between angles of intersecting lines.
- Investigate, describe and use the Pythagorean relationship for right triangles.
- Create and solve problems that involve lines, angles and right triangles.

Chapter Sections:

10.1: Exploring Points on a Circle
10.2: Intersecting Lines, Parallel Lines and Transversals
10.3: Angles in a Triangle
10.4: Exploring Quadrilaterals
10.5: Exploring Right Triangles
10.6: Applying the Pythagorean Theorem
10.7: Solve Problems Using Logical Reasoning

What Makes this so Cool?

- This is fundamental for high school geometry
- Because you build your logical reasoning skills, which are useful everywhere
- Helps determine location (like on a map)
- Building houses (construction, architecture)
- It adds a new way to approach problems
- Theatre technicians use it, pilots use it, anyone who needs to measure uses it

Mini-Quiz

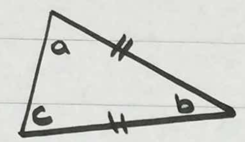
① Determine each square root.

(a) $\sqrt{64}$ (c) $\sqrt{49}$ (e) $\sqrt{24}$

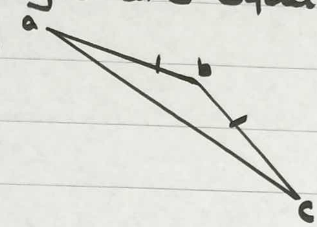
(b) $\sqrt{9}$ (d) $\sqrt{10}$ (f) $\sqrt{2}$

② Which angles in each triangle are equal?

(a)

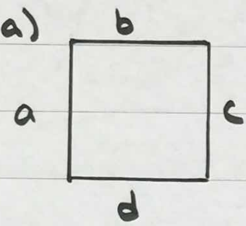


(b)

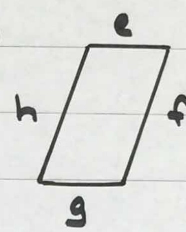


③ Identify the parallel and perpendicular lines.

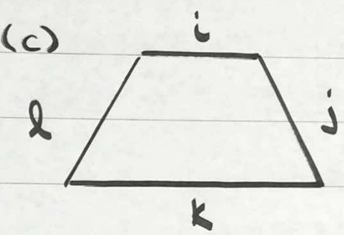
(a)



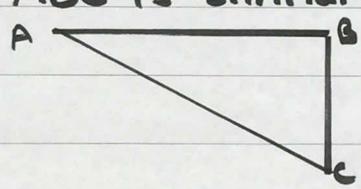
(b)

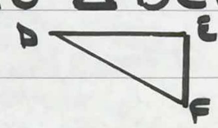


(c)



④ ΔABC is similar to ΔDEF.





(a) Which angle has the same measure as $\angle A$?

(b) Which other pairs of angles are equal?

⑤ Draw an example of each triangle.

(a) a Δ with a 90° angle and a 4 cm side.

(b) a Δ with a 3 cm side between a 30° angle and a 70° angle

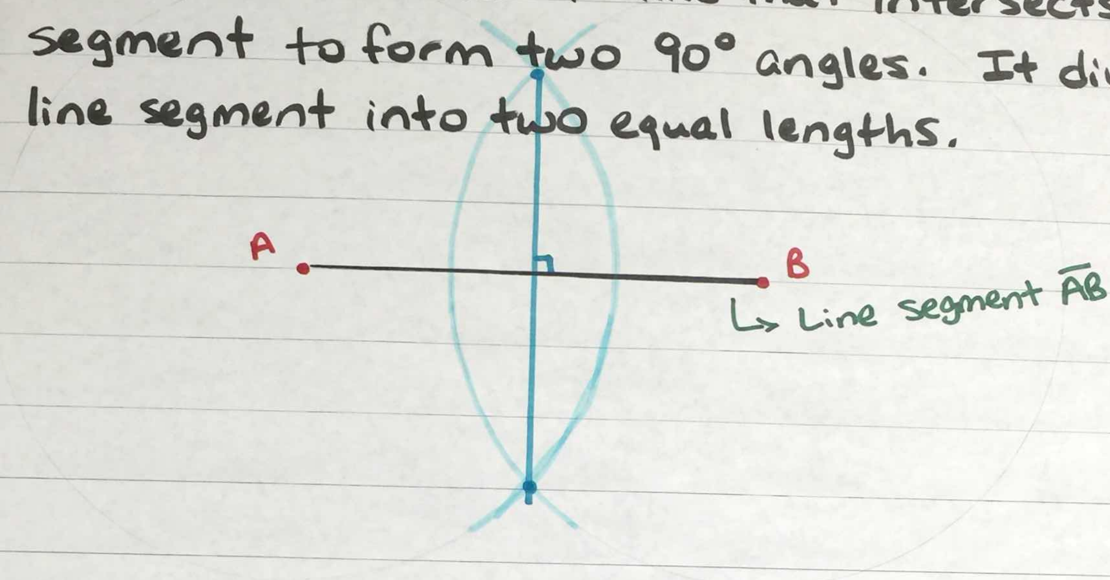
(c) a Δ with a 5 cm side, a 40° angle and a 60° angle

10.1: Exploring Points on a Circle

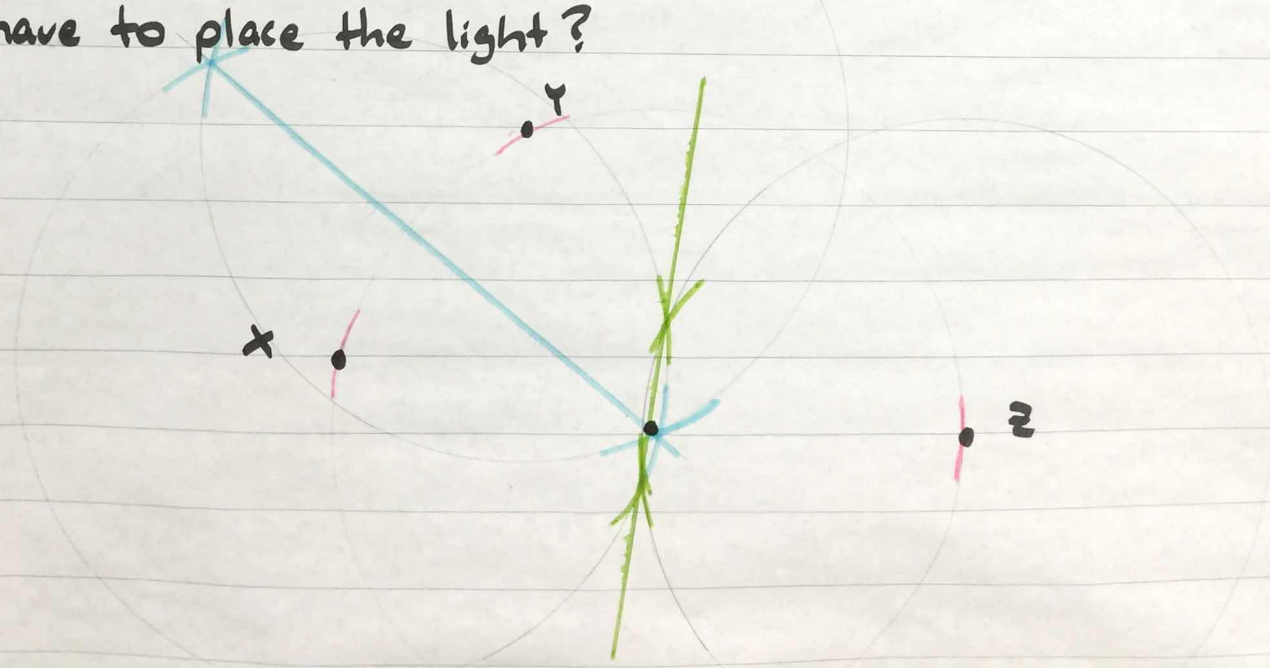
Equidistant: The same distance

Learning Goal: Locate the centre of a circle, given three points on a circle.

Perpendicular Bisector: A line that intersects a line segment to form two 90° angles. It divides this line segment into two equal lengths.



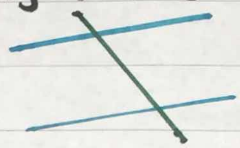
Example #1: Three houses want to share the cost for a new street light. To be fair, they decide to place the light equidistantly from all three houses. Where do they have to place the light?



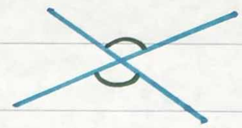
10.2: Intersecting Lines, Parallel Lines and Transversals

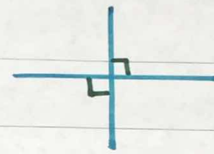
Learning Goal: Identify and apply the relationships between the measure of angles formed by intersecting lines.

Transversal: A straight line that intersects two or more lines.

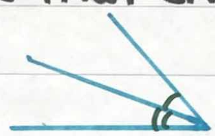


Opposite angles: Non adjacent angles that are formed by two intersecting lines.

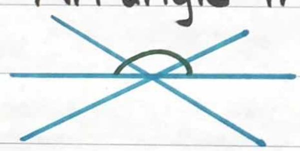




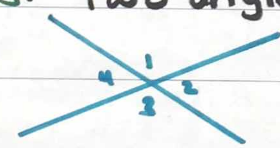
Adjacent angles: Angles that share a common vertex and a common arm.



Straight angle: An angle that measures 180°.

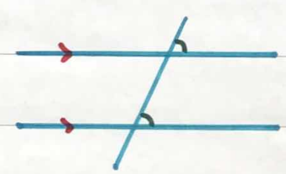


Supplementary angles: Two angles whose sum is 180°.

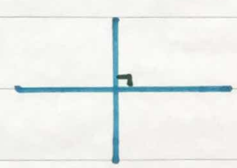


$\angle 1 + \angle 2 = 180°$

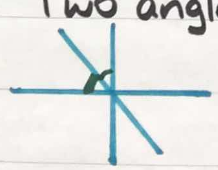
Corresponding angles: Matching angles that are formed by a transversal and two parallel lines



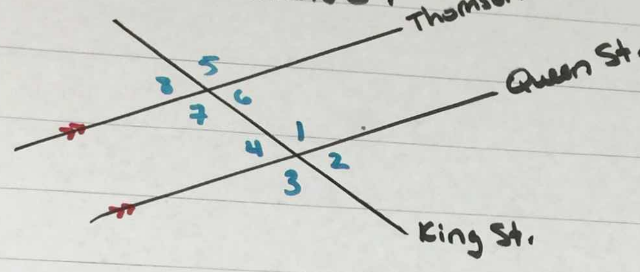
Right angle: An angle that measures 90°.



Complementary angles: Two angles whose sum is 90°.



Example #1: How are the angles formed by these intersecting streets related?



(A) Which street is the transversal? King St.

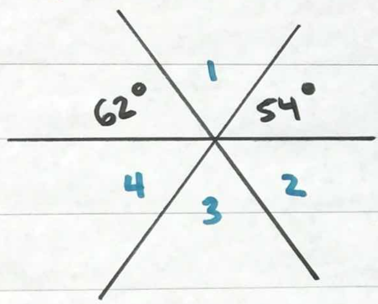
(B) Which angle corresponds to ∠5? ∠1

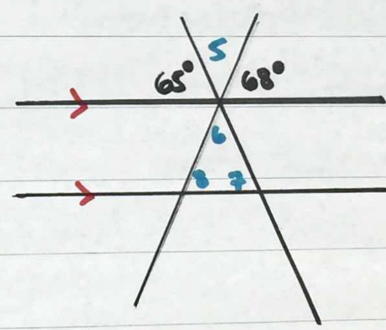
(C) Name an adjacent angle to ∠7. ∠6, ∠8

(D) List all eight pairs of adjacent supplementary angles.
∠1 + ∠2   ∠3 + ∠4   ∠5 + ∠6   ∠7 + ∠8
∠2 + ∠3   ∠1 + ∠4   ∠6 + ∠7   ∠5 + ∠8

(E) Which angle is opposite ∠4? ∠2

Example #2: Determine the measure of the following angles.





∠1 = 180° − 62° − 54° = 64°

∠2 = 62° (opposite)

∠3 = 64° (opposite ∠1)

∠4 = 54°

∠5 = 180° − 65° − 68° = 47°

∠6 = 47° (opposite ∠5)

∠7 = 65° (corresponding)

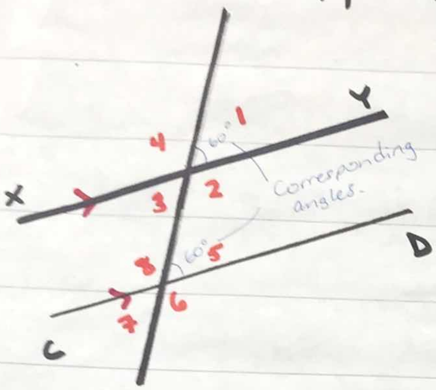
∠8 = 68°

Support Questions:

p. 342, # 4 - 13

Section 10.2 Support Questions, p. 343

(9)

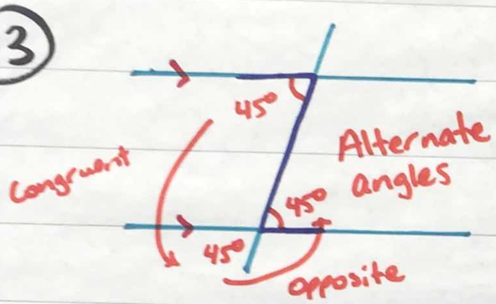


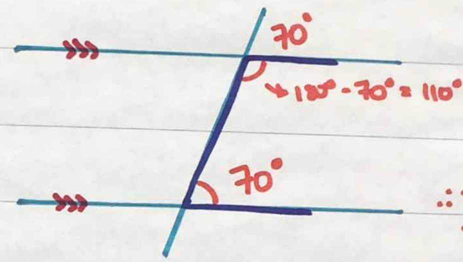
Corresponding Angles

∠1 and ∠5 ∠3 and ∠7

∠2 and ∠6 ∠4 and ∠8

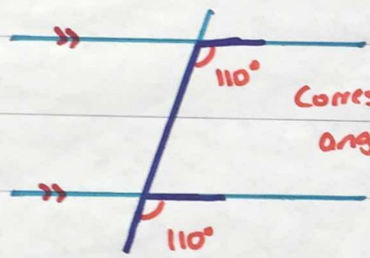
(13)



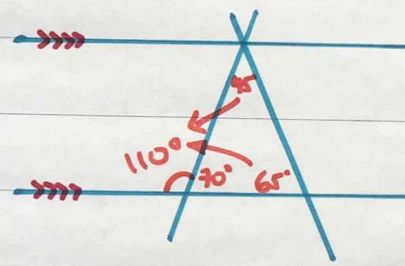


$$\begin{array}{r} 110° \\ +\ 70° \\ \hline 180° \end{array}$$

∴ The angles are supplementary



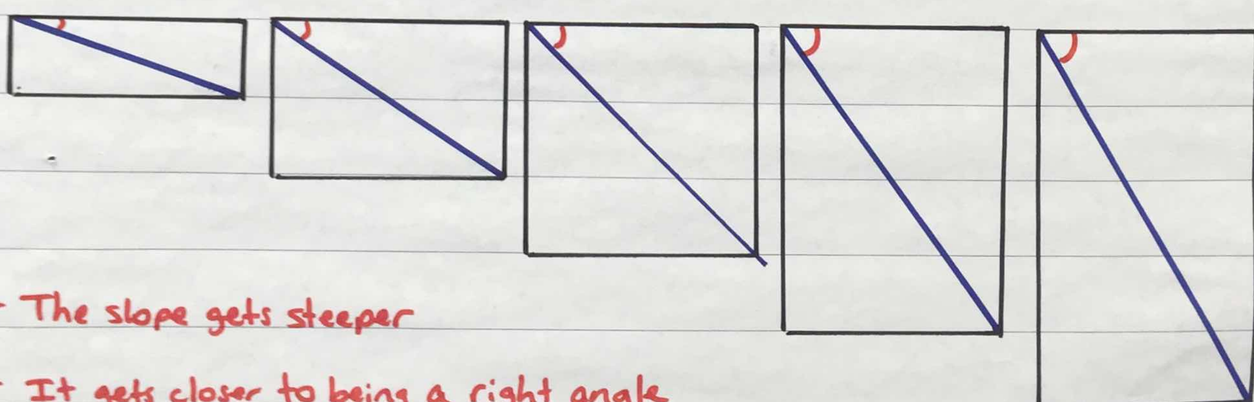
Corresponding angles



Exterior angle

$$\begin{array}{r} 45° \\ +65° \\ \hline 110° \end{array}$$

(14)



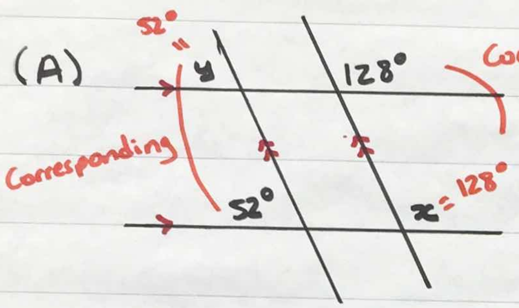
- The slope gets steeper
- It gets closer to being a right angle
- It's a diagonal, so the slope will never hit 90°, but it will get closer and closer.

10.3: Angles in a Triangle

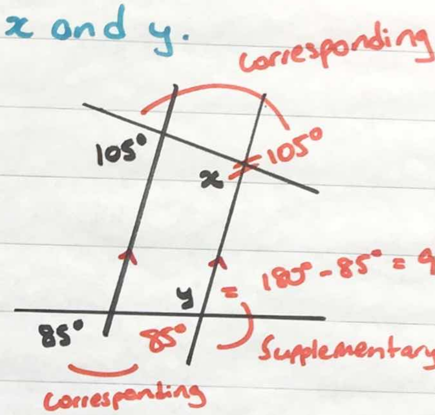
Learning Goal: Determine the sum of angles in a triangle.

EXAMPLE #1: Find the values of x and y.

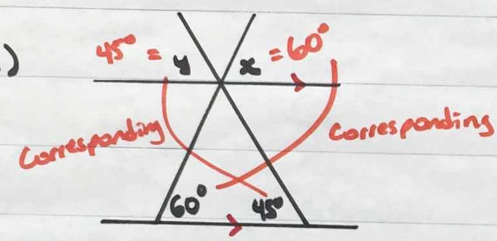
(A)



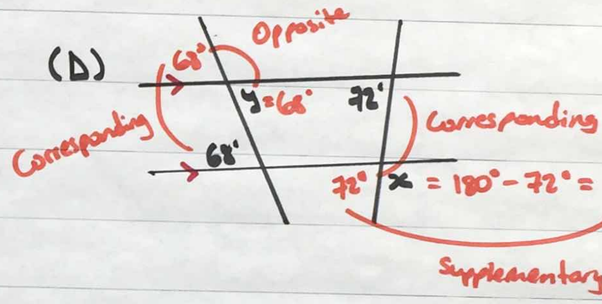
(B)



(C)

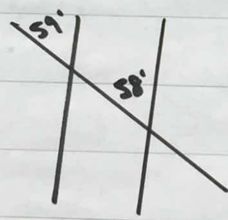


(D)



EXAMPLE #2: Find the parallel lines.

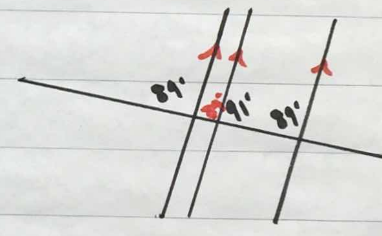
(A)



No parallel lines.

- If the lines were parallel, with a transversal, the angles for 58° and 59° would be congruent. Since they aren't, the lines aren't ||.

(B)



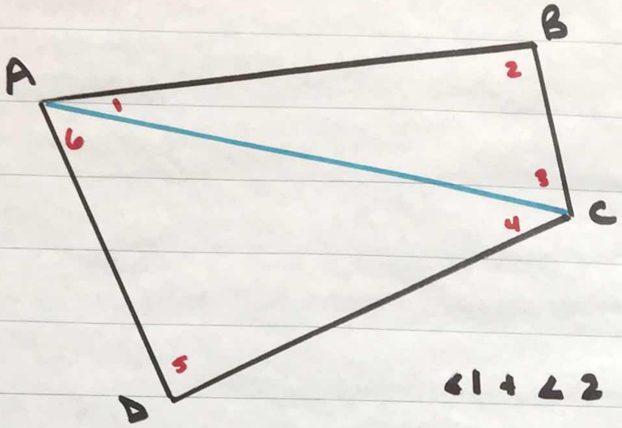
- The two 89° are congruent, so their lines are parallel.
- The supplementary angle to 91° is 89° (91° + 81° = 180°). This is corresponding ~~complementary~~ to the other 89° angles. So this line is parallel to the other two.

Support Questions:

p. 346, #3-14, 16, 17

Section 10.3: Support Questions, p. 347

(16)



Sum of the interior angles of

$\triangle ABC = 180^\circ$

$\triangle ACD = 180^\circ$

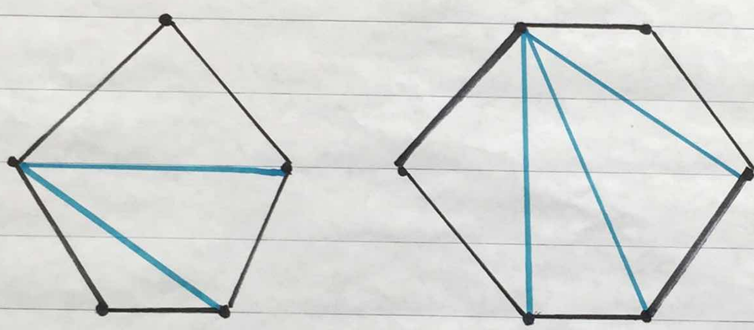
$\angle 1 + \angle 2 + \angle 3 = 180^\circ$

$\angle 4 + \angle 5 + \angle 6 = 180^\circ$

$\therefore \underbrace{\angle 1 + \angle 2}_{\angle A} + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 180^\circ + 180^\circ$

$= 360^\circ$

(17)



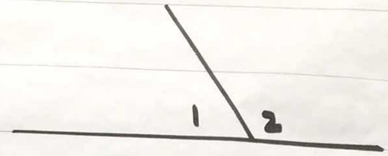
# of sides	# of triangles	# of degrees
5	3	180x 3 = 540°
6	4	180x4 = 720°

180° x 3 triangles: 540°   180° x 4 triangles: 720°

Sum of the interior angles of an n-sided polygon = $(n-2)180^\circ$

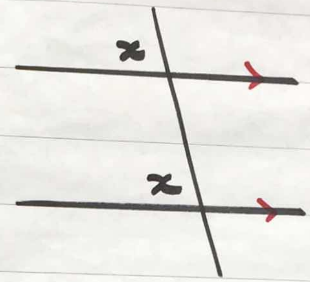
Solving Problems Using Angle Properties (G8-26)

Supplementary Angles (SA):

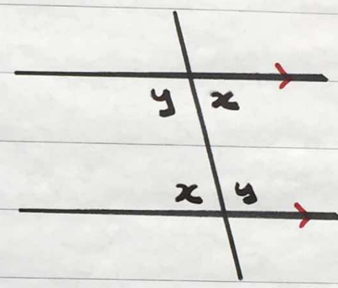


$\angle 1 + \angle 2 = 180^\circ$

Corresponding Angle Theorem (CAT):



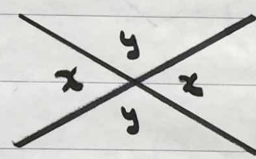
Alternate Angle Theorem (AAT):



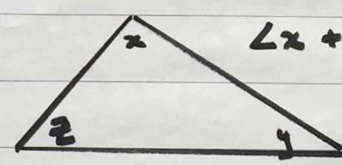
540°

420°

Opposite Angle Theorem (OAT):



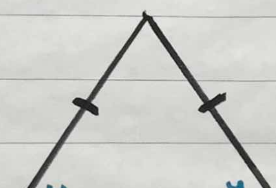
Sum of the Angles in a Triangle Theorem (SATT):



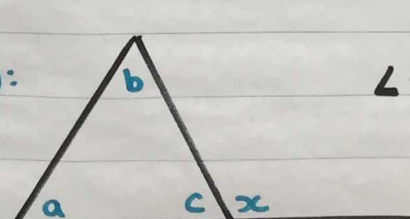
$\angle x + \angle y + \angle z = 180^\circ$

Isoceles Triangle Theorem (ITT):

The base angles in an isoceles triangle are equal.



Exterior Angle Theorem (EAT):



$\angle x = \angle a + \angle b$

$\angle x = \boxed{180^\circ - \angle c}$

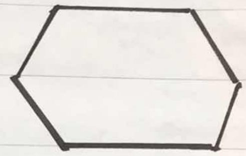
$\angle a + \angle b + \angle c = 180^\circ$

$\angle x = (\angle a + \angle b + \angle c) - \angle c$

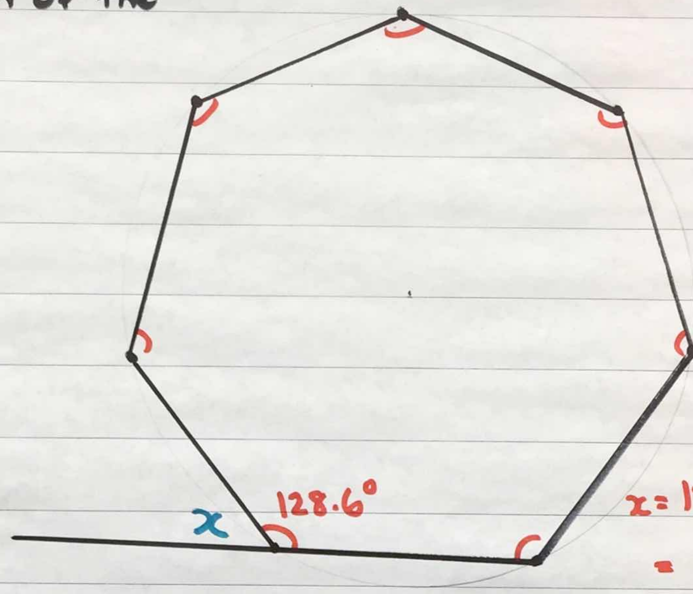
$\angle x = \angle a + \angle b$

Sum of the Angles in a Polygon Theorem (SAPT):

The sum of the interor angles of a polygon is a function of the number of sides



$(n-2)180°$



There are 7 equal angles.

$\frac{900°}{7} = 128.6°$

EXAMPLE #1:

$(n-2)180$

$=(7-2)180$

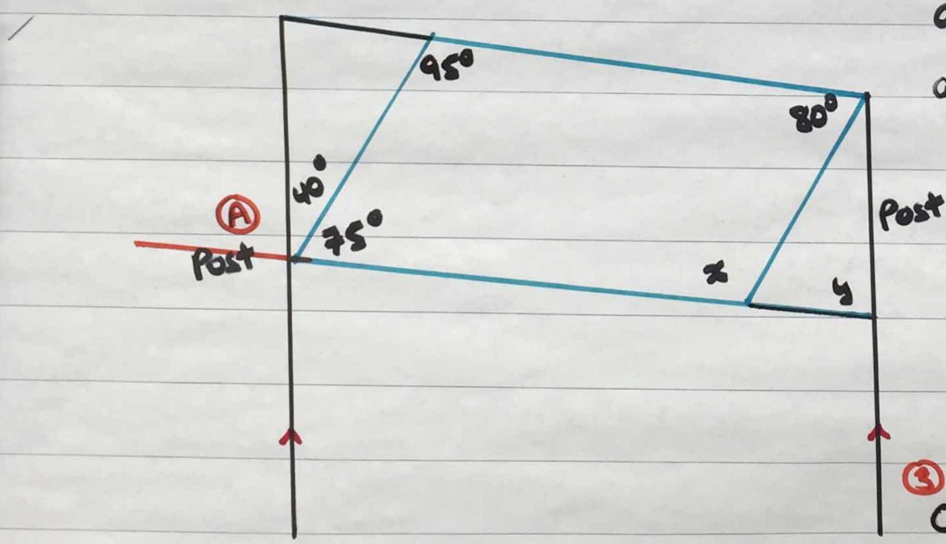
$=5 \cdot 180$

$=900°$ (SAPT)

$x = 180° - 128.6°$

$= 51.4°$ (SA)

EXAMPLE #2:



① The sign is a quadrilateral, and so the sum of the inside angles is 360°. (SAPT)

②

$75 + 95 + 80 + x = 360$

$\quad -250 \qquad\qquad -250$

$250 + x = 360$

$x = 110°$

③ On the bottom left, $40° + 75° = 115°$.

④ $\angle A = 180° - 115°$

$= 65°$ (SA)

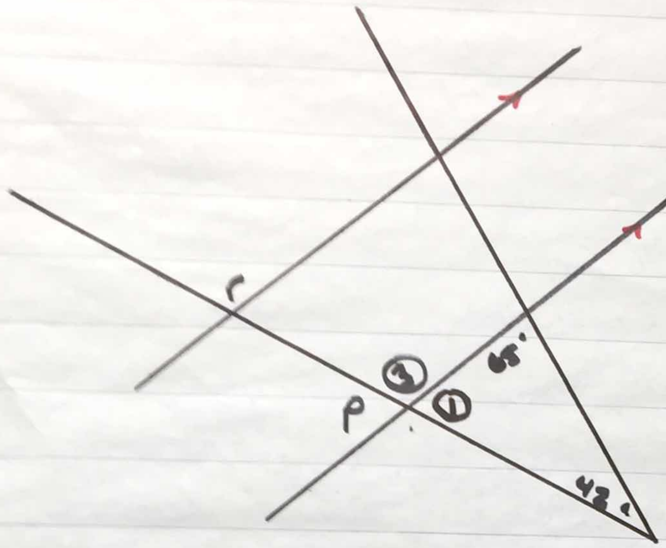
⑤ $\angle A = \angle y$ (CAT)

(The posts are parallel)

⑥ $\therefore \angle y = 65°$

EXAMPLE # 3: "Angle, Angle"

Determine the measure of angles p and r.



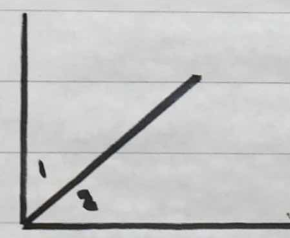
1. $65 + 42 + \angle 1 = 180$ (SATT)
   $107 + \angle 1 = 180$
   $\angle 1 = 73°$

2. $\angle p = \angle 1$ (OAT)
   $\angle p = 73°$

3. $\angle 3 = 180 - \angle 1$ (SA)
   $\angle 3 = 180 - 73$
   $\angle 3 = 107°$

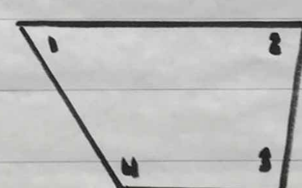
4. $\angle 3 = r$ (CAT)
   $107° = r$

Complementary Angles (CA):



$\angle 1 + \angle 2 = 90°$

Sum of the Angles in a Quadrilateral Theorem (SAQT)



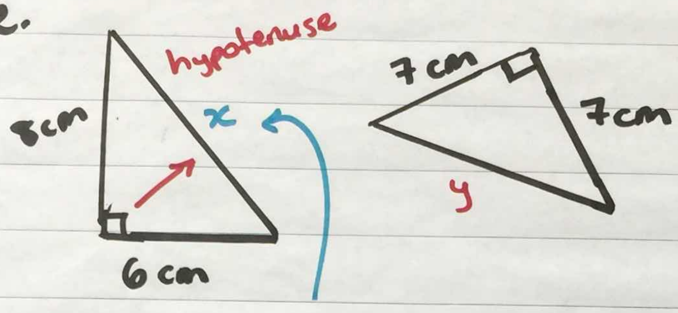
$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$

*This is really just a particular case of the "Sum of the Angles in a Polygon Theorem" (SAPT). But having this as its own theorem saves time, so here you go!

10.5 Exploring Right Triangles

Learning Goal: Relate the lengths of the sides of right triangles.

Right Triangle: A triangle that has one right angle.



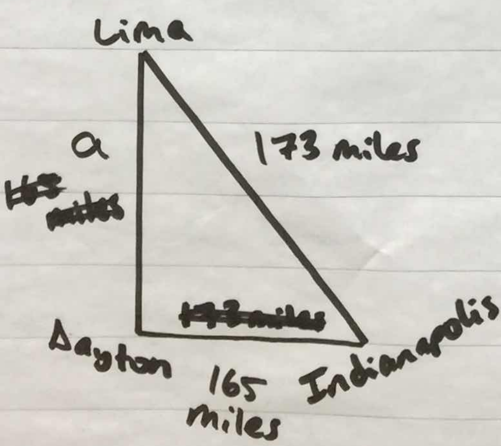
Pythagorean Theorem

$$a^2 + b^2 = c^2$$
$$8^2 + 6^2 = x^2$$
$$64 + 36 = x^2$$
$$\sqrt{100} = \sqrt{x^2}$$
$$10\text{ cm} = x$$

$$a^2 + b^2 = c^2$$
$$7^2 + 7^2 = y^2$$
$$49 + 49 = y^2$$
$$98 = y^2$$
$$\sqrt{98} = \sqrt{y^2}$$
$$9.9 \approx y$$

Meg lives here

EXAMPLE 1: Dayton is 165 miles ~~east~~ west of Indianapolis, Lima is due north of Dayton, Indianapolis is 173 miles away from Lima. How many more miles would Meg drive if she stopped in Dayton first, than if she drove straight to Lima?



Distance from Lima to Dayton

$$a^2 + b^2 = c^2$$
$$a^2 + 165^2 = 173^2$$
$$a^2 + 27225 \underset{-27225}{=} 29929 - 27225$$
$$\sqrt{a^2} = \sqrt{2704}$$
$$a \approx 52\text{ miles}$$

I → D → L

165 + 52

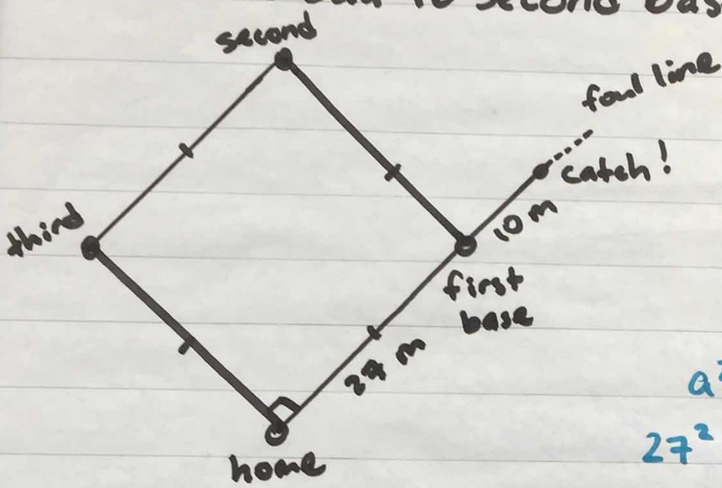
= 217 miles

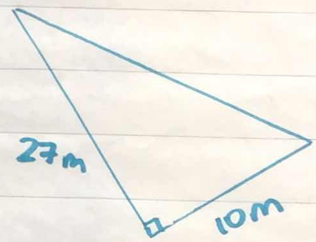
I → L

173 miles

∴ She would drive 44 more miles.

EXAMPLE 2: In regulation baseball, the distance between adjacent bases is 90 feet. This is about 27 m. Suppose that the right fielder catches the ball while standing on the foul line, 10 m from first base. Then she throws the ball to second base. How far does she throw?





$a^2 + b^2 = c^2$

$27^2 + 10^2 = c^2$

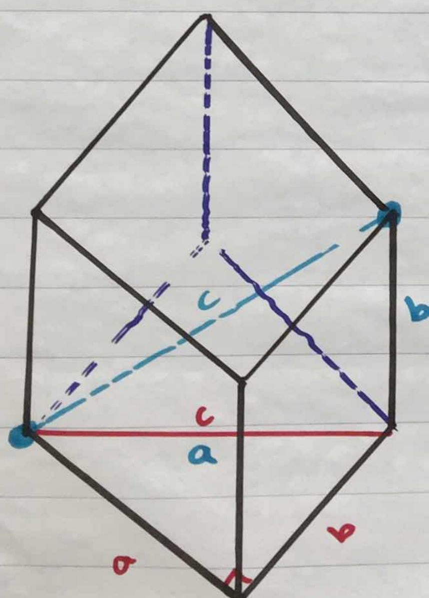
$729 + 100 = c^2$

$\sqrt{829} = \sqrt{c^2}$

$28.8 \text{ m} \approx c$

∴ She has to throw the ball 28.8 m.

Example 3: A cube has 10 cm sides. What is the distance between any two opposite corners?



Step 1: Length of base

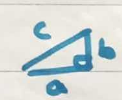
$a^2 + b^2 = c^2$

$10^2 + 10^2 = c^2$

$100 + 100 = c^2$

$\sqrt{200} = \sqrt{c^2}$

$14.14 = c$



Step 2: Length of diagonal

$a^2 + b^2 = c^2$

$14.14^2 + 10^2 = c^2$

$199.94 + 100 = c^2$

$\sqrt{299.94} = \sqrt{c^2}$

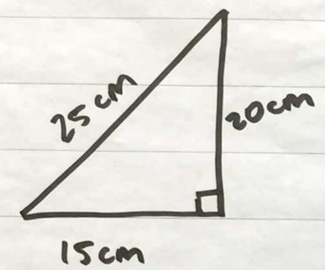
$17.32 = c$

∴ The length of the diagonal is 17.32 cm.

Support Questions: p. 356, All

Implications of the Pythagorean Theorem

Example 1: A triangle has side lengths 15 cm, 20 cm and 25 cm. Is it a right-angle triangle?



$a^2 + b^2 = c^2$

$15^2 + 20^2 = 25^2$

$225 + 400 = 625$

$625 = 625$ True

$\therefore$ It's a right triangle

Example 2: The hypotenuse of a right scalene Δ is 7 cm. What must be the length of one of the legs?

(a) 8 cm　　(c) 5.5 cm

(b) 4.95 cm　　(d) 7 cm

The hypotenuse is the longest side, so eliminate (a).
The Δ is scalene, so eliminate (d).

Try (b): $a^2 + b^2 = c^2$

$4.95^2 + b^2 = 7^2$

$24.5 + b^2 = 49$

$b^2 = 24.5$

$b = 4.95$

This doesn't work because it's a scalene Δ.

$\therefore$ (c) Must be the answer.

Check $a^2 + b^2 = c^2$

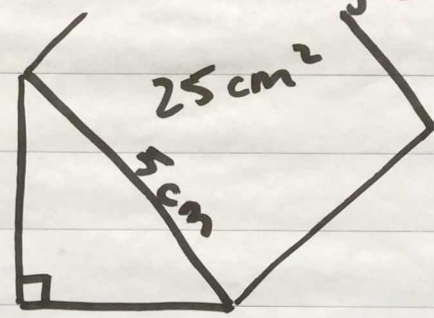
$5.5^2 + b^2 = 7^2$

$30.25 + b^2 = 49$

$b^2 = 18.75$

$b = 4.3$ cm

Example 3: A square is drawn on the hypotenuse of a right-angle triangle. Its area is 25 cm². What are possible lengths of the other sides of the triangle?



Pythagorean Theorem

$a^2 + b^2 = c^2$

a	b	c
1	4.9	5
2	4.6	5
3	4	5
4	3	5
5	0	5 } Not a triangle

When a = 1

$a^2 + b^2 = c^2$

$1^2 + b^2 = 5^2$

$1 + b^2 = 25$

$b^2 = 24$

$b \approx 4.9$ cm

When a = 2 cm

$b \approx 4.6$ cm

When a = 3 cm

$b \approx 4$ cm

When a = 4 cm

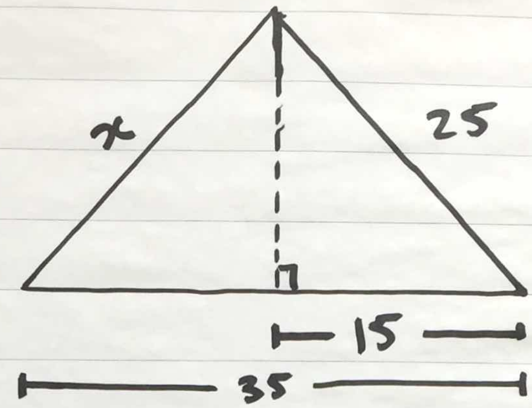
$b \approx 3$ cm

Support Questions:

- p. 360-61, #4, 6
- p. 363, #1-10
- p. 365, #1-8

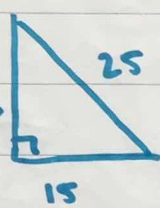
Pythagorean Thm. & Algebra

Example # 1: Find the value of x. (Not to scale.)



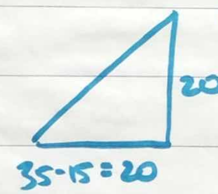
Round off to the nearest tenth of a metre.

<u>Find height of triangle</u>



$$a^2 + b^2 = c^2$$
$$15^2 + b^2 = 25^2$$
$$225 + b^2 = 625$$
$$-225 \qquad\qquad -225$$
$$\sqrt{b^2} = \sqrt{400}$$
$$b = 20$$

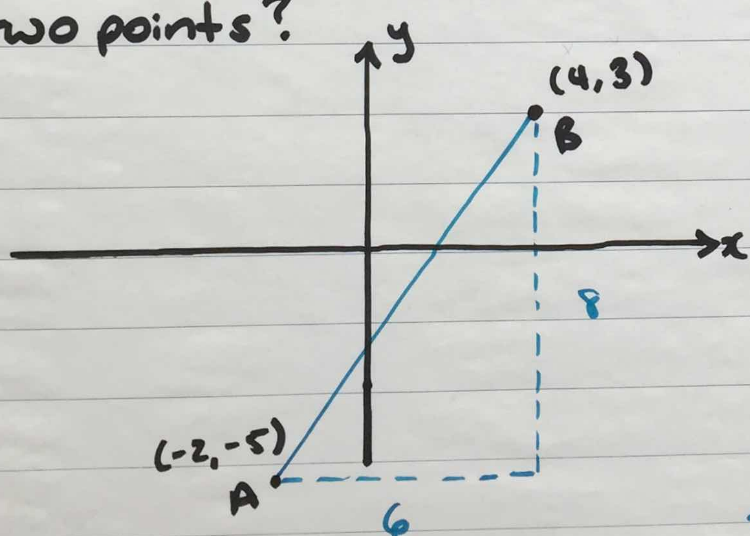
<u>Finding x</u>



$$a^2 + b^2 = c^2$$
$$20^2 + 20^2 = c^2$$
$$400 + 400 = c^2$$
$$\sqrt{800} = \sqrt{c^2}$$
$$28.3 = c$$

$\therefore x$ is 28.3 m.

Example # 2: What is the distance between these two points?



<u>Finding the distance</u>

$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = c^2$$
$$36 + 64 = c^2$$
$$\sqrt{100} = \sqrt{c^2}$$
$$10 = c$$

$\therefore$ The distance between the two points is 10 units.

Chapter 10 Cahier Success Criteria

- [ ] All chapter 10 notes are copied
- [ ] Most questions in each section are finished
- [x] – p. 342 (4), ~~p. 343 (4)~~, p. 356 (7), p. 360-61 (1),
- [x] p. 363 (6), p. 365 ~~[illegible]~~ (5)
- [ ] Finished the 10 Pythagorean Thm. worksheet questio[ns]
- [ ] Everything is neat and organized.